CONDUCTING A WHOLE CLASS DISCUSSION ABOUT AN INSTANCE OF STUDENT MATHEMATICAL THINKING

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Productive use of student mathematical thinking is a critical aspect of effective teaching that is not yet fully understood. We have previously conceptualized the teaching practice of building on student mathematical thinking and the four elements that comprise it. In this paper we begin to unpack this complex practice by looking closely at its third element, Conduct. Based on an analysis of secondary mathematics teachers' enactments of building, we describe the critical aspects of conducting a whole-class discussion that is focused on making sense of a high-leverage student contribution.

Keywords: Classroom Discourse, Communication, Instructional Activities and Practices.

The mathematics education community has advocated using students' mathematical thinking as a critical component of whole-class instruction (e.g., Association of Mathematics Teacher Educators, 2017; National Council of Teachers of Mathematics, 2014). Elsewhere, we have argued that some instances of in-the-moment student thinking are particularly high-leverage and, if taken advantage of productively, can be used to engage students in a whole-class discussion focused on making sense of the important mathematical ideas that underlie the student thinking (Leatham et al., 2015). Taking advantage of such instances, however, requires that a teacher coordinate a collection of teaching actions into a coherent practice. Furthermore, certain aspects of this practice do not occur naturally in whole-class instruction (Stockero et al., 2020). Thus, taking advantage of such instances is a complex practice.

Grossman and colleagues (2009) have described the decomposition of teaching practices into their "constituent parts" (p. 2069) for the purpose of better understanding and improving teachers' ability to engage in complex practices. We have previously conceptualized the teaching practice of *building on student mathematical thinking* and decomposed it into four elements that comprise the practice: "(1) *Establish* the student mathematics of the MOST so that the object to be discussed is clear; (2) *Grapple Toss* that object in a way that positions the class to make sense of it; (3) *Conduct* a whole-class discussion that supports the students in making sense of the student mathematics of the MOST; and (4) *Make Explicit* the important mathematical idea from the discussion" (Leatham et al., 2021, p. 1393). In this paper we further decompose the practice of building on student thinking by looking closely at its third element, Conduct. Specifically, we examine this research question: What are critical aspects of the Conduct element of building as revealed through teachers' attempts to enact the practice?

Literature Review

Conducting a whole-class discussion that engages students in making sense of an instance of student mathematical thinking is a complex teaching practice that requires deliberate actions by the teacher. Research on teacher moves during whole-class discussion sheds some light on a range of teacher actions that can support student sense making. Some actions prompt students to make sense of an individual student contribution by, for example, pressing for justification (e.g. Drageset, 2014; Ellis et al., 2019), asking probing questions that encourage other students to engage with specific details of the contribution (e.g., Webb et al., 2019), requesting that students evaluate the correctness of an idea (e.g., Drageset, 2014; Bishop et al., 2016), or asking students to reflect on an idea to advance their mathematical understanding (e.g., Ellis et al., 2019). Other teacher actions support students in making sense of how ideas are related, such as positioning one student contribution relative to another (Webb et al., 2019) and requesting that students make connections among two or more contributions (Lineback, 2015; Bishop et al., 2016). Still other actions keep students focused on the contribution that they are making sense of, such as putting aside contributions that are unrelated to the idea at hand (Dragset, 2014), and redirecting students' attention (Lineback, 2015) to the idea that is the focus of the sense-making discussion.

Although all of these individual moves have potential, in general, to support student sense making, we have come to understand that conducting a whole-class discussion focused on making sense of a particular student contribution requires a coordinated collection of teacher moves. Smith and Stein (2018) articulated one such coordinated collection of moves for orchestrating a whole-class discussion around a high-cognitive demand task. Their 5 Practices have been widely used to support teachers in facilitating mathematical discussions when the teacher has an opportunity to monitor students' work, and then intentionally select and sequence the solutions they wish to incorporate in the follow-up discussion. When a high-leverage instance of student thinking emerges in the moment during a whole-class discussion, however, a different collection of actions is needed to conduct a whole-class discussion around that student contribution. We define this collection of actions as the teaching practice of building on student mathematical thinking (Leatham et al., 2021; Van Zoest et al., 2016).

