HOW A TEACHING PRACTICE THAT BUILDS ON STUDENT THINKING HELPS TEACHERS DRAW OUT CONCEPTUAL CONNECTIONS

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Past research has identified factors that help maintain the cognitive demand of tasks, including drawing conceptual connections. We investigated whether teachers who were engaging in the teaching practice of building—and thus focusing the class on collaboratively making sense of their peers' high-leverage mathematical contributions—drew conceptual connections at a higher rate than has been found in previous work. The rate was notably higher (54% compared to 14%). By comparing multiple enactments of the same task, we found that this higher rate of drawing conceptual connections seemed to be supported by (1) eliciting student utterances that delve more deeply into the underlying mathematics, (2) giving students more time to explore the underlying math, and (3) using previously learned abstractions to help move the class toward understanding the new abstract concepts underlying a task.

Keywords: Classroom Discourse, Instructional Activities and Practices

The important role that high-cognitive-demand tasks play in student learning is ubiquitous in mathematics education (e.g., NCTM, 2014). Unfortunately, the high cognitive demand of tasks is often not maintained as those tasks are enacted in classrooms (e.g., Henningsen & Stein, 1997). As a result, much work has been done to understand the complexity of maintaining high levels of cognitive demand during task enactments. For example, Stein et al. (1996) identified factors that maintain and lower cognitive demand. These factors have been utilized by other studies (e.g., Sullivan, 2019) to better understand the maintenance of cognitive demand. One of these factors that has not received much attention is *drawing conceptual connections*. This is despite the fact that developing conceptual connections is at the core of the type of student learning envisioned by NCTM (1989, 2000, 2014).

The *teaching practice of building* (e.g., Leatham et al., 2022; hereafter referred to as *building*) is a teaching practice designed to take full advantage of MOSTs (Mathematical Opportunities in Student Thinking)—high-leverage student mathematical contributions that "provide an in-the-moment opportunity to engage the class in joint sense making about that contribution to better understand the important mathematics within it" (Van Zoest et al., in press). This important mathematics that students can come to better understand—the *mathematical point* (MP) of the contribution (e.g., Van Zoest et al., 2016) is central to building. The teacher identifies the MP when deciding if a student contribution is a MOST, keeps it in mind throughout the joint sense making discussion, and finally, ensures that the MP—the mathematics that the students had the opportunity to learn as a result of the discussion—is made explicit as the discussion concludes (e.g., Leatham et al., 2022).

It seems that the emphasis that building places on MPs and student thinking may support teachers to draw out conceptual conceptions, and thus to better maintain the high cognitive demand of tasks as they are enacted in their classrooms. To better understand the phenomenon of conceptual connections and how building might support drawing them out, this study

investigated how teachers who were attempting to build on MOSTs in their classroom engaged in drawing out conceptual connections from their students.

Literature Review

Cognitively demanding tasks are challenging problems, or sets of problems, that require students to use their existing knowledge, sometimes in new and unique ways, along solution pathways that are not immediately clear (Stein et al., 1996). The use of such tasks can lead to student learning gains (Stein & Lane, 1996). As mentioned above, the high cognitive demand of these tasks is often not maintained as they are enacted in classrooms (Henningsen & Stein, 1997), and as a result, much work has been done to understand the complexity of maintaining high levels of cognitive demand during task enactment. The foundation of that work is Stein et al.'s (1996) study of 520 task enactments by teachers utilizing reform-based teaching practices, which led to identifying seven factors that maintain, and six factors that lower, cognitive demand. *Drawing conceptual connections* is one of the factors that help maintain cognitive demand.

