# CONCEPTUALIZING THE TEACHING PRACTICE OF BUILDING ON STUDENT MATHEMATICAL THINKING

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*An important aspect of effective teaching is taking advantage of in-the-moment expressions of student thinking that, by becoming the object of class discussion, can help students better understand important mathematical ideas. We call these high-potential instances of student thinking MOSTs and the productive use of them building. The purpose of this paper is to conceptualize the teaching practice of building on MOSTs as a first step toward developing a common language for and an understanding of productive use of high-potential instances of student thinking. We situate this work in the existing literature, introduce core principles that underlie our conception of building, and present a prototype of the teaching practice of building on MOSTs that includes four sub-practices. We conclude by discussing the need for future research and our research agenda for studying the building prototype.*

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Mathematics education researchers recognize the important role student mathematical thinking plays in crafting and carrying out quality mathematics instruction (e.g., Fennema et al., 1996; Stein & Lane, 1996). The field has begun to understand how to effectively use written records of student work to support mathematics learning (e.g., Smith & Stein, 2011), but much less is known about how to effectively use in-the-moment student thinking that emerges during whole-class discourse, often fertile ground for valuable student mathematical thinking (Van Zoest et al., 2015a, 2015b). In fact, research has documented that many teachers fail to notice or act on opportunities to capitalize on such thinking to further students' mathematical understanding (Peterson & Leatham, 2009; Stockero, Van Zoest, & Taylor, 2010).

Although research in mathematics teacher education suggests the benefits of instruction that uses student thinking (e.g., Franke & Kazemi, 2001), what it means to "use student thinking" is not well defined. For example, our interviews with secondary school mathematics teachers about productive use of student thinking revealed a range of perceptions. Some teachers viewed validation of student participation as productive use, others felt a discussion of student errors was not productive because it would confuse students, and still others saw productive use occurring when student thinking (correct or incorrect) was made the object of consideration for other students in the class (Leatham, Van Zoest, Stockero, & Peterson, 2014). These results highlight a need to develop a common understanding of and vocabulary for talking about productive use of student thinking in order for the field to better communicate about this important aspect of effective teaching. These interviews also revealed that some teachers felt differently about productive use of student thinking depending on the grade and ability level of their students; these teachers felt that it was possible to make student thinking the object of consideration for their advanced classes, but not for their beginning or remedial classes. Given what we know about the benefits of engaging students with each other's thinking, this perspective creates an inappropriate restriction on students' opportunities to engage in considering

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each other's thinking. A challenge to the field is to prompt teachers to question such artificial borders and to provide them with tools that support them to productively use the thinking of all their students.

Not all student thinking warrants the same consideration by the class, however, since it is not all about mathematical ideas, nor does it always provide leverage for accomplishing mathematical goals. Leatham, Peterson, Stockero, and Van Zoest (2015) described a framework to identify those instances of student thinking—MOSTs—that provide such leverage. To move work related to the teaching practice of using student thinking forward, it is critical that teachers and teacher educators develop an understanding of productive use of high-leverage instances of student mathematical thinking—what Leatham et al. (2015) called *building* on MOSTs. As a first step toward developing this understanding and providing a common language for the field, the purpose of this paper is to conceptualize *the teaching practice of building* on MOSTs.

## Theoretical Framework

Our theorizing about the teaching practice of *building* on student thinking takes as its foundation the framework for identifying student thinking worth building on developed by the MOST research group (Leatham et al., 2015; Stockero, Peterson, Leatham, & Van Zoest, 2014; Van Zoest, Leatham, Peterson, & Stockero, 2013). We defined MOSTs—**M**athematically Significant Pedagogical **O**pportunities to Build on **S**tudent **T**hinking—as occurring in the intersection of three critical characteristics of classroom instances: student mathematical thinking, significant mathematics, and pedagogical opportunities. For each characteristic, two criteria were provided to determine whether an instance of student thinking embodies that characteristic. For student mathematical thinking the criteria are: "(a) one can observe student action that provides sufficient evidence to make reasonable inferences about *student mathematics* and (b) one can articulate a mathematical idea that is closely related to the student mathematics of the instance—what we call a *mathematical point*" (p. 92). The criteria for significant mathematics are: "(a) the mathematical point is *appropriate* for the mathematical development level of the students and (b) the mathematical point is *central* to mathematical goals for their learning" (p. 96). Finally, "an instance embodies a pedagogical opportunity when it meets two key criteria: (a) the student thinking of the instance creates an *opening* to build on that thinking toward the mathematical point of the instance and (b) the *timing* is right to take advantage of the opening at the moment the thinking surfaces during the lesson" (p. 99). When an instance satisfies all six criteria, it embodies the three requisite characteristics and is a MOST.