Theoretical Framework

When we discuss the teaching practice of building, we are focused on building on a high-leverage instance of student mathematical thinking that emerges during whole-class instruction. More specifically, we focus on building on MOSTs (**M**athematically **O**pportunities in **S**tudent **T**hinking), which are described in Leatham et al. (2015). MOSTs are instances of student mathematical thinking that are related to significant mathematics and that create an opportunity in that moment they are shared to engage the class in joint sense making of the mathematics embedded in the instance.

When we say *building on a MOST* we mean the teaching practice that takes advantage of the opportunity that a MOST provides (Van Zoest et al., 2016). We define *building on a MOST* (hereafter referred to as *building*) as making a MOST "the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea" (Van Zoest et al., 2017, p. 36). As mentioned earlier, *building* is comprised of four elements: (1) *Establish*; (2) *Grapple Toss*; (3) *Conduct*; and (4) *Make Explicit* (Leatham et al., 2021). This paper focuses on the third element, Conduct, but we frame our discussion of this element by first sharing some detail about the first two elements. Two important aspects of Establish are establishing precision by taking steps to assure that the MOST is "clear, complete, and concise" enough for the students to engage in making sense of it (Leatham et al., 2021, p. 1395) and then establishing the precise MOST as an object to be discussed. Once the MOST is established, it is then grapple tossed—offered to the class in a way that positions them to make sense of it. The key aspects of a Grapple Toss are an explicitly established object and an action that students are to apply to that object—actions such as justify or make sense of.

Lischka, A. E., Dyer, E. B., Jones, R. S., Lovett, J. N., Strayer, J., & Drown, S. (2022). Proceedings of the forty-fourth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Middle Tennessee State University.

Methods

Twelve practicing secondary school mathematics teachers worked with us to study the theorized practice of building. These teacher researchers used four mini tasks (see Figure 1) in their classrooms that were designed to elicit particular MOSTs; this allowed them to enact the practice of building in a context where there would be a predictable MOST. We compared the 27 resulting videotaped enactments to our theorized conceptualization of building by coding the enactments for teacher actions that seemed to either facilitate or hinder the overall practice of building. Our analysis included identifying critical aspects of each element, as well as subtleties of each. In this paper we report what we learned about the Conduct element of building, specifically the aspects of this element and actions teachers might take to increase the likelihood that students will productively engage in a collaborative discussion focused on making sense of a peer's contribution.

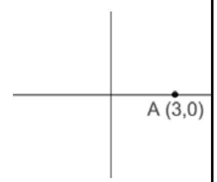
(a) Percent Discount The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?	**(c) Points on a Line** Is it possible to select a point B on the y-axis so that the line $x + y = 6$ goes through both points A and B? Explain why or why not. 
(b) Variables Which is larger, x or $x + x$? Explain your reasoning.	**(d) Bike Ride** On Blake's morning bike ride, he averaged 3 miles per hour (mph) riding a trail up a hill and 15 mph returning back down that same trail. What was his average speed for his whole ride?

Figure 1: The Four Mini Tasks Used to Create Instantiations of Building

Context of the Conduct Element

Once a MOST has emerged during a class discussion, been established as a precise object, and grapple tossed to the class, the teacher enacts the Conduct element— perhaps the most complex element of building. The overarching goal of Conduct is to keep the whole-class discussion focused on the MOST as the teacher facilitates the class's movement toward making sense of the mathematics of the MOST. In essence, making sense of the MOST and the mathematics associated with it becomes a new mini-discussion that the class becomes temporarily engaged with before returning to the larger discussion. Thus, conducting a discussion about a MOST requires a different collection of coordinated actions than those that a teacher might engage in other circumstances (such as after they have students work on a task and then collect ideas about that task). Once a teacher decides to build on a MOST, we have found that it is unproductive to continue to collect ideas about a broader task or anything other than ideas related to the MOST. In fact, our data suggest that doing so is detrimental to taking advantage of the opportunity the MOST provided to make sense of important mathematics.

