TEACHERS' INITIAL RESPONSES TO HIGH-LEVERAGE INSTANCES OF STUDENT MATHEMATICAL THINKING

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We investigate teachers' initial in-the-moment responses to instances of high-potential student mathematical thinking (SMT) during whole class discussion to understand what it means to productively incorporate SMT into instruction. Teachers' initial responses were coded using the Teacher Response Coding scheme, which disentangles the teacher action, who the response is directed to, and the degree to which the SMT is honored. We found that teachers incorporated students' actions and ideas in their response, but tended to address the SMT themselves and did not fully take advantage of the SMT. We consider the productivity of teachers' initial responses in relation to principles of productive use of SMT and compare the results to those of a previous study of teachers' hypothetical initial responses to SMT in an interview setting.

The incorporation of student mathematical thinking (SMT) into classroom instruction has been a consistent focus of recommendations for effective mathematics instruction (e.g., National Council of Teachers of Mathematics, 2000, 2014) and has been recognized as a central component of what has been referred to as *ambitious* (Lampert et al., 2013) or *responsive teaching* (Robertson, Atkins, Levin, & Richards, 2016). Research has begun to help us understand how to effectively incorporate students' thinking about a high-level task that they have worked on (e.g., Stein, Engle, Smith, & Hughes, 2008), but much less is known about how to effectively respond to and incorporate SMT that emerges during mathematics classroom discourse. Research has shown, however, that teacher responses matter; the ways in which teachers respond to SMT affects student learning in the classroom and can support very different types of instruction. For example, Kazemi and Stipek (2001) identified teacher responses that contributed to high-press and low-press interactions, with the high-press interactions supporting classroom instruction driven by SMT. Ing et al. (2015) studied teacher responses that encouraged students to engage with each other around SMT (e.g., asking students to explain each other's strategies and to discuss differences among strategies), finding such responses to be correlated with increased student participation and higher achievement. Correnti et al. (2015) identified the use of *uptake* moves—a move to "extend, deepen, clarify, or elaborate the discussion" (p. 308)—as a key difference in the responses of two teachers who enacted instruction aligned with traditional versus ambitious mathematics teaching, with this move accounting for four times as many responses in the classroom aligned with ambitious instruction. This suggest that the use of this move could be a key difference between such classrooms.

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One issue related to responding to SMT is that not all student thinking has the same potential to provide leverage for accomplishing mathematical goals, and thus, does not all warrant the same response. We are interested in understanding how teachers might most productively respond to particular instances of student thinking that have high potential to advance student learning, those we have identified as MOSTs—**M**athematically Significant Pedagogical **O**pportunities to Build on **S**tudent **T**hinking (Leatham, Peterson, Stockero, & Van Zoest, 2015). In prior work we have developed the MOST Analytic Framework, a tool to identify instances of student thinking that are MOSTs (Leatham et al., 2015), but we are only beginning to understand effective teacher responses to MOSTs. In Stockero, Van Zoest, Peterson, Leatham, & Rougée (2017) we investigated teachers' descriptions of how they would respond to a common set of MOSTs in a scenario-based interview. This study provided some important insights into initial teacher responses, including that such responses most often were directed to the student who had contributed the MOST, aimed to develop or justify the student contribution, and incorporated the students' words and ideas. However, we do not know the extent to which these hypothetical responses that teachers described in the scenario interview reflect teachers' actual responses during instruction. Thus, the study reported here addresses the question: What are the characteristics of teachers' initial in-the-moment responses to MOSTs that surfaced during their instruction? We use these characteristics to discuss the productivity of various teacher responses. We also compare teachers' responses during instruction to a set of hypothetical responses during a scenario interview. Better understanding teacher responses will contribute to our understanding of the current state of the practice of responding to MOSTs, as well as what it means to productively incorporate high potential student thinking into instruction.

THEORETICAL FRAMEWORK

MOSTs are instances of SMT that are particularly worth noticing and acting upon since they have high potential to help students better understand important mathematical ideas if made the object of consideration by the class. As we have described elsewhere in greater detail (Leatham et al., 2015), MOSTs simultaneously satisfy three critical characteristics of student contributions: student mathematical thinking, significant mathematics, and pedagogical opportunity. We see these instances as those worth *building* on—that is, "student thinking worth making the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea" (Van Zoest et al., 2017, p. 36). As elaborated elsewhere (Stockero et al., 2017), our conception of productive use of MOSTs is grounded in four core principles of quality mathematics instruction (Figure 1) that we distilled from current research and calls for reform (e.g., NCTM, 2014). Thus, to determine the productiveness of a teacher response to a MOST, we focus on the extent to which the response effectively coordinates these core principles.

Mathematics Principle: The student mathematics of the MOST is at the forefront. **Legitimacy Principle**: Students are positioned as legitimate mathematical thinkers. **Sense-making Principle:** Students are engaged in sense making. **Collaboration Principle**: Students are working collaboratively.

