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Teachers' Orientations around Using Student Mathematical Thinking as a Resource during Whole-class Discussion

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Rationale and Purpose

Mathematical Opportunities in Student Thinking

- Using student mathematical thinking during instruction
  - Valued and widely discussed (e.g., NCTM,1989; 2000; 2014)
  - Difficult to enact well (Scherrer & Stein, 2013; Peterson & Leatham, 2009).
- The field must improve its understanding of both how and why teachers use student thinking.
- The field has begun to understand how (e.g., Lineback, 2015), but less is known about why.

Purpose: To characterize teachers' orientations toward using student mathematical thinking as a resource during whole-class discussion.

Conceptualization of Orientations

- Teacher responses must always be viewed as sensible to the teacher enacting (or proposing) the response (Leatham, 2006).
  - Simon and Tzur (1999): "the reasonableness of all the teacher's observed actions" (p. 255).
  - Herbst and Chazan (2012): *practical rationality*
  - Schoenfeld (2011): *theory of goal-oriented decision making*
- We focus on the orientation and resource components of Schoenfeld's (2011) theory
- Teachers' orientations must be inferred (Leatham, 2006)

Literature

- Found no research explicitly reporting on thinking-as-a-resource orientations
- Evidence of such orientations fell under three themes:
  - Student mathematical capability
  - Student thinking informing instruction
  - The utility of student errors in instruction

Conceptualization of Student Thinking as a Resource: Three Frameworks

Mathematical Opportunities in Student Thinking

- Core **principles** of quality mathematics instruction
- High-leverage instances of student thinking **(MOSTs)**
- Teaching practice of **building** on student mathematical thinking

Core Principles Underlying Productive Use of Student Thinking

Mathematical Opportunities in Student Thinking

- **Mathematics Principle**: The mathematics of the MOST is at the forefront
- **Legitimacy Principle**: Students are positioned as legitimate mathematical thinkers
- **Sense-making Principle**: Students are engaged in sense making
- **Collaboration Principle**: Students are working collaboratively

(Drawn from NCTM's Principles to Action, 2014)

High-Leverage Student Thinking

- **M**athematical **O**pportunities in **S**tudent **T**hinking (MOSTs)
  - **Student Mathematical Thinking**
    - Student Mathematics
    - Mathematical Point
  - **Significant Mathematics**
    - Appropriate
    - Central
  - **Pedagogical Opportunity**
    - Opening
    - Timing
- In-the-moment student thinking worth *building* on
  - Worth making the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea

Building Sub-practices

Mathematical Opportunities in Student Thinking

The teaching practice of building involves a sequence of sub-practices that coordinate the core principles.

1. Make the object of consideration clear (**Make Precise**)
2. Turn the object of consideration over to the students with parameters that put them in a sense-making situation (**Grapple Toss**)
3. Orchestrate a whole-class discussion in which students collaboratively make sense of the object of consideration (**Orchestrate**)
4. Facilitate the extraction and articulation of the mathematical point of the object of consideration (**Make Explicit**)

Scenario Interview

Mathematical Opportunities in Student Thinking

- Interviewee situated as the teacher
- Presented with statements from eight individual students representing a range of student thinking
- Asked what they might do next if the statement were to occur during whole-class discussion in their classroom
- Contextual information not initially provided, but interviewee is provided an opportunity to ask questions about the context

Context	Instance	MOST Classification
Students were sharing their solutions to the following task (a corresponding picture was on the board). Given two concentric circles, radii 5cm and 3cm, what is the area of the band between the circles?	Chris shared his solution: "The radius of the big circle is 5 and the radius of the little circle is 3, so the gap is 2, so the area of the band is 4π cm^2."	MOST
	Before the teacher had a chance to respond to Chris, Pat says, "I also got 4π cm^2, but I did it a different way."	SM

Analysis: Inferring Orientations

Mathematical Opportunities in Student Thinking

Evidence for potential orientations from teacher B's responses
"I'm thinking I still have a lot of kids in the classroom who haven't had a chance to resolve the dilemma between these two and actually work through another problem....I want every student in the classroom to have a chance to look through it and go, "How am I going to process this?
"Because if Sam's going to present his method right now, then all the kids who are slower workers are going to go, 'Phff. I'm done. Why did I even try?" "
"But I still want the rest of the kids to have some think time before he gets to justify his reasoning."

