LeveragingMOSTs.org

Teachers' Responses to a Common Set of High Potential Instances of Student Mathematical Thinking

Shari Stockero – Michigan Technological University

Laura R. Van Zoest – Western Michigan University

Blake E. Peterson – Brigham Young University

Keith Leatham – Brigham Young University

Annick O. T. Rougée – Washington State University

Supported by

Leveraging MOSTs: Developing a Theory of Productive Use of Student Mathematical Thinking

a 5-year collaborative research project funded by the US National Science Foundation (DRL-1220141, DRL-1220357, DRL-1220148)

Teacher responses to student mathematical thinking

Mathematical Opportunities in Student Thinking

- affect student learning (e.g., Fenemma et al., 1996)
- a feature of effective mathematics instruction that undergirds classroom mathematical discourse (e.g. Franke, Kazemi, & Battey, 2007; NCTM, 2014; Van Zoest, Peterson, Leatham, & Stockero, 2016)
- researchers have
  - characterized teacher responses (e.g., Lineback, 2015; Conner, 2014)
  - investigated changes in teacher responses as a result of professional development (e.g., Brodie, 2011)
  - investigated responses to different kinds of student thinking (e.g., Drageset, 2015)
- do not know how teachers respond to a common set of high-leverage, in-the-moment instances

High-Leverage Student Thinking

Mathematical Opportunities in Student Thinking

MOSTs

- **M**athematical **O**pportunities in **S**tudent **T**hinking
- instances of in-the-moment student thinking worth *building* on
  - worth making the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea

Research Question:

To what extent do teacher responses to MOSTs accomplish the purpose of *building* on them?

Principles Underlying Productive Use of MOSTs

Mathematical Opportunities in Student Thinking

- The mathematics of the MOST is at the forefront.
- Students are positioned as legitimate mathematical thinkers.
- Students are engaged in sense making.
- Students are working collaboratively.

(Drawn from NCTM's Principles to Action, 2014)

We conceptualize *building* as the coordination of these actions in response to a MOST.

Building

Mathematical Opportunities in Student Thinking

Sequence of subpractices of the teaching practice of *building* on MOSTs

1. Make the object of consideration clear (make precise)
2. Turn the object of consideration over to the students with parameters that put them in a sense-making situation (grapple toss)
3. Orchestrate a whole-class discussion in which students collaboratively make sense of the object of consideration (orchestrate)
4. Facilitate the extraction and articulation of the mathematical point of the object of consideration (make explicit)

Methodology

Mathematical Opportunities in Student Thinking

- video recorded scenario interviews
  - 4 scenarios

Scenario Interview

Mathematical Opportunities in Student Thinking

Scenario	Context	MOST
G1	Students were sharing their solutions to the following task (a corresponding picture was on the board). Given two concentric circles, radii 5cm and 3cm, what is the area of the band between the circles?	Chris shared his solution: "The radius of the big circle is 5 and the radius of the little circle is 3, so the gap is 2, so the area of the band is 4π cm^2."
A2	Students had been discussing the following task and had come up with the equation $y = 10x + 25$. Jenny received $25 for her birthday that she deposited into a savings account. She has a babysitting job that pays $10 per week, which she deposits into her account each week. Write an equation that she can use to predict how much she will have saved after any number of weeks.	Casey said, "You could also change the story so the number in front of the x is negative."

Methodology

- video recorded scenario interviews
  - 4 scenarios
  - 25 secondary school mathematics teachers from across the USA
  - total of 99 teacher responses
- teacher response
  - *the collection of actions that a teacher describes they would take immediately following an instance of SMT*
  - *includes any elaboration they provide in response to additional interviewer questioning*
- *Teacher Response Coding Scheme* (TRC)

Teacher Response Coding Scheme (TRC)