Before discussing the specifics of the Conduct element, we begin by providing a "big picture" overview of the teacher actions that take place while effectively carrying out this element of the building practice. Note that this overview is not meant to oversimplify the complex activity of conducting a discussion about a MOST, but to help the reader think about how specific teacher actions might coordinate to carry out the nuances of the work. When a student contributes an idea during Conduct, the teacher first determines whether the mathematics underlying that contribution will help students make sense of the MOST–whether it is a MOST-related contribution (MRC). The subsequent sequence of teacher actions depends on that determination. If a teacher determines that the contribution is not MOST-related (a non-MRC), we have found that a productive teacher action is to gracefully put aside the idea (Drageset, 2014) and recenter the MOST as the focus of the discussion. If the teacher determines that the idea is an MRC, we have found a productive sequence of actions is to first establish the MRC and then make a move that invites students to connect the MRC to the MOST itself—to make sense of how the new idea helps to make sense of the MOST. In essence, the flow of the Conduct element is a repetition of this pattern, since in doing the work of responding to either a MRC or non-MRC, the teacher sets students up to continue to contribute their ideas to make sense of the MOST. We elaborate on the details of these teacher actions in the following sections.

Is the Student Contribution MOST-related?

When conducting a discussion around a MOST, it would be ideal if all of the contributions that students add to the discussion helped to make sense of the MOST. Students do not always do or say what a teacher might want or expect, however. In order to keep the discussion focused on making sense of the MOST, each time a new contribution is shared during the Conduct element, the teacher needs to begin their work by determining whether the contribution is actually a MOST-related contribution (MRC). This determination informs the teacher's subsequent actions.

Determining whether a contribution is an MRC may require the teacher to clarify what the student has said or expand the contribution to get a sense of the student's reasoning. After the teacher has sufficient information to infer the mathematics of the contribution, they then must ask themself, "Is the mathematics of this contribution directly related to the MOST?" (rather than, for example, a contribution that shares another idea about a broader task that students have worked on) and "Will it help students make sense of the MOST?" If both answers are yes, the contribution is an MRC. Otherwise, it is not. In general, for a contribution to be MOST-related, it needs to have the same underlying mathematics as the MOST.

In the following sections, we elaborate on the work that our data suggest a teacher needs to engage in after they determine whether a student contribution is an MRC. We first discuss actions around contributions that are not MRCs, and then actions that support the class's sense-making around an MRC. As we discuss these actions, we will share examples from our data based on whole- class discussions of the tasks designed to elicit MOSTs in Figure 1.

The Student Contribution is Not an MRC

If the teacher determines that a student contribution is not an MRC, our data suggests that it is best to redirect the conversation back to the MOST by *putting aside* the unrelated contribution and recentering the MOST as the focus of the conversation. The action of *putting aside* is adapted from the work of Drageset (2014) who stated that "redirecting actions might be used to put aside suggestions without too much discussion to keep the class concentrated and in order not to lose the line of thought" (p. 300). Recentering the MOST draws on the work of Lineback (2015) who talks about a type of "focus redirection" in which the teacher "refocus[es] the students on an earlier discussion topic" (p. 432). In the case of building, the MOST that is under consideration would be the earlier discussion topic. In this section we elaborate on the ways that the putting aside and recentering might occur.

One way we saw teachers put aside a non-MRC and recenter the MOST was by simply reminding students that they are to be making sense of the MOST. For example, in the Points on a Line task, a common shared solution was "Yes. Point B is $(0,3)$ because you get $3 + 3 = 6$," including the explanation that they took the x-value for the equation $x + y = 6$ from point A and the y-value from point B. Despite being asked to make sense of this particular student

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contribution, in the ensuing discussion students often offered other approaches to solve the task itself, such as "If you write the equation in slope-intercept form, the y-intercept is (0, 6)." This contribution is a non-MRC because the underlying mathematics related to slope-intercept form is different from the mathematics of the MOST (related to what it means for a point to be a solution to an equation). In this situation, the teachers said things like, "That's another interesting approach, but right now we want to be making sense of the claim that Point B is $(0,3)$ because $3 + 3 = 6$." This statement validates the contribution but immediately puts it aside and recenters the MOST as the focus of the discussion. In our data, we have found that it is essential to not only put aside a non-MRC, but to also redirect the conversation by recentering the MOST; this recentering provides guidance to focus students' subsequent contributions. Other teacher actions that supported recentering included gesturing to the public record of the MOST while reminding the class that the MOST is what they are currently discussing, and re-grapple tossing the MOST to re-engage students in sense-making around that contribution. These actions for handling a non-MRC are seen on the left side of Figure 2.