Analysis: Inferring Orientations

Mathematical Opportunities in Student Thinking

Evidence of potential orientations from teacher X's responses	Statements of Potential Orientation
"I'm thinking I still have a lot of kids in the classroom who haven't had a chance to resolve the dilemma between these two and actually work through another problem....I want every student in the classroom to have a chance to look through it and go, "How am I going to process this?	The teacher wants to give students time to think through a problem before going over it as a class.
"Because if Sam's going to present his method right now, then all the kids who are slower workers are going to go, 'Phff. I'm done. Why did I even try?" "	The teacher thinks that if someone else gives the answer before other students have a chance to think about it, kids might not see a point to trying.
"But I still want the rest of the kids to have some think time before he gets to justify his reasoning."	The teacher wants every student to have a chance to think through a problem.

Analysis: Inferring Orientations

Mathematical Opportunities in Student Thinking

Evidence of potential orientations from teacher X's responses	Statements of Potential Orientation	Thinking-as-a-resource Orientation
"I'm thinking I still have a lot of kids in the classroom who haven't had a chance to resolve the dilemma between these two and actually work through another problem....I want every student in the classroom to have a chance to look through it and go, "How am I going to process this?	The teacher wants to give students time to think through a problem before going over it as a class.	It is important for students to have an opportunity to think about mathematical work.
"Because if Sam's going to present his method right now, then all the kids who are slower workers are going to go, 'Phff. I'm done. Why did I even try?" "	The teacher thinks that if someone else gives the answer before other students have a chance to think about it, kids might not see a point to trying.	
"But I still want the rest of the kids to have some think time before he gets to justify his reasoning."	The teacher wants every student to have a chance to think through a problem.	

A total of 173 thinking-as-a-resource orientations were inferred.

Analysis: Examining Alignment

Mathematical Opportunities in Student Thinking

Building-related Constructs
Core Principles
MOST Criteria
Building Sub-practices

Analysis: Examining Alignment

Mathematical Opportunities in Student Thinking

Building-related Constructs	Construct-related Codes
Core Principles	Mathematics
MOST Criteria	Student Mathematics (SM)
Building Sub-practices	Grapple Toss

Analysis: Examining Alignment

Mathematical Opportunities in Student Thinking

Building-related Constructs	Construct-related Codes	Key Questions: If this orientation were motivating a teacher's action in response to a MOST….	Potential Evidence of Alignment between Orientation and Construct
Core Principles	Mathematics	...would the student mathematics of the instance be likely to be the focus of the class discussion?	• Students' ideas are made public • Students are oriented to each other's thinking
MOST Criteria	Student Mathematics (SM)	...would the teacher be likely to accurately infer the SM?	• Students' ideas are clarified when necessary • The reasoning behind students' responses is sought
Building Sub-practices	Grapple Toss	...would the student mathematics of the instance likely be turned over to the class for consideration in a way that necessitates sense making of the idea?	• Students are invited to discuss one another's ideas • Students are oriented to each other's thinking

Results: Illustrating Variation

- Orientation alignment with the Core Principles
- Orientation potential to support the practice of building

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- **Orientation alignment with the Core Principles**
- Orientation potential to support the practice of building

Mathematics Principle

Mathematical Opportunities in Student Thinking

Aligned orientation: *It is valuable for students to see and hear other students' mathematical explanations.*

Misaligned orientation: *The teacher should respond to student thinking by explaining, showing, using examples, and demonstrating mathematical ideas to students.*

Sense-making Principle

Mathematical Opportunities in Student Thinking

Aligned orientation: *It is important for students to support, critique, listen to, and see other students' work and to explain their own thinking to others.*

Misaligned orientation: *If a student's answer is correct but the mathematics of their statement is incomplete, the student probably just got lucky.*

Collaboration Principle

Mathematical Opportunities in Student Thinking

Aligned orientation: *It is preferable to direct a question related to a student's response to the whole class.*

Misaligned orientation: *Students understand and retain better when they are taught one-on-one.*

Legitimacy Principle

Mathematical Opportunities in Student Thinking

Aligned orientation: *Students can identify mistakes and question the shared work of fellow students without the teacher intervening to ask questions.*

Misaligned orientation: *It is the teacher's responsibility to correct student mistakes and misconceptions as quickly as possible.*