Mathematical Opportunities in Student Thinking

Category	Coding Category Description	Codes
Actor	Who is publicly asked to consider the student thinking	teacher, same student(s), other student(s), whole class
Recognition Action	The degree to which the teacher response uses the student action, either verbal (words) or non-verbal (gestures or work)	explicit, implicit, or not
Recognition Idea	The extent to which the student is likely to recognize their idea in the teacher response	core, peripheral, other, cannot infer, not applicable
Move	What the actor is doing or being asked to do with respect to the instance of student thinking	adjourn, allow, check-in, clarify, collect, connect, correct, develop, dismiss, evaluate, justify, literal, repeat, validate

Peterson, B. E., Van Zoest, L. R., Rougée, A. O. T., Freeburn, B., Stockero, S. L., & Leatham, K. R. (2017). Beyond the "move": A scheme for coding teachers' responses to student mathematical thinking. In Kaur, B., Ho, W.K., Toh, T.L., & Choy, B.H. (Eds.). *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education, Vol. 4* (pp. 17-24). Singapore: PME.

Recognition of Student Actions and Ideas

Mathematical Opportunities in Student Thinking

		Student Ideas			
		Core	Peripheral	CNI, Other, N/A	TOTAL
Student Actions	Explicit	43	10	1	54
	Implicit	26	4	2	32
	Not	5	1	7	13
	TOTAL	74	15	10	99

Recognition of Student Actions

MOST Mathematical Opportunities in Student Thinking

		Student Ideas			
		Core	Peripheral	CNI, Other, N/A	TOTAL
Student Actions	Explicit	43	10	1	54
	Implicit	26	4	2	32
	Not	5	1	7	13
	TOTAL	74	15	10	99

Recognition of Student Ideas

Mathematical Opportunities in Student Thinking

		Student Ideas			
		Core	Peripheral	CNI, Other, N/A	TOTAL
Student Actions	Explicit	43	10	1	54
	Implicit	26	4	2	32
	Not	5	1	7	13
	TOTAL	74	15	10	99

Example: Explicit & Core

Mathematical Opportunities in Student Thinking

Scenario G1. Chris shared his solution: "The radius of the big circle is 5 and the radius of the little circle is 3, so the gap is 2, so the area of the band is 4π cm^2."

- *"I would want to know what he means by gap. Um, and maybe have him illustrate that visually, just to kind of picture that as a class," (T4)*
  - *explicit* because it incorporates the student's words (gap)
  - *core* because it incorporates the student's ideas (having him illustrate his idea visually)
- Aligns with the principles underlying productive use of MOSTs
  - keeps the students' mathematics at the forefront
  - positions the student as a legitimate mathematical thinker

Move and Actor

Mathematical Opportunities in Student Thinking

	Same Student	Whole Class	Teacher	Other Student(s)	TOTAL
Adjourn	0	0	3	0	3
Allow	0	5	0	1	6
Clarify	5	0	0	0	5
Collect	2	4	0	1	7
Connect	1	4	0	1	6
Correct	1	0	0	0	1
Develop	32	5	0	0	37
Dismiss	0	0	1	0	1
Evaluate	0	4	0	0	4
Justify	16	2	0	0	18
Literal	4	2	0	0	6
Repeat	4	0	0	1	5
TOTAL	65 (66%)	26 (26%)	4 (4%)	4 (4%)	99 (100%)

Move

Mathematical Opportunities in Student Thinking

	Same Student	Whole Class	Teacher	Other Student(s)	TOTAL
Adjourn	0	0	3	0	3
Allow	0	5	0	1	6
Clarify	5	0	0	0	5
Collect	2	4	0	1	7
Connect	1	4	0	1	6
Correct	1	0	0	0	1
Develop	32	5	0	0	37
Dismiss	0	0	1	0	1
Evaluate	0	4	0	0	4
Justify	16	2	0	0	18
Literal	4	2	0	0	6
Repeat	4	0	0	1	5
TOTAL	65	26	4	4	99

Example: Move

Mathematical Opportunities in Student Thinking

Scenario A2. Casey said, "You could also change the story so the number in front of the x is negative."

- Nearly two-thirds of the instances of *develop* moves (20 of 32) occurred in response to this scenario.
  - Most common teacher move was to ask Casey to explain how they would change the story: "*Well what do you mean? What sort of an equation, or what sort of a real life situation can you think of where that would be a negative?*" (Teacher 6 [T6]).
- Moves such as this position students as legitimate mathematical thinkers.