MOSTs are instances of student thinking worth *building* on—that is, "student thinking worth making the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea" (Van Zoest et al., 2015b, p. 4). Such use encapsulates the core ideas of current thinking about effective teaching and learning of mathematics. Thus, building on MOSTs is a particularly productive way for teachers to engage students in meaningful mathematical learning. After discussing related literature, we share our current conceptualization of the teaching practice of building on MOSTs.

## Related Literature

We see our work as connecting and contributing to research both on professional noticing—attending to, interpreting, and deciding how to respond to student mathematical thinking (Jacobs, Lamb, & Philipp, 2010)—and on teaching practices that enact and coordinate these decisions. This section elaborates on these two areas of contribution.

We see studies of professional noticing as generally falling into two categories: (a) noticing *within* an instance of student thinking, and (b) noticing *among* instances (Stockero, Leatham, Van Zoest, & Peterson, in press). Noticing *within* studies include interventions in which teachers (or prospective teachers) are given a specific instance of student thinking that they are asked to analyze, using media such as one-on-one student interviews (e.g., Schack et al., 2013) or student written work (e.g., Fernández, Llinares, & Valls, 2013). In such studies, the task of the teacher is not to identify which instances to analyze, but instead, to notice what is happening *within* the instance of student thinking they are provided. Noticing *among* studies typically use classroom video (e.g., Sherin & van Es, 2009; van Es, 2011) as a medium for teachers to select the instances they deem important, and to write about or explain why those particular instances were important or interesting. Most of these studies, however, do not assign value to any particular instances, beyond the broad category of student thinking. We contend that, in order to productively use student mathematical thinking, teachers need to learn how to coordinate the skills of noticing *within* and noticing *among* instances of student mathematical thinking. The MOST Analytic Framework (Leatham et al., 2015) provides a tool for attuning teachers and researchers to this coordination and our current conceptualization of productive use of student thinking provides a theoretical characterization of what this coordination might look like in practice.

The theorizing we are doing about the teaching practice of building on MOSTs has the potential to contribute in substantial ways to understanding the third component of noticing, *deciding how to respond* (Jacobs et al., 2010). Most studies of noticing, particularly those in the noticing *within* category, ask teachers to propose a "next move." There is not yet a common understanding in the field, however, of what makes those next moves more or less productive, particularly during a whole-class discussion. In the case of one-on-one student interviews, asking a question or proposing a follow-up task to the student that is based on evidence of the student's understanding would generally be considered a productive move (see, e.g., Jacobs et al., 2010). In a whole-class setting, however, where the teacher needs to work toward engaging many students in making sense of the mathematics on the table, what makes a next move productive is more complicated. In fact, our work suggests that this practice requires a series of teacher moves. Unpacking and clearly articulating this collection of moves, of which our theorizing is a first step, is requisite for helping teachers to improve their use of student thinking in the classroom.

Prior work related to teacher responses to student mathematical thinking has also influenced our efforts to theorize the teaching practice of building on MOSTs. Research has identified a number of patterns in mathematics teachers' responses to student thinking. Mehan (1979) coined *IRE*—Initiation, Response, Evaluation—to describe a common pattern of classroom interaction where the teacher's main response to elicited student thinking is to evaluate it. An IRE interaction is an example of what Wood (1998) referred to as *funneling,* where the teacher's response is intended to corral students' thinking within predetermined and often narrowly-defined parameters. By contrast, Wood characterized certain other teacher responses as *focusing*, in which, for example, a teacher might ask "clarifying questions to keep attention focused on the discriminating aspects of the solution" (p. 175). Clarifying questions are just one example of the many types of productive questions teachers might ask in response to student mathematical thinking. Unfortunately, Boaler and Brodie (2004) found that questions geared toward "gathering information, leading students through a method" (p. 777)—funneling questions—characterized the vast majority of questions in US classrooms using a traditional mathematics curriculum. Although 60-75% of the questions asked by teachers using an NSF-funded reform curriculum were also of this type, the remaining 25-40% covered a broader range of more productive questions. Boaler and Brodie's (2004) findings highlight the additional learning opportunities afforded students who are exposed to a broader range of questions. These findings also point toward the value in teaching practices that engage with and capitalize on student mathematical thinking.