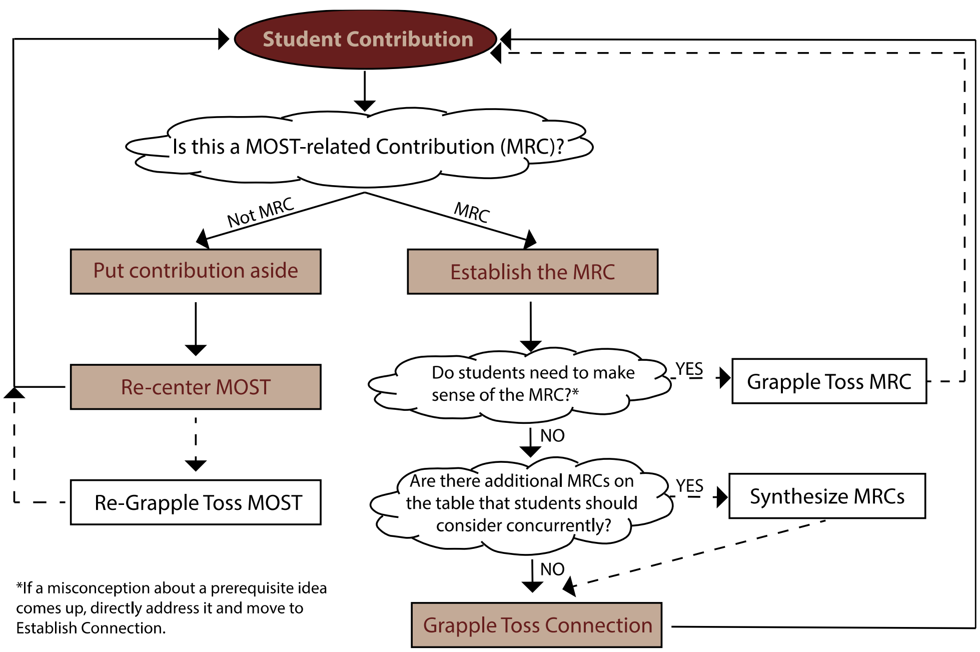

Figure 2: The Conduct Element of Building

The Student Contribution is an MRC

If it is determined that a student contribution is an MRC, the teacher's focus should be on weaving this MRC into the conversation about the MOST. Prior to doing this weaving, we have found that the teacher needs to ensure that everyone in the class understands the essence of the MRC they will consider. Our data indicate that this involves two essential actions—establishing the MRC, and positioning students to connect the MRC to the MOST in a way that allows them to use the MRC to help make sense of the MOST. We first talk about these two essential actions when working with an MRC (the tan boxes on the right side of Figure 2) and then discuss other considerations when working with an MRC (the white boxes on the right side of Figure 2).

Establishing the MRC. Our data suggests that prior to weaving an MRC into the discussion, similar to establishing the MOST, the teacher should first establish the MRC as clear, complete, and concise (Leatham et al., 2021). For example, recall that in the Points on a Line task, the

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MOST that emerged was that the point B (0, 3) was the point that satisfied the conditions of the problem because $3 + 3 = 6$. In the discussion of the MOST, Joelle (all names are pseudonyms) said, "Well, I mean, it doesn't ever, I don't know, I just feel like there's no, it doesn't say that the x and y are coming from different points, I just don't know how that would work." With so much hedging, it was necessary for the teacher to hone in on the key aspect of what Joelle said to make her statement concise, and clarify her statement in the process. He did so by saying "Okay so you're describing this [the teacher points at the MOST written on the board, 'Yes, use (0,3) because 3+3=6'] as taking x and y from different points." The teacher made this statement with a tone indicating that he was seeking confirmation from Joelle that he had interpreted her statement correctly. His gesture to the board also clarified that Joelle's contribution was describing the approach used in the MOST. Teacher moves to establish the MRC have the impact of making the MRC clear, complete and concise for the rest of the students in the class.