Drawing conceptual connections occurs when a teacher or student explicitly makes connections between a task and its underlying mathematical concepts. Stein et al.'s (1996) study found drawing conceptual connections in only 13% of tasks where cognitive demand was maintained during enactment. Henningsen and Stein (1997) looked specifically at tasks that began at the highest level of cognitive demand—doing math—and similarly found that drawing conceptual connections occurred in only 14% of such tasks for which the cognitive demand was maintained. In another study, Sullivan (2019) provided a group of 16 teachers with over 300 hours of professional development (PD) to help them maintain high levels of cognitive demand and share mathematical authority through high-quality classroom discourse. They analyzed the teacher-identified "best" enactments using the Instructional Quality Assessment (IQA; Boston, 2012), a toolkit that assesses elements of ambitious instruction in mathematics. On the IQA Mathematical Residue Rubric, which considers the extent to which conceptual connections are drawn during whole-group discussion, the teachers in Sullivan's study had an average score of 2.08 (out of 4). A rating of 2 reflects "the teacher is telling students what connections should have been made" or "students make superficial contributions that are taken over by the teacher" (Boston, 2017, Mathematics Residue Rubric). Smith and Stein (2018) created 5 practices for orchestrating productive mathematics discussions as a way to help teachers maintain the cognitive demand of task enactments, and described the 5th practice "connecting different students' responses and connecting the responses to key mathematical ideas" (Smith & Stein, 2018, p. 10) as "the most challenging of the five practices because it calls on the teacher to craft questions that will make the mathematics visible and understandable" (p. 70). Perhaps because drawing conceptual connections is the least prominent factor that helps maintain cognitive demand, less research has been done to understand it than many of the other factors, and consequently less is known about it as well.

We can infer some things about drawing conceptual connections from Hiebert and Wearne (1993), who looked at classroom discourse to understand the maintenance of cognitive demand. They found that some types of questions teachers ask, such as "Why does this procedure work?" or "What's going on with this strategy?", helped maintain cognitive demand. Similarly, increased length of student utterances during whole class work also positively correlated with increased cognitive demand. Essentially, a teacher asking probing questions that require students to give explanations, rather than one-word answers, is an indicator of maintaining cognitive

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demand. Although Hiebert and Wearne did not discuss drawing conceptual connections specifically, when teachers ask students to explain the nature of a problem using descriptive answers, for example, they are creating a path that can lead to students making conceptual connections.

To better understand how teachers draw conceptual connections, we investigate these questions: (1) Do teachers who are attempting to build on MOSTs in the context of enacting a high cognitive demand task rate higher on drawing conceptual connections than those who are not? (2) What can we learn about drawing conceptual connections from analyzing their instruction?

Theoretical Framework

We approach this work from a *Knowledge in Pieces* epistemological perspective (e.g., diSessa & Sherin, 1998; diSessa, Sherin, & Levin, 2016). Knowledge in Pieces (KiP) models knowledge as "a complex system of many local abstractions of experience" (Walkoe & Levin, 2020, p. 28). Harlow & Blanchini (2020) describe key ways in which the KiP perspective influences teaching, including teachers: (1) recognizing students' initial ideas as "useful and productive for building understanding that is consistent with canonical knowledge" (p. 397) and (2) designing their instruction around their students' ideas, often in the midst of their instruction. MOSTs are often incomplete or non-canonical thinking (Van Zoest et al., 2017) and building is a teaching practice that supports teachers to act in ways consistent with a KiP perspective because it centers student thinking and provides a pathway for facilitating collaborative sense-making about that thinking (Leatham et al., in press). Thus building on MOSTs is a teaching practice consistent with the KiP epistemology.

Building on MOSTs is comprised of four elements (Leatham et al., 2021, p. 1393):

- 1. establish the student mathematics of the MOST as the object to be discussed;
- 2. grapple toss that object in a way that positions the class to make sense of it;
- 3. *conduct* a whole-class discussion that supports the students in making sense of the student mathematics of the MOST; and
- 4. *make explicit* the important mathematical idea from the discussion [the mathematical point (MP) of the MOST].

Although the centrality of the MP to building heightens opportunities to draw conceptual connections throughout the practice, the focus of the fourth element, make explicit, is drawing a conceptual connection between the whole-class discussion and the MP of the MOST. For this reason, the instruction of teachers who are attempting to build on MOSTs seems a fruitful context for investigating the way in which teachers draw conceptual connections.