As we have argued elsewhere (Leatham et al., 2015), work on the cognitive demand of tasks (e.g., Stein, Smith, Henningsen, & Silver, 2009) and on orchestrating classroom discussions around rich mathematical tasks (e.g., Smith & Stein, 2011) has also been influential in conceptualizing the productive use of student mathematical thinking in classrooms. The MOST framework and our theorizing about the teaching practice of building on MOSTs is related to Smith and Stein's (2011) practices of monitoring, selecting and planning how to sequence student thinking that is observed as they work on a task, but extends Stein and Smith's work by focusing on *recognizing* and *responding* to potentially productive student thinking in the moment that it occurs. There are two particularly important differences between the MOST work and that of Stein, Smith, and colleagues. First, the MOST work focuses a broader range of contexts in which student thinking can emerge, including student questions that arise during a lecture or student comments that emerge during a discussion of homework. Although high cognitive demand tasks certainly create opportunities for student thinking to occur, we have also found high-potential instances of student mathematical thinking in classrooms that lack rich tasks. The MOST framework applies to the broad range of instructional situations in which student thinking might emerge in mathematics classrooms. Second, our work on building focuses on responding to student thinking *at the moment in which it occurs* during a lesson, rather than on monitoring and selecting student work and then purposefully sequencing the presentation of that work later in the lesson (see Smith & Stein, 2011). Although student thinking can be valuable to use at a later point in a lesson, our work has convinced us that there are certain instances of student thinking that lose their instructional value if they are not acted on immediately and in particular ways. The purpose of this paper is to conceptualize a productive response to MOSTs—the teaching practice of building.

## Our Current Conceptualization of The Teaching Practice of Building

We base our conception of *building* on core principles of quality mathematics instruction that we distilled from current research and calls for reform. The NCTM, for example, in their *Principles to Actions* document (2014), states that students "construct knowledge socially, through discourse, activity, and interaction related to meaningful problems" (p. 9), and that "effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (p. 10). We see embedded in these statements four core principles of quality mathematics instruction: mathematics is at the forefront, students are positioned as legitimate mathematical thinkers, students are engaged in sense-making, and students work collaboratively. We have applied these principles to the productive use of MOSTs in Figure 1. An important aspect of this application is that in the first principle, the mathematics that is at the forefront is the mathematics of the MOST—mathematics closely connected to the student thinking under consideration. We use these core principles to determine whether a given use of a MOST is productive.

Principles Underlying our Conception of Productive Use of MOSTs

1. The mathematics of the MOST is at the forefront.
2. Students are positioned as legitimate mathematical thinkers.
3. Students are engaged in sense making.
4. Students are working collaboratively.

**Figure 1.** Principles underlying our conception of productive use of MOSTs**.**

In our initial work identifying productive use of MOSTs, we focused on the productivity of a single teacher move that followed a MOST, but we quickly realized that this approach was insufficient. Because teaching is a complex system (Stiger & Hiebert, 1999), one needs to look beyond single actions, such as inviting students to share solutions at the board, to characterize effective teaching. Similarly, trying to ascertain productive use of MOSTs by only focusing on discrete teacher moves misses the real purpose of those moves. To evaluate productive use, one needs to consider the combining and coordinating of teacher moves. We thus conceptualize the teaching practice of *building* as several teacher moves woven together to engage students in the intellectual work of making connections between ideas and abstracting mathematical concepts from consideration of their peers' mathematical thinking.

To highlight the distinction between the teaching practice of building and the moves that may or may not be part of the practice, consider a teacher who invites two students to share their different, but both correct, solution strategies for a problem. The initial move of inviting the students to share their thinking could be the start of building because the teacher is inviting the whole class to consider the two students' mathematical thinking. Consider two different moves the teacher could make once the strategies have been shared: (a) the teacher says to the class, "See, there are many correct ways to solve problems like this and you can use whichever method makes the most sense to you," and moves on to the next problem; and (b) the teacher asks the class, "What similarities and differences do you notice in these two strategies?" and engages the students in a discussion about those noticings. Although the initial setup move was the same (making the two student solutions public), we see here that the follow-up moves vary significantly in their potential to accomplish the building goal of students coming to understand important mathematical ideas. Specifically, variations in follow-up moves might cause the resulting practice to deviate from any of the four principles underlying our conception of productive use of MOSTs listed in Figure 1: (1) the mathematics of the instance could be abandoned, (2) the teacher could trivialize students' contributions, (3) the overall practice (regardless of the actor) could focus on recall of facts or on procedural steps rather than on making sense of the underlying structure of the mathematics, or (4) the teacher could limit or eliminate engagement with the idea beyond the individual who contributed the instance. Thus, although a move that makes student thinking public *may* be part of a broader building practice, such a move does not imply engagement in the practice—building is more than a single move.

