Using a Public Record to Anchor Joint Sense Making of Mathematics

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with
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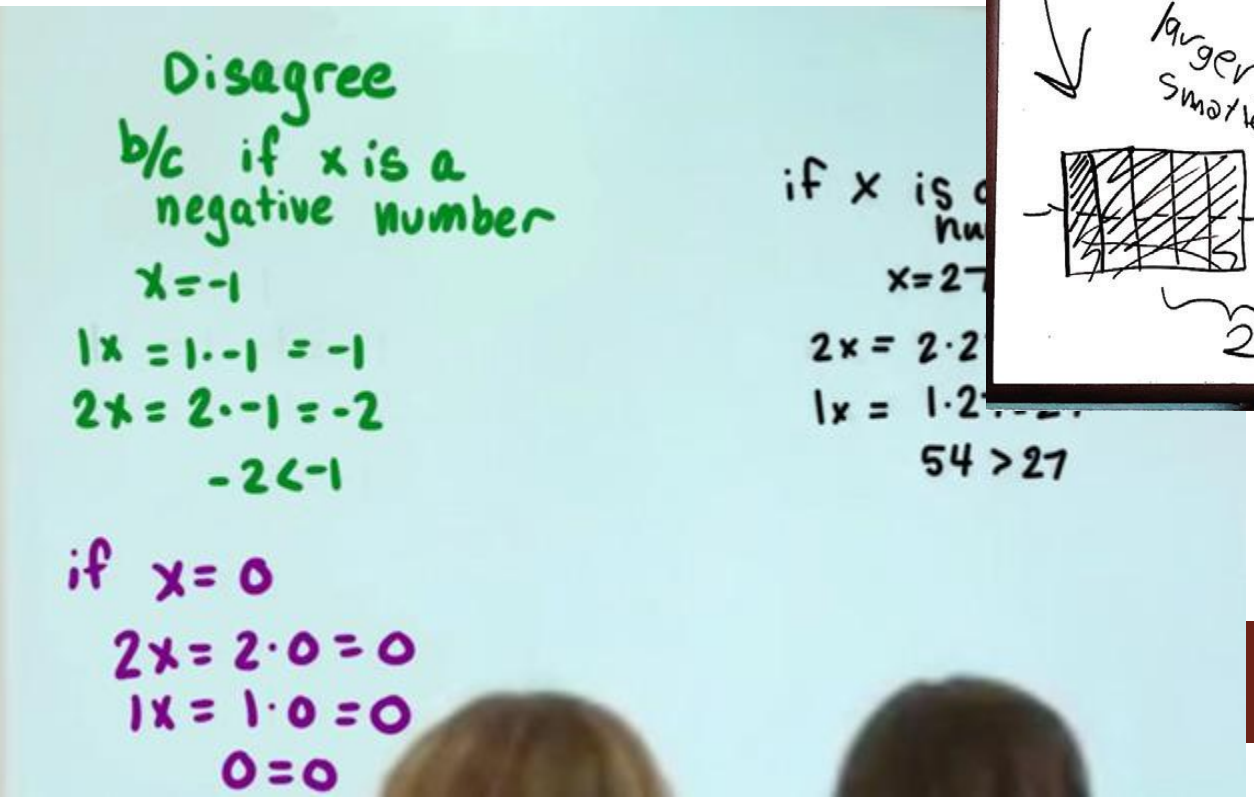
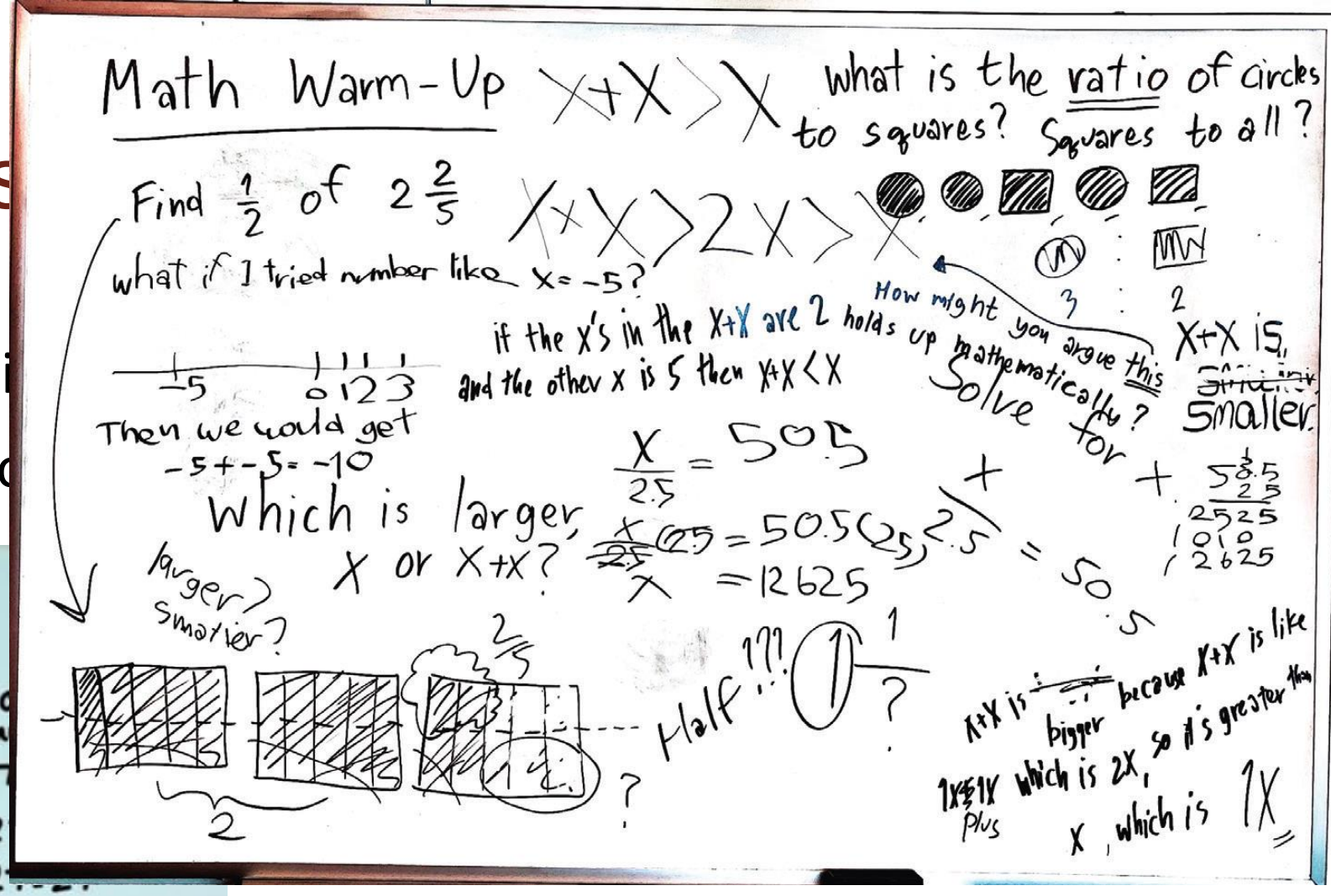
A public record is