Actor

M O S T — Mathematical Opportunities in Student Thinking

	Same Student	Whole Class	Teacher	Other Student(s)	TOTAL
Adjourn	0	0	3	0	3
Allow	0	5	0	1	6
Clarify	5	0	0	0	5
Collect	2	4	0	1	7
Connect	1	4	0	1	6
Correct	1	0	0	0	1
Develop	32	5	0	0	37
Dismiss	0	0	1	0	1
Evaluate	0	4	0	0	4
Justify	16	2	0	0	18
Literal	4	2	0	0	6
Repeat	4	0	0	1	5
TOTAL	65 (66%)	26 (26%)	4 (4%)	4 (4%)	99 (100%)

Example: Actor

Scenario A2. Casey said, "You could also change the story so the number in front of the x is negative."

- Most common teacher move was to ask Casey to explain how they would change the story: *"Well what do you mean? What sort of an equation, or what sort of a real life situation can you think of where that would be a negative?"* (Teacher 6 [T6]).
  - Contrast this response with a similar one directed to the whole class: *"Interesting comment… who can come up with a story, a situation that would match what Casey is saying?"* (T7).
- Directing the response to the whole class better adheres to the principles
  - Puts the students' mathematics at the forefront
  - Positions students as legitimate mathematical thinkers
  - Provides all students the opportunity to collaboratively engage in making sense of the mathematics of the MOST

A Caveat

- Goal of building on MOSTs is to have the whole class consider the student mathematics of the instance, **BUT** there are some cases where directing the initial teacher response back to the same student might be desirable.

- Example: Student gives a long or complicated explanation
  - Quite possible that other students in the class would not initially understand the explanation
  - A common teacher response in our data, "*ask him to explain by using…pictures and words, like how he came up with the [his answer]*" (Scenario G1, T18) may be the teacher helping to make the students' idea precise before other students are asked to consider it

- This is an instantiation of the first subpractice of building (make precise)—an important first step in setting the teacher up to engage in the next subpractice (grapple toss), in which they turn the now-precise student thinking over to the class for consideration.

Conclusions

Mathematical Opportunities in Student Thinking

- Teachers most often responded to MOSTs by making a *develop* or *justify* move that stayed *core* to the ideas in the student thinking and often *explicitly* incorporated the students' actions.
  - Signal that the teacher values the students' contributions.
  - Position the students as legitimate mathematical thinkers who can make valid contributions to the development of the mathematics in the classroom.
  - **The words and idea(s) teachers use in their responses to students' ideas could matter in terms of how students are positioned in the classroom.**
- Most teacher responses were directed to the *same student* who had shared the initial thinking
  - Could prevent teachers from enacting the building practice.
  - ***Tossing* the student thinking to the whole class provides all students an opportunity to collaboratively make sense of the mathematics.**

Why is this important?

Mathematical Opportunities in Student Thinking

- Decomposing teacher responses in the way we have in this study has the potential to help teacher educators and researchers focus their development efforts.

  Since the majority of teacher responses honored student thinking, but engaged only the student who contributed the instance, it seems that professional development work should focus specifically on helping teachers understand the potential in directing a response to the whole class, and when it would and would not be appropriate to do so.

- Such focused efforts would allow professional developers to leverage teachers' strengths and thus develop teachers' practice more effectively.

Contact Information

Mathematical Opportunities in Student Thinking

LeveragingMOSTs.org

Example: Core & Implicit

Mathematical Opportunities in Student Thinking

- Many teacher responses that are *core* to the student ideas and *implicitly* incorporate student actions also adhere to the legitimacy principle.
- May be problematic, however, in that it may not be clear to the student(s) what mathematics is under consideration.
- Example: In response to another scenario (A3), a teacher said, *"So I would want to ask her, 'Why did you do this? What are you thinking? Tell us a little bit more.'"* (T24)
  - Fails to specify what mathematics the teacher wants to know more about.
  - In this case, there were two competing options (why the student subtracted or why they chose to select the numbers that they did).