Figure 1: Principles underlying productive use of MOSTs (Stockero et al., 2017)

METHODOLOGY

This study is part of a larger project focused on understanding what it means for teachers to build on SMT during classroom instruction (see BuildingOnMOSTs.org). The larger MOST project analysed 11 videotaped mathematics lessons from 6-12th grade mathematics classrooms from different geographical regions of US that reflected diversity of teachers, students, mathematics topics, and curricula. None of the teachers had training specific to responding to MOSTs.

The data analysis of these lessons for this study focused on two different units of analysis: an instance of student thinking and the teacher's initial response to that instance. In prior work, we analysed each instance of student thinking that occurred during whole-class interaction to identify those that were MOSTs. In the current study, we analysed the initial teacher response, our second unit of analysis, to each MOST found in the data. We operationalized a *teacher response* as *the collection of observable teacher actions that begins as a given instance of SMT ends and concludes when the initial teacher turn ends or there is a clear shift to a different activity.*

In the 11 video-taped classroom lessons, we identified 251 MOSTs for which a teacher response was inferable. We applied the Teacher Response Coding Scheme (TRC) (Peterson et al., 2017) to each of these responses. Figure 2 provides the TRC coding categories, definitions and codes.

RESULTS

Our analysis revealed that some aspects of teachers' initial responses to MOSTs aligned well with the principles underlying productive use of MOSTs while others were less aligned with these principles. We begin by discussing the Student Recognition aspects of teacher responses, which generally showed good alignment with the principles. We then discuss the most common Actor and Action codes in the data and discuss how, in many cases, the alignment of these aspects of teacher responses to the principles could be enhanced with variations in the responses.

Student Recognition of Actions and Ideas

We capture the extent to which teachers honoured student thinking in their initial response to a MOST using the categories Recognition-Student Action and Recognition-Student Idea. Recognition-Student Action (as described in Figure 2) is the

Category	Coding Category Description	Codes
Actor	Who is publicly given the opportunity to consider the instance of student mathematical thinking	teacher, same student(s), other student(s), whole class
Student Recognition	Recognition-Student Action: The degree to which the teacher response uses the student action, either verbal (words) or non-verbal (gestures or work)	explicit, implicit, not
	Recognition-Student Idea: The extent to which the student is likely to recognize their idea in the teacher response	core, peripheral, other, not applicable
Action	What the actor is doing or being asked to do with respect to the instance of student thinking	adjourn, allow, check-in, clarify, collect, connect, correct, develop, dismiss, evaluate, justify, literal, repeat, validate

Figure 2: Teacher Response Coding Scheme (TRC).

extent to which the teacher response uses the student action, either verbal (words) or non-verbal (gestures or written work). About 70% of the teacher responses to MOSTs used either the student's specific actions (coded as explicit; see example below) or referred to the student's actions using pronouns or other referents (coded as implicit; see Table 1). The remaining 30% of the teacher responses did not explicitly or implicitly use the student actions (coded as Not).

		Student Ideas				
		Core	Peripheral	NA	Other	Total
	Explicit	95	15	3	2	115 (46%)
Student	Implicit	54	5	5	1	65 (26%)
Actions	Not	23	14	31	3	71 (28%)
	Total	172 (69%)	34 (14%)	39 (16%)	6 (2%)	251

Table 1: Recognition of student actions and ideas.

In the Recognition-Student Idea category, about two thirds of the teachers' initial responses were coded as core to the student mathematics of the MOST (see example below). This means that the teacher response focused on a main idea of the MOST in a way that the student who contributed the instance would likely recognize the idea as their own. Only 2% of the teacher responses for Recognition-Student Ideas focused on an idea that did not seem to be related in any way to a main idea of the MOST.

It is worth noting that about 82% (95 out of 115) of teacher responses to MOSTs that were coded as *Explicit* for Recognition-Student Action were also coded *Core* for Recognition-Student Idea. Similarly, 83% (54 of 65) of the responses that were coded as *Implicit* for Recognition-Student Action were also coded *Core* for Recognition-Student Idea. As an example of a response that is *Explicit* and *Core*, consider an instance where students were finding the lengths of the sides of similar triangles. A student said, "Oh wait, you can have any number for side A. You just have to use the corresponding… you just have to use the rules to get sides B and C." In response, the teacher asked the student who contributed this idea, "So what are the rules to get sides B and C if you start with side A?" This teacher response used the student's words and focused on a main idea of the MOST in a way that the student would likely recognize the idea as their own. Our results indicate that teachers are honouring students' thinking to a great extent in their responses by using the student actions and focusing on the students' ideas. This is important because such responses align with the first principle of productive use of MOSTs (see Figure 1), the Mathematics Principle, since the mathematics of the MOST is at the forefront.