Mode of Inquiry

Six middle school teacher-researchers (TRs) in the larger MOST project, which was focused on conceptualizing the teaching practice of building (for more details, see Leatham et al., 2022), provided 24 videotaped classroom task enactments (each teacher enacted two tasks twice; see Figure 1 below for the tasks). The enactments were analyzed using The Instructional Quality Assessment (IQA; Boston, 2012) by researchers trained in using the IQA who had no connection to the project. We also analyzed these enactments with the Reorganized Factors that Undermine or Keep Cognitive Demand (RUK; Ruk, 2020). The RUK is a succinct tool designed to measure

the factors that maintain and lower cognitive demand (as identified by Stein et al., 1996) so that such measurements can be compared across studies. The Conceptual Connections category of the RUK looks at the extent to which conceptual connections to underlying concepts were made and seemed to be understood by students. Our analysis of these results was compared to what is known about this factor and used to create hypotheses about how the teachers drew conceptual connections during the task enactments. Based on these hypotheses, questions for an online survey and teacher interviews were created. For example, teachers were asked "When enacting this task, what would you ideally like to hear students say to show you that they understood the underlying mathematics of this task?" These questions were given to the five teachers who were able to participate in this part of the data collection. Their responses allowed us to verify or disprove the hypotheses. For more details about the larger study on the maintenance of high cognitive demand, see Ruk (2021).

Results & Discussion

The teachers in our study (TRs), who were attempting to build on MOSTs in the context of enacting a high-cognitive demand task, rated higher on drawing conceptual connections than those in past research studies. Specifically, the TRs drew conceptual connections during 54% of the enactments, compared to 13% and 14% in Stein et al. (1996) and Henningson and Stein's (1997) studies, respectively. Furthermore, on the IQA Mathematical Residue Rubric (similar to the RUK Conceptual Connections category), the TRs had an average score of 2.83, compared to 2.07 in Sullivan's (2019) study. It is important to note that both of these groups were middle school mathematics teachers who were committed to NCTM-Standards-based teaching, thus this difference is quite striking. The higher ratings for drawing conceptual connections when attempting to build on MOSTs are likely due to the structure of the building practice and the focus on drawing out student contributions related to the underlying mathematics of the MOST that is being built on. This specificity may have done more to draw out conceptual connections than a general focus on reform-based teaching practices (Henningsen & Stein, 1997), or high-quality classroom discourse (Sullivan, 2019).

We now turn to what we can learn about drawing conceptual connections from analyzing these teachers' enactments. When teachers attempt to draw conceptual connections, they attempt to surface connections between in-class work and the mathematical concepts underlying this work. The RUK looks not only at this attempt but also at how well these connections appear to be understood by students. Figure 1 shows each teacher's score on the RUK Conceptual Connections category for each of their enactments. The variability across enactments both within and between teachers may reflect the fact that these teachers were attempting a new practice. This variation, however, allowed us to analyze differences in teacher actions between the highest-rated (those rated 4) and lowest-rated (those rated 1) enactments. In the interest of space, we draw our examples from the Variables task (Figure 1a).

Task		RUK Conceptual Connections Rating by TR					
		TR1	TR2	TR3	TR4	TR5	TR6
a. Variables: Which is larger, x or $x + x$? Explain your	1 st	1	4	4	3	3	2
reasoning.	2 nd	3	1	3	3	4	1
b. Percent Discount: The price of a necklace was first		3	1	4	1	4	1
increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?	2 nd	1	2	4	3	2	1

Figure 1: Conceptual Connections Rating by Teacher-Researcher for Each Enactment

Comparing task enactments rated highest and lowest on the RUK Conceptual Connections category led to the identification of three main differences. First, during whole-class discussion for the highest-rated enactments students themselves made utterances that delved more deeply into the underlying mathematics, even without the teacher needing to push them. In contrast, during the lowest-rated enactments, teachers tried to push students to explore the underlying mathematics of the task more deeply but were less successful. Second, students in the lowestrated enactments were not given time to explore the underlying mathematics after it surfaced. In highest-rated enactments, students were given additional time to do this. Third, the exploration of previously learned abstractions seems to be a needed precursor for understanding a task's underlying mathematics. For the highest-rated enactments, this exploration was used mainly to help move the class toward understanding the new abstract concepts underlying a task. For the lowest-rated enactments, this exploration was used partially for this purpose, but mostly for solidifying abstract concepts needed to engage with the task in the first place. In the following, we describe each of these three patterns. Before doing so, it is important to note that there was no indication that students in highest-rated enactments were different from students in the lowestrated enactments in ways that would affect the study.