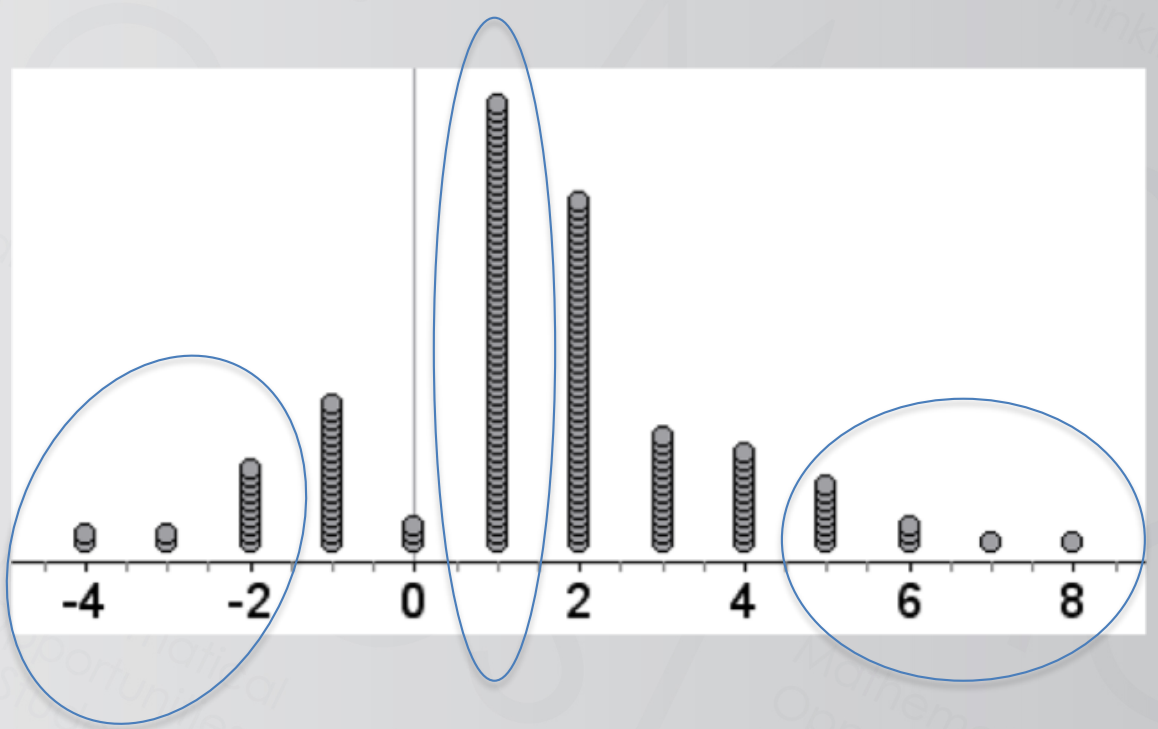
Results: Illustrating Variation

- Orientation alignment with the Core Principles
- **Orientation potential to support the practice of building**

Continuum of Potential to Support Building

Mathematical Opportunities in Student Thinking

- Ranged from 8 to -4 alignment tallies
  - High Potential
    - 5 or more tallies
    - 13 orientations
  - Low Potential
    - Only one tally
    - 55 orientations
  - Likely to Hinder
    - Negative tallies
    - 14 orientations



High-potential Orientations

Mathematical Opportunities in Student Thinking

Mathematics
The class commenting on the student thinking suggests that the student mathematics is at the forefront.

Legitimacy
The student with the initial idea is positioned as having a legitimate idea.

Sense making
Students commenting and asking questions of the student whose thinking has been shared suggests that students are making sense of one another's ideas.

Collaboration
Using student ideas to promote learning suggests collaborative engagement with the ideas.

It is important to find out what students are thinking or understanding by having the class comment on or ask questions of the student whose thinking has been shared.

Student Mathematics
Indicates a desire to understand what students are really trying to say, which might support the teacher in inferring the Student Mathematics (SM).

Make Precise
Students' questioning and commenting on the student thinking would create space to make that thinking precise.

Grapple Toss
Having the class comment and ask questions suggests turning over the student thinking to the class.

Orchestrate
The class commenting and asking questions suggests that there will be space for discussion.

Low-potential Orientations

Mathematical Opportunities in Student Thinking

It is important for students to justify their reasoning.

Legitimacy
Having students justify their reasoning positions students as legitimate mathematical thinkers.

It is important to keep track of who is or is not following/ understanding what is going on in class.

Appropriate
Monitoring students' understanding of what is gong on in the class would support determining the appropriateness of the mathematics.