- A visual representati
- Publicly accessible t





Joint sense making of mathematics looks like…

- Students
  - grappling with a mathematical situation
  - responding to their peers' thinking about that situation
- Teachers
  - helping to facilitate that activity
  - avoiding the temptation to do the sense making for the students

Public records of student thinking provide…

- Permanence
  - physical objects
  - can refer to and operate on
- Focus
  - establish and sustain common ground
  - demarcate ideas

Classroom Vignette

Middle school, whole-class discussion

The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?”

Mr. L and three students: Shari, Blake, and Laura

Blake: The prices will be the same because the price would increase and then decrease by the same amount.

final ? = original

Mr. L: Well, class, what do you think? How does Blake's claim hold up mathematically?

Shari: I think Blake's claim is wrong. The prices wouldn't go up and down by the same amount.

Mr. L: Okay, Shari. Why not?

Shari: Because if a necklace is 100 dollars, and it increases by 50 percent, then it goes up to 150 dollars. And if it decreases by 50 percent, it goes down to 75 dollars. Cause you're decreasing by half of the amount of the 50 percent more.

necklace = $100
inc by 50% = 50% of 100 = 50
100 + 50 = $150
50% of 150 = $75

Mr. L: Interesting. How did you get the 150 dollars?

Shari: I added 50 percent of 100—that's 50—to 100 and got 150.

Mr. L: And how did you get the 75 dollars?

Shari: I took 50 percent of 150—that's 75.

Blake: Well, I used an example too, but I started with 20 dollars.

Mr. L: Great. Could you share your example?

Blake: Yup. I got 20 dollars plus 10 is 30 and then I got 30 dollars minus 10 is 20.

Mr. L: Okay, class. Let's think about what we have here. How does Shari's approach relates to Blake's approach?

Laura: If you used Blake's approach with Shari's example you'd get 50 percent of 20 dollars is 10 and then 20 dollars plus 10 is 30. Then you'd take 50 percent of 30 to get 15 dollars and 30 minus 15 is 15 dollars.

Mr. L: Now, that's interesting. We have two ways of using 20 dollars and we're getting different answers. What's going on here?

final $\stackrel{?}{=}$ original

Yes

The price will increase and decrease by the same amount

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necklace = $20 + 10 = 30

$30 - 10 = 20

50% of $20 = 10

$20 + 10 = 30

50% of 30 = $15

30 - 15 = $15

Creating Quality Public Records

1. Make the public record precise
2. Purposefully organize the public record
3. Take advantage of the public record

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Make the public record precise

- Clear
- Complete
- Concise

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- Concise

50 percent of it is 10.

50 percent of what is 10?

What did you take 50 percent of to get 10?

They aren’t the same.

What aren’t the same?

What are you claiming aren’t the same?

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Mr. L: How did you get 150 dollars?

Shari: I added 50 percent of 100—that's 50—to 100 and got 150.

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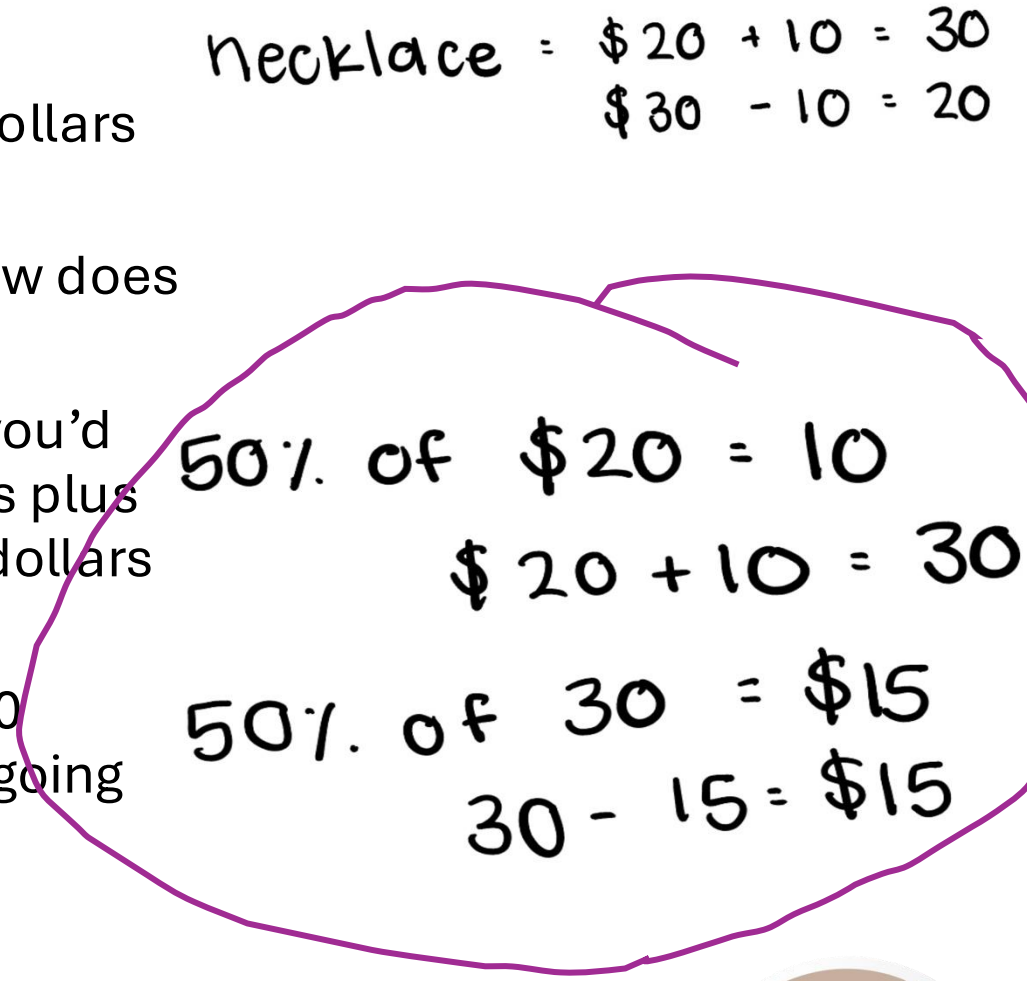