Utterances of the Underlying Mathematics

All of the task enactments analyzed contained student utterances that helped move the class toward surfacing connections to the underlying mathematical concept, but only those in the highest-rated enactments actually surfaced the concept. Figure 2 shows the underlying mathematics concept for the Variables task and two examples of student utterances that surfaced the underlying concept. The highest-rated task enactments also contained teacher comments that elicited student utterances that delved more deeply into the underlying mathematics by specifically asking them to make connections between the discussion and the underlying mathematics. Here are two examples:

- Okay, because some of you initially just said that x+x is bigger 'cause it's double the size but now you guys are saying it really depends on the value of the variable. So, if you go back to the original statement, what did we figure out? Can somebody summarize for us what we just learned?
- It depends on the number that x is? Can somebody restate what we just learned in this one problem combining everyone's thinking? What should we consider when we are comparing two expressions?

Variables task	Underlying math concept	Examples of student utterances that surfaced the underlying concept
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x or x + x? Explain your	must be considered to determine relative values of	"If the x is a positive then x+x is bigger. If x is a negative, then x is bigger and then if x is zero then they're equal." "It just depends on what x is, if the x is negative, then x is negative, if positive then x+x is positive."
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Figure 2: Task, Underlying Concept, and the Utterances that Surfaced this Concept

In contrast, the lowest-rated enactments were not as effective at eliciting student utterances that delved more deeply into the underlying mathematics. For example, in one lowest-rated enactment students had not mentioned the case of x=0 (which is not needed to answer the question), so the teacher asked, "Is there anything else we should look at, are there any other numbers out there besides positives and negatives?" After a student said zero, the teacher led them to expand their statement to say "[if x is zero] none of them are any bigger than the other," and then the teacher extrapolated this to mean that x and x+x are equal. The teacher then tried to surface the underlying mathematics by asking students to recite all of the cases (x positive, negative, and zero). Rather than saying anything clearly related to domain, students made vague utterances such as "they're both right and they're both wrong" or "try with both negative and positive numbers if it's a variable." Students never uttered an encompassing statement of the underlying math, and the teacher made an attempt themself by saying, "be more general, just think [of] different numbers for x. You might have to try other values. Not just positives and negatives." It seems that although the teacher could see the connection between zero and the problem, the students did not. Thus, when the teacher focused on zero without reorienting the students, they were no longer in a sense-making position and appeared to lose the sense that they had already made. This points to the importance of teachers being explicit about grounding their questions in the discussion—something we saw examples of teachers in the highest-rated enactments doing in the bulleted questions above.

Time to Explore

Directly after the teacher's utterance of the underlying mathematics, the teacher ended the task by saying, "Ok, make sure your name is on your paper." This abrupt ending gave the students no time to discuss the underlying mathematics. This was problematic because these particular cases (positive, negative, and zero) do not generalize to all variable comparisons. Similar patterns were present in other lowest-rated enactments as well. For example, in one enactment it seemed students could have uncovered the underlying mathematics themselves, as one student said, "[I]t depends on the value of x." However, the teacher did not allow time to consider this idea further. Had the teacher allowed the class to consider the student's idea, they likely would have connected it to the context of the problem, since they had discussed positive, negative, and zero earlier. However, across the lowest-rated enactments, students were not given time to discuss the underlying mathematics. Thus, students again had to make the connections between the contextual examples from this problem and the underlying mathematics on their own. To contrast this, we return to the highest-rated task enactments. During the whole-class discussion for these enactments students said things like, "[I]t will depend on x. If it is a positive, x+x will be greater, but if it is a negative x, x it will be greater. Or if it was a 0, they'd be equal," and "[It] depends on the x-value, if it's positive, negative, or zero." These utterances discussed positive, negative, and zero—the three components needed to fully consider the domain in the context of this task. Additionally, these utterances occurred earlier during whole-class discussion than any utterances considering zero during the lowest-rated enactments. After these initial utterances considering negative values and zero, students in the highest-rated enactments were