As we have begun to think about what collection of teacher actions meet the requirements of building, we have theorized that there are four sequential sub-practices of building, each consisting of a move or collection of moves, as well as some prerequisites. Before teachers can build, they must have completed two prerequisite actions: (1) invited or allowed students to share their mathematical thinking; that is, elicited student mathematical thinking, and (2) recognized that an instance of student thinking is a MOST—a high-potential instance of student thinking. In addition, the success of a teacher's enactment of the building practice is influenced by the norms present in the classroom. One such prerequisite norm is that students listen to and make sense of each other's thinking. The absence of that norm would greatly inhibit successful building.

Once these prerequisites are satisfied, we hypothesize that there are four sub-practices of the teaching practice of building; our current building prototype is outlined in Figure 2. The first sub-practice of building is to ensure that the student mathematics of the MOST—the object of consideration—is clear. We say the teacher should *make precise* what it is that students are meant to consider. Sometimes a MOST has been communicated in such a way that both the object and the need to engage with it are obvious and no further action is needed, but often the teacher must focus the class on what student thinking is to become the object of consideration. The second sub-practice turns the object of consideration—the student mathematical thinking—over to the students. We use the term *grapple toss* because it captures two key aspects of this sub-practice—the teacher must "toss" the student thinking of the MOST over to the students to be considered, and they must do so in such a way that the students are positioned to "grapple" with the object of consideration in order to make sense of it. The third sub-practice involves orchestrating the students' process of making sense of the MOST. We use *orchestrate* to mean, "arrange or direct the elements of (a situation) to produce a desired effect, especially surreptitiously" ("Orchestrate," n.d.). Although this orchestration could require only a few teacher moves, this sub-practice could easily consist of a large and complex collection of moves. The fourth sub-practice is to facilitate the extraction and articulation of the important mathematical idea from the discussion; that is, to *make explicit* that idea.

| **Sequence of Sub-Practices of the Teaching Practice of Building on MOSTs** |
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| 1. Make the object of consideration clear (make precise)<br>2. Turn the object of consideration over to the students with parameters that put them in a sense-making situation (grapple toss)<br>3. Orchestrate a whole-class discussion in which students collaboratively make sense of the object of consideration (orchestrate)<br>4. Facilitate the extraction and articulation of the mathematical point of the object of consideration (make explicit) |

**Figure 2.** Building prototype: Our current conception of the teaching practice of building**.**

Given that our understanding of this practice is primarily theoretical at this point, further research is needed to study the building prototype. Our plans for future research include studying the prototype by (1) analyzing current teacher responses to MOSTs to see the extent to which those responses coordinate our core principles; and (2) generating instantiations of the building prototype and engaging in a similar analysis of these responses. Specifically, this direction for future research will allow us to refine our building prototype by addressing three primary research questions: (1) What teaching practice(s) coordinate the core principles underlying productive use of MOSTs? (2) How do teachers' responses to MOSTs align with the core principles underlying productive use of MOSTs? and (3) In what ways, if any, do teachers' responses to MOSTs empower or disenfranchise students (particularly those from traditionally underrepresented populations) with respect to mathematics? Once the teaching practice of building on MOSTs is better understood, it will be possible to design professional development to support teachers in improving their abilities to build on MOSTs.

## Conclusion

Conceptualizing the teaching practice of building is a first step towards achieving the goal of productively using students' mathematical thinking during instruction—a central tenet of effective teaching (e.g., NCTM, 2014). There is little research about the complex but essential practice of responding to student thinking *in the moment*, yet this is something that teachers face every day. Our conceptualization of building contributes to a common understanding of and vocabulary for talking about productive use of student thinking that will support the field in communicating about this important aspect of effective teaching. Better understanding the in-the-moment practice of building on MOSTs—particularly opportune instances of student thinking—has the potential to significantly impact mathematics instruction for all students. Focusing attention on student thinking and how it can be built on supports teachers in looking for the mathematics present in instances of student thinking, and thus helps to avoid deficit thinking (e.g., Frade, Acioly-Régnier, & Jun, 2013) and making judgments based on student characteristics rather than the content of their thinking. Having a systematic way to interpret student mathematical thinking (i.e., the MOST Analytic Framework, Leatham et al., 2015) and a mechanism for responding to MOSTs (the teaching practice of building) positions teachers to question artificial borders that prevent them from engaging all their students—no matter what their ability or experience level—in these important learning opportunities.

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