All students need to be at the same place in their understanding of what is on the table.

Make Precise
Ensuring everyone is clear about the idea on the table would support a focus on making precise the student mathematics under consideration.

Hindering Orientations

Mathematical Opportunities in Student Thinking

Mathematics
The teacher would pursue their own mathematics more often than the mathematics that is intrinsic in the shared student mathematical thinking.

Legitimacy
The teacher is the one who is doing the thinking rather than students. Therefore students are not positioned as legitimate thinkers of mathematics.

Sense making
The teacher is placed in the role of making sense of the mathematics rather than the students.

The teacher should respond to student thinking by explaining, showing, using examples, and demonstrating mathematical ideas to students.

Grapple toss
The teacher is unlikely to turn over the student thinking to the class.

Conclusions

- Continuum of thinking-as-a-resource orientations
  - Broadened
  - Fleshed out
- Substantially different teacher readiness to develop the practice of building

Discussion Questions

Mathematical Opportunities in Student Thinking

- How might this continuum of thinking-as-a-resource orientations inform our practice as mathematics teacher educators?
- How might these results help us to work with teachers in their efforts to develop productive practices related to using student thinking?

Discussion Questions (cont.)

Mathematical Opportunities in Student Thinking

- Nature of orientation analysis
  - Practice
  - Instrument
  - Frameworks
  - Alignment and Potential
- Extending the orientation analysis
  - Other teaching practices
  - Other types of instruments
  - Other lenses (what's valued, principles)
  - Other sets of orientations
  - Other types of resources

Thank You!

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List of High-potential Orientations

Mathematical Opportunities in Student Thinking

1. It is important to find out what students are thinking/understanding by having the class comment on or ask questions of the student whose thinking has been shared.
2. It is important to monitor student work in order to direct class discussion and consider the flow and progression of student contributions.
3. Hearing the student thinking behind a response may help the class make sense of the mathematical idea underlying the response.
4. It is preferable to direct a question related to a student's response to the whole class.
5. Monitoring student work and sequencing are an important part of teaching a lesson.
6. Student thinking is valuable and it is preferable that student's ideas are tossed out for other students in the class to see and discuss, rather than the teacher being the one to evaluate those ideas.
7. It is important for students to support, critique, listen to, and see other students' work and to explain their own thinking to others.
8. Students learn by comparing their mathematical work, which in some cases could include an alternative solution or method provided by the teacher.
9. Students can identify mistakes and question the shared work of fellow students without the teacher intervening to ask questions.
10. Digging into a student's thinking or having a student clarify their thinking can surface more reasoning and is beneficial to the class, so the main purpose/power of using student thinking is to promote learning for all students and to get them engaged.
11. Student thinking should be made public so that students can discuss, correct, and improve their understanding of the student thinking.
12. It is valuable for students to see and hear other students' mathematical explanations.
13. A teacher's actions related to student thinking, including their reasoning about selecting, sequencing, pursuing or dismissing student thinking, depends on the teacher's goals and the task at hand.

List of Hindering Orientations

Mathematical Opportunities in Student Thinking

1. The teacher should respond to student thinking by explaining, showing, using examples, and demonstrating mathematical ideas to students.
2. Students who claim they used a different method than another student may not have understood the original method that had been shared, so their method may be the same.
3. Student thinking should come from highly-scaffolded instruction that minimizes student struggle.
4. If a student's answer is correct but the mathematics of their statement was incomplete, the student got lucky.
5. For problems where a diagram may support the process of finding a solution, it is always important for students to first have an accurate diagram before working on the problem.
6. Students are unlikely to make sense of a situation involving an idea that has not been addressed in class.
7. Students who provide numbers without reasoning are likely trying to guess and check and would rather do this and wait for the teacher's validation than reason through the problem.
8. Students stop making sense of a problem once they have the right answer.
9. If students believe that a particular student is smart, they assume the student is right and repeat that student's thinking rather than share their own.
10. Students should share their ideas one at a time and the teacher should resolve each idea before another idea is shared.
11. It is the teacher's responsibility to correct student mistakes and misconceptions as quickly as possible.
12. If student thinking reveals a lack of understanding, the teacher should help that student understand by asking them guiding questions or going back to previously taught ideas.
13. It is better to address a student's thinking one on one rather than in a whole class setting because students will understand and retain better.
14. It is difficult for a teacher to keep track of students' thinking all at once, particularly in large classes.