Make the public record precise

- Clear – clear enough
- Complete – don’t go too far down the sense-making path
- Concise – use a “check-in” move

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Purposefully organize the public record

- How do these shared ideas fit into the ongoing argument?
- How might these ideas help the class move forward in their joint sense making?
- How might the recording of these ideas help scaffold the class as they move forward in this joint sense making?

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1. Distinguish between ideas
2. Consider the placement of ideas
3. Seek parallelism of ideas

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Distinguish between ideas

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$30 − 10 = 20

No

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$20 + 10 = $30
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30 − 15 = $15

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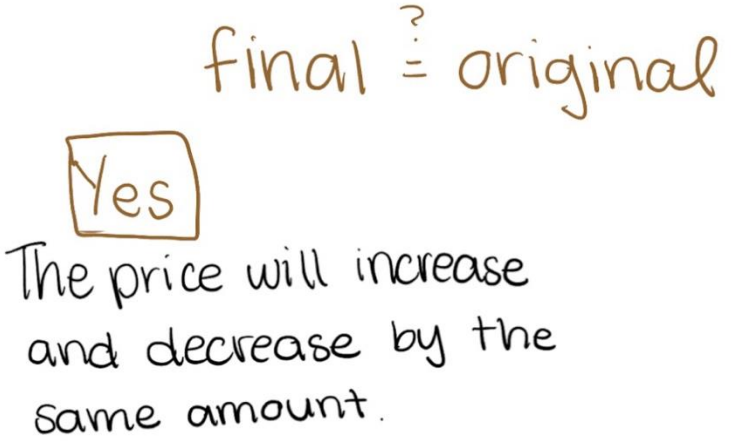

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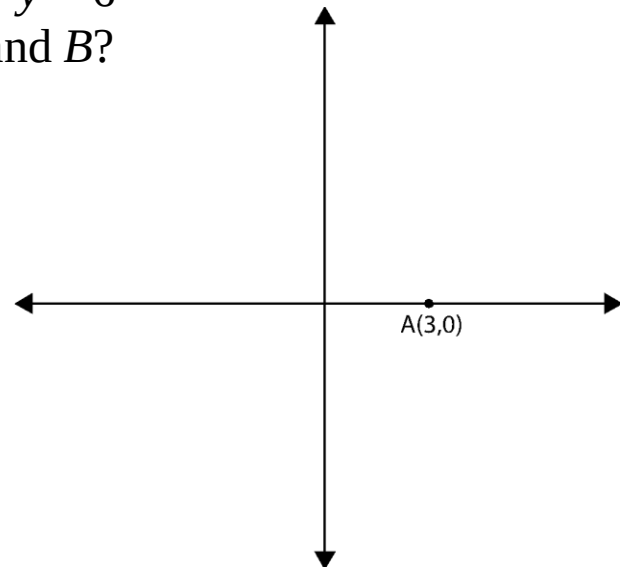
Now you try…