Grapple Toss Connection. Asking students to consider how an MRC helps them make sense of the MOST is an important aspect of the Conduct element. When there is an MRC on the table, the teacher needs to make a move to position not only the MOST, but also the MRC, as the object of the discussion. The *grapple toss connection* tells students the action they are to take on the MRC and the MOST—to connect the MRC to the MOST in a way that positions the students to use the MRC to help make sense of the MOST. Because a connection necessarily implies tying two things together, we have found that an important aspect of the grapple toss connection is clearly identifying the two ends of the connection, the established MRC and the MOST. Without clearly defining both ends of the connection, some students may not know what object they are to consider. In addition, we have found that the teacher needs to clearly define the action that students are to take on that object. Failing to do so often leaves students wondering what they are to think about, rather than putting them in a sense-making situation. The reason this is called a grapple toss connection is because the action-defining move resembles the Grapple Toss of the MOST, but the object that is tossed is the connection between the MRC and the MOST.

We have seen some variations in how a teacher connects an MRC to the MOST. One of the most productive connecting actions we have seen in terms of helping students use the MRC to make sense of the MOST—the purpose of the Conduct element—is explicitly asking students to relate the new information (the MRC) to the MOST. For example, in the Percent Discount Task, the common MOST is the response that the original and final price are the same because you are adding and subtracting 50%. In the discussion of the MOST, Kaleb contributed the following MRC using a sample necklace price of $32, "If you add the half of 32, which is 16, it would equal 48. [t*eacher writes "If you add ½ of the 32, 32+16 = 48" on the whiteboard*.] But, if you subtract the half of 48, it would give you 24 which is not the original price." The teacher responded to this MRC by saying, "Okay, so, if you add half of the original, 16, that would give you 48 and then if you subtract half of the 48, it would be 24? So what does that have to do with the original claim here? How does that prove or disprove it?" Here we see the teacher identify both ends of the connection by restating Kaleb's contribution and asking how it relates to "the original claim"— the MOST. The fact that the original claim is recorded on the whiteboard strengthens the connection because students can refer to the public record to recall exactly what the original claim was. Not only does the teacher clearly identify both ends of the connection but they also identify an action, "prove or disprove", that the students are expected to take as they consider the connection between the MRC and the MOST. This grapple toss connection clearly positions students to make sense of the connection between the MRC and the MOST.

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Another way we have seen teachers connect the MRC and the MOST is to ask students to consider conflicting information. In this case, the teacher typically highlights that there is in fact a conflict, and then asks students to make sense of that conflict. For example, after an MRC surfaced in the discussion of the Percent Discount task, the teacher said, "So in Bruce's argument, [the initial and final price are] not the same but Jaden was saying it would be the same cause you would just add 5 and minus 5. So how do we know which one we're supposed to do?" In this response, we see the teacher connecting the MRC (Bruce's argument) to Jaden's claim (the MOST) by highlighting the contradiction that is now on the table and asking students the action-defining question, figuring out "which one we're supposed to do." Pointing to Jaden's claim on the board, restating a portion of the claim, and explicitly referencing Bruce's argument when contrasting it with Jaden's all support having a clear object of consideration.

In summary, when integrating MRCs into the discussion of a MOST, we have found that explicitly defining both ends of the connection puts students in a position where they are able to use the new information provided in the MRC to make sense of the MOST. A second important aspect of the grapple toss connection is to define some type of mental action the students are to carry out when considering how the MRC helps them make sense of the MOST.

Other Considerations. After an MRC is established, but prior to the grapple toss connection, we have found that additional teacher actions are sometimes needed. These moves–grapple tossing the MRC itself and synthesizing the ideas that are on the table–only seem necessary when it is clear that they will better position students for the grapple toss connection.

Grapple Tossing the MRC. Before they are asked to make sense of the connection between the established MRC and the MOST, we have found that sometimes students need to spend some time making sense of the MRC itself. This is important when the students do not yet appear to understand the MRC. If the teacher determines that this is the case, our data suggest that the best move is to Grapple Toss the MRC itself to the class by asking a question that positions students to make sense of that contribution (seen on the right side of the flowchart in Figure 2).