Actor and Action

Six moves occurred most frequently in the data (clarify, develop, dismiss, evaluate, literal and repeat), collectively accounting for two-thirds of all teacher initial responses (Table 2). Three of these actions—dismiss, literal and repeat—clearly do not position students to make sense of the SMT because they either let the instance pass by (dismiss), or require only a minimal response (literal, repeat). Yet, they made up about 38% of the data. For example, in a calculus class a student said that they had used the quotient rule on $\int \frac{e^x}{1+e^x}\,dx$ and got $\frac{e^x}{1+e^x}$. The teacher responded to this incorrect answer by directing the whole class, "OK, the quotient rule. Tell her when you use the quotient rule." Rather than directly asking the other students to engage with the student's mathematics, the teacher instead asked them to give a literal response—factual information about the use of the quotient rule. Additionally, of the 95 response that fell into these three categories of actions, 56 (59%) also had a teacher actor, meaning the teacher did not use the action to engage students in the instance at all. Given that the instances of SMT in our data were all MOSTs, the abundance of such responses results in a lot of lost opportunities for students to make sense of important mathematics, and thus do not support the Sense-making Principle.

The other three predominant actions in the data have more potential to support building. Clarify actions (11% of the data) seek to make the instance of SMT more precise. Develop actions (10%) provide or ask for an expansion of the instance that goes beyond a simple clarification but does not require a justification. Evaluate actions (9.3%) provide or ask for a determination of the correctness of the SMT. These actions have potential to position the class to make sense of the SMT (and thus support the Sense-making Principle), either by making clearer the student thinking they are going to consider or by asking them to engage with the thinking in some way. As with the

	Actor			
Move	Same Student	Whole Class	Teacher	Total in Data Set
Literal	8	20	17	46 (18%)
Clarify	10	1	16	28 (11%)
Repeat	8	2	16	26 (10%)
Develop	11	5	9	25 (10%)
Dismiss	0	0	23	23 (9%)
Evaluate	3	14	6	23 (9%)
Total in Data Set	56 (23%)	69 (27%)	116 (46%)	251

Table 2: Summary of subset of actor and moves codes discussed in the paper.

actions discussed previously, however, clarify and develop most often had a same student or teacher actor, meaning that the engagement that would result from the action would likely be quite limited. Evaluate moves were the only one of these potentially-productive moves that most often engaged the whole class.

To illustrate why it is more productive for teachers to direct their initial responses to MOSTs to the whole class, consider the previously-discussed instance related to finding the lengths of the sides of similar triangles. The teacher response, “So what are the rules to get sides B and C if you start with side A?" was directed to the student who contributed the idea. In directing the response in this way, it engaged the contributing student in developing their idea, but it limited the opportunity for other students in the class to engage with the important mathematics of similarity, and thus did not support the Collaboration Principle. This broader engagement could have easily been achieved had the teacher instead directed the same question to the whole class.

DISCUSSION AND CONCLUSION

The high level of honouring student thinking that we observed in the use of the student actions and the incorporation of student ideas aligns with the principles underlying productive use of MOSTs (Figure 1) by putting the student mathematics of the MOST at the forefront, and by positioning students as legitimate mathematical thinkers. Also, with nearly half of the initial responses to MOSTs engaging either the same student or the whole class, the data indicates that students are being given the opportunity to engage with each other’s thinking—a component of the Collaboration Principle. Since the instances to which the teachers are responding to are MOSTs, however, they are instances that the whole class could engage with, making the move of going back to the same student a missed opportunity for students to work collaboratively and thus for the response to fully align with the Collaboration Principle.

A comparison of the results of the actor coding in this study to the hypothetical responses to the scenario interview discussed in our previous work (Stockero, et al., 2017) provides additional insight into the extent to which teachers' initial responses align with the principles underlying productive use of MOSTs. In both settings, teacher responses were directed to the whole class about one fourth of the time. The teachers' hypothetical responses had a teacher actor 6% and a same student actor 65% of the time, but in the actual classrooms reported in this study, the teacher was the actor 46% and the same student was the actor 23% of the time. In both settings, we only examined teacher responses to MOSTs—instances which could be productively turned over to the whole class. Thus, regularly having the same student as the primary actor in the hypothetical study or the teacher as the primary actor in this classroom study leaves the whole class actor as a distant second choice in both situations, implying many missed opportunities for the class to engage in collaborative sense making (Collaboration Principle). Additionally, having the teacher as the actor in nearly half of the instances in this study misaligns with the Legitimacy Principle, since responses with this actor do not position students as legitimate mathematical thinkers.

Also, in our hypothetical work, over half of the initial teacher responses asked students to develop or justify their thinking–two actions that are of importance when attempting to engage students in sense making. Unfortunately, in this study, only about a quarter of responses included these two types of actions. The three most common actions in the classroom study, which account for just over half of teacher responses, were literal, dismiss, and evaluate–moves that do not align with the Sense-making Principle.

While the results of this study have some positive aspects in the ways SMT is used, we can see that, as a whole, the initial teacher responses in our data are not as well aligned with ambitious or responsive teaching as the responses that teachers gave to interview scenarios in our prior work. We acknowledge that this weak alignment may have resulted from our methodological choice of only examining the initial teacher response and that subsequent teacher turns may align better—an area for future research. This weak initial alignment, however, suggests that the ways that teachers want to or think they should respond to SMT are difficult to enact in the moment that decisions about responding are made, highlighting the complexity of enacting ambitious teaching. An area for future research could be to examine possible reasons for this disconnect.

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