Is it possible to select a point B on the y-axis so that the line $x + y = 6$ goes through both points A and B? Explain why or why not.



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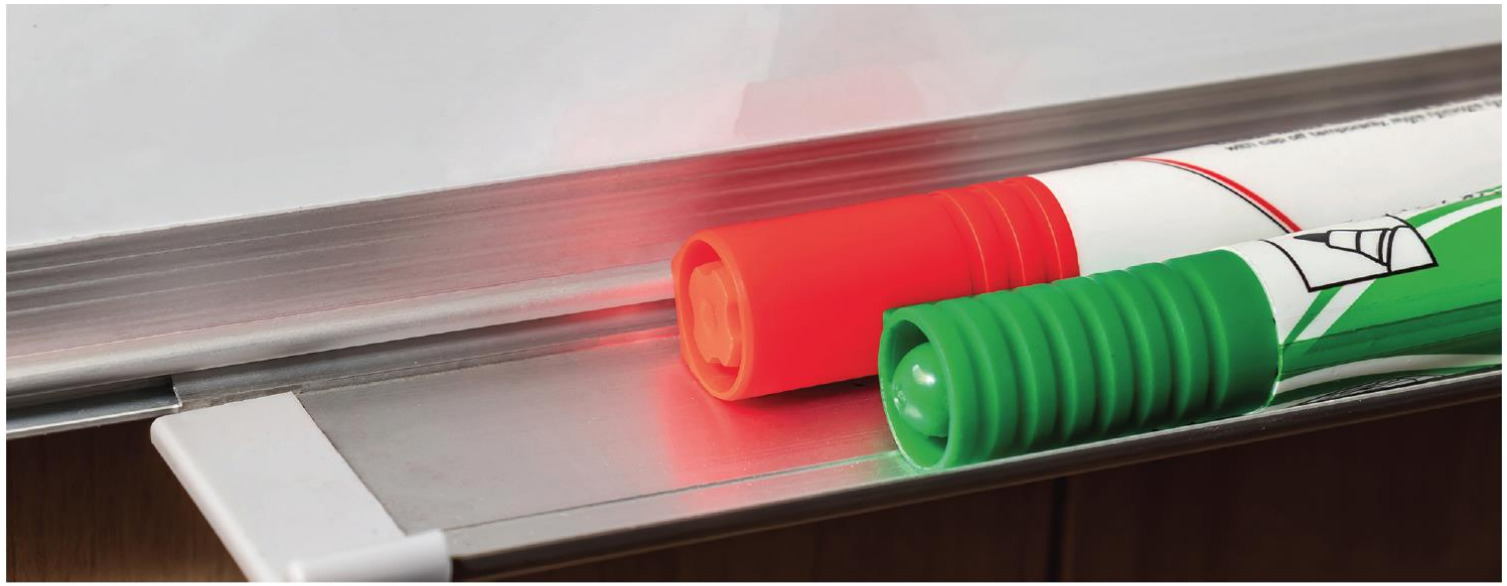

Using Public Records to Scaffold Joint Sense Making

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Session: *362 - Tackling Tangential Student Contributions*

Time: 11:00 AM - 12:00 PM

Location: McCormick Place, S404 A

Session: *434 - Wait, What Are We Talking About? (Re)focusing Students During Whole-Class Discussion*

Time: 2:30 PM - 3:30 PM

Location: McCormick Place, S406 B

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