In the discussion of why the simple average of 9 mph (the MOST) is not correct in the Bike Ride task, Corbin contributed the MRC, "You're going 3 miles per hour for the same length of time as you're going 15 miles." Recognizing the potential value of Corbin's thinking to make sense of the MOST, the teacher responded, "Do you guys believe that Blake rode the bike for the same amount of time going 3 miles per hour and the same amount of time at 15 miles an hour?" In this case, Corbin's thinking was a productive contribution to the discussion because knowing the lengths of time at each speed would help students make sense of the MOST, but it wouldn't be productive to connect it to the MOST until students have had a chance to make sense of the validity of this claim. In this instance, grapple tossing just the MRC was a productive move prior to asking students to connect the MRC to the MOST because it put students in a better position to think about how the MRC might help them make sense of the MOST.

Synthesizing. After an MRC has been established but before the grapple toss connection, a teacher needs to ask themselves whether there are prior MRCs on the table that, if considered in conjunction with the new MRC, would better position students to make sense of the MOST. If so, then some synthesis needs to take place that highlights the salient aspects of the MRCs that students will be asked to collectively consider. In this case, the grapple toss connection might position the class to consider the relationship among multiple MRCs and the MOST.

For the Variables task, the common MOST is for students to say that $x + x$ is greater than x because $x + x$ is $2x$ and that is twice as big as x. After several MRCs had emerged, the teacher synthesized with the following response: "So here we have Andre's thinking that x + x is gonna

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be larger than x because x + x is double so it makes everything larger, right? Somebody else said 5 plus 5, would be greater than just 5, right? And Briana, now you're saying that if x were -9, then -9 is greater than -9 plus -9." In this synthesis, the teacher has tied the concrete example of x = 5 to the statement of "x+x is larger than x" which is written on the board and then highlights the MRC where x = -9 by writing "-9 > -9 + -9". Note, however, that they stop short of writing or saying $x > x + x$ so the students are left to make that connection and see the contradiction. In this example, the teacher highlights the salient aspects of two MRCs and also ties one MRC to the MOST. With the appropriate grapple toss connection, the students are in a position to make sense of the MOST with a new perspective provided by the MRCs.

Discussion and Conclusion

Engaging students in whole-class discussion focused on making sense of the mathematics in an instance of student thinking is a complex teaching practice. Although past research has identified individual teacher actions that support student sense making, our work suggests that conducting such a discussion requires a coordinated collection of teacher decisions and actions. Each time a new student contribution emerges, the teacher needs to assess whether the contribution will support the joint sense making, or take students in an unproductive direction. This critical initial decision determines the teachers' subsequent decision points and actions.

There are two additional insights about the Conduct element of building that we wish to share. First is the importance of keeping the discussion focused on making sense of the object of discussion, the MOST. In our work, we have seen teachers pursue every student idea that surfaces, resulting in a discussion that meanders among ideas. Although it seems that teachers do this with good intent as a means of honoring students and their ideas, we argue that putting some ideas aside better honors the student who contributed the MOST by maintaining a focus on the important mathematics that they initially brought to the discussion.

Second, we have come to realize the importance of ensuring that students have a clear understanding of the object they are to focus on and how they are to engage with that object at any point in a discussion. During the practice of building, the discussion begins by positioning the MOST as the object that students are to make sense of. In the Conduct element, for each MRC that is contributed, students first need to understand the MRC itself, and then need to shift their attention to considering a connected object—the MRC and the MOST—and how the MRC helps to make sense of the MOST. Without a clear sense of these shifts in focus, students are unlikely to engage in a focused sense-making discussion. Thus, an important part of the teacher's work during Conduct is explicitly helping students productively focus their attention.

Decomposing complex teaching practices is an important first step in supporting teacher learning of such practices (Grossman et al., 2009). Unpacking the Conduct element of building has allowed us to better understand the complexity of the practice of building on MOSTs and positions us to move to the next stage of our work in supporting teachers to develop their abilities to productively use student mathematical thinking.

Acknowledgements

This research report is based on work supported by the U.S. National Science Foundation (NSF) under Grant Nos. DRL-1720410, DRL-1720566, and DRL-1720613. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the NSF. We thank the teacher-researchers for opening up their practice to help us learn about building, and the undergraduate and graduate students who have helped us to collect, process, and analyze building enactments.

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