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allowed additional discussion time, and several other students made similar utterances, so ultimately multiple students uttered complete examples of the underlying mathematical concept. Again, this is in comparison to the lowest-rated enactments where only the teacher uttered such complete examples, and the whole-class discussion ended shortly thereafter. Overall, teachers in the highest-rated enactments supported students to provide robust utterances of the underlying math, as opposed to the lowest-rated enactments where teachers allowed for only superficial utterances.

Previously Learned Abstractions

For all 12 of the Variables task enactments, students gave a concrete example of x having a negative value. Students said things like: "-9 plus -9 is -18, and -18 would be less than -9." For the highest-rated enactments, such examples emerged after more abstract utterances that if x is negative, then x plus x is less than x, and for the lowest-rated enactments at least one concrete example came before an abstract utterance. This shows a different pattern for whole-class discussion in the highest- and lowest-rated enactments. Overall, the lowest-rated enactments started with misconceptions, moved to concrete examples, then to an abstraction of those concrete examples, and concluded with a brief statement related to the underlying mathematics that was not discussed further by the class. For these enactments, concrete examples were, at least partially, used to understand the mathematical concepts needed to engage with the task. For example, if a student said, "Isn't negative 3 plus negative 3 equal to negative 6," they were not saying this strictly as an example of the more abstract concept that if x is negative, then x plus x is less than x, but rather as a way to verify their understanding of adding negative numbers. Conversely, the highest-rated task enactments started with an abstraction of the mathematics needed to find a solution, moved to concrete examples of this abstraction, and ended with discussion of, a new abstraction of the tasks' underlying mathematics, which seemed to be understood by the majority of the class. For these enactments, concrete examples were in service of understanding the underlying mathematics of the task, as opposed to focusing on understanding prerequisite concepts.

These observations suggest that exploration of previously learned abstractions may be a needed prerequisite to understanding a task's underlying mathematics. However, for the highest-rated enactments, this exploration was predominantly used to move toward understanding new abstract mathematical concepts underlying the task—the underlying mathematics of the task. But for the lowest-rated enactments, this exploration primarily focused on solidifying an abstract concept needed to engage with the task—such as adding negative numbers.

Implications

This study found higher rates of drawing conceptual connections than has been found in previous work. The difference is likely due to the building practice calling for explicit utterances of the underlying mathematics. Thus, even though building was not specifically developed to support the maintenance of high cognitive demand tasks, because aspects of the practice aligned with factors that support the maintenance of cognitive demand, teachers who attempted to build in the context of using a high-cognitive demand task by default increased their ability to maintain the high-cognitive demand of the task. This finding suggests that developing teaching practices that support factors that have been found to maintain high cognitive demand may be an important way to increase the maintenance of high cognitive demand tasks during enactments. Our results also showed that conceptual connections were drawn out at higher levels when multiple students made complete utterances of the underlying mathematics and the class was

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given sufficient time to explore the ideas that surfaced. Furthermore, the exploration of previously learned abstractions seems a necessary prerequisite to understanding a task's underlying mathematics, and this exploration is more productive if it is used to move towards understanding new abstract mathematical concepts underlying the task rather than to solidify abstract concepts needed to engage with the task.

Overall, to draw conceptual connections, teachers need to explicitly ask for them and allow students enough time to discuss the underlying mathematics so that it is uttered by numerous students, and make sure students are taking what they learn from engaging with the task and using it to move towards new abstractions, rather than simply rehashing previously learned abstractions. Most importantly, the results of this study show that teachers' actions are needed to help students draw conceptual connections. If students are allowed to explore underlying mathematics, as opposed to a teacher directly telling them what it is, it appears that all students have the ability to draw conceptual connections.

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