The Nature of Student Thinking Available in a Secondary Mathematics Classroom

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Mathematical Opportunities in Student Thinking

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Background

- The value of using student thinking in instruction is widely accepted (e.g., NCTM, 2014; AMTE, 2017) but effectively using student thinking is challenging for teachers (e.g., Ball & Cohen, 1999).
- Better understanding variations of student thinking that are available during whole-class instruction may give more traction to efforts to support teachers in effectively using student thinking in their instruction.
- To help understand the variations in student thinking we utilize the MOST analytic framework (Leatham et al., 2015) for high leverage instances (see Figure 1 below) to examine all instances of student thinking that emerged during whole-class instruction in a secondary mathematics classroom.

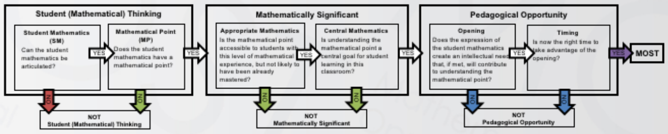
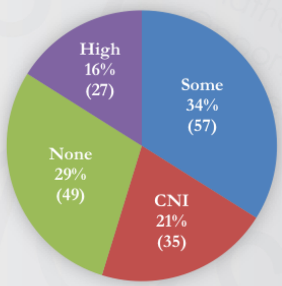


Figure 1: The MOST Analytic Framework (Leatham et al., 2015)

Methodology

- Data: Video of a 90-minute, junior-level, geometry lesson taught by a beginning teacher in an urban high school where 99% of the students received free lunch.
- The lesson was about finding the surface area and volume of a sphere.
- The rate of instances of student thinking per minute and MOSTs per minute of the 45-minutes of whole-class instruction in this lesson were at the medians of the larger data set (Van Zoest et al., 2016).
- 180 *instances of student thinking*—observable student action or small collections of connected action—were identified from the 45-minutes of whole-class instruction.
- The instances were categorized using the MOST Analytic Framework in Figure 1.
- Instances that were non-mathematical as well as instances of repetitive student thinking were exempted from further analysis leaving 168 instances that were analyzed.
- Instances were categorized by their *Building Potential* into four categories—*high, some, none and cannot infer (CNI).*
- ***Building Potential* refers to *the potential for learning to occur if the student thinking of the instance is made the object of consideration during whole-class discussion.***

Building Potential Category Frequencies



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Illustrating the *Building Potential* Categories

High Building Potential (MOSTs)

A student shows their work for finding the surface area of a sphere on the whiteboard (Fig. 2).

Instance of student thinking: Figure 2.

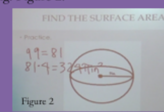

Figure 2

SM: I found the surface area of the sphere with a great circle of radius 9 in. by doing 9*9=81 then 81*4=324 π in².

MP: The equal sign is not a command for an answer, but represents the equality of the expressions on each side of the equal sign.

- *High potential* instances are MOSTs. Making these instances of student thinking an object of consideration for the class would capitalize on an opportunity to further the class's learning.
- In this example, the teacher could *toss* this thinking to the class by asking, "What do you think about the mathematics that has been presented?"

No Building Potential

The teacher asks the class where 9*9 in Fig. 2 came from and a student replies, "That came from the radius."

Instance of student thinking: ***"That came from the radius."***

SM: The 9*9 came from the radius.

MP: In finding the surface area of a sphere and using the formula $SA=4\pi r^2$, the r^2 means that you multiply the value of the radius by itself.

- Making an instance that has *no potential* the object of consideration by the class would be counterproductive.
- In this example, the most productive thing that the teacher could do would be to simply move on with the discussion.

Some Building Potential

The teacher asks the class what dimension they are looking at when talking about surface area and a student replies, "Two dimensional."

Instance of student thinking: ***"Two dimensional"***

SM: Surface area is two dimensional.

MP: Surface area is a 2-dimensional measure.

- Instances that have *some potential* could be used to further student thinking, but it would not be a missed opportunity were they not made the object of consideration for the class. These instances lack the leverage of high potential instances.
- In this example, the teacher could simply continue the conversation about appropriate units for the kind of problems the class is working on.
- Another subcategory of examples would be instances of student thinking that contain mathematical ideas that the class is not ready to engage with at that time. For such instances the teacher could acknowledge the student thinking, and indicate that it will be revisited later.

Cannot Infer the Building Potential (CNI)

On the heels of students claiming the area of the face of the cube covered by the hemisphere (Fig. 3) is 9 a student shouts out, "Subtract it."

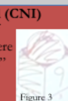

Figure 3

Instance of student thinking: ***"Subtract it"***

SM: Cannot Infer.

- Instances of student thinking that are *CNI* have unknown potential, and need to be clarified to determine the potential.
- In this example, the teacher would need to first ask the student to clarify what would be subtracted from what.

Implications

- It is important that teachers consider the potential that student thinking has when they attend to, and make decisions about how to respond to, that thinking.
- Teachers may need support to develop the ability to identify the potential in student thinking and respond to the thinking in ways that support students' learning of mathematical ideas and optimize limited instructional time.

References

Available on a handout.

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