Using Public Records to Support Class Discussion

Using Public Records When Building on MOSTs

Blake Peterson, Keith Leatham, Shari Stockero, Laura Van Zoest

Working with Public Records of Students Mathematical Thinking Routine

Christina Koehne, Eva Thanheiser, Kate Melhuish

Structuring Boardspace to Facilitate Repeated Reasoning

Bill DeLeeuw, Samuel Otten, Ruveyda Karaman Dundar

Representing

Michael Hicks, Christina Koehne, Mai Bui, Jessica Bishop

Common Understanding of Public Record

- A public record is ...
  - a physical or visual representation of student mathematical thinking
  - publicly accessible to all participants within the classroom
- A public record is used ...
  - as an object of discussion during class discourse
  - to support collective problem solving

Related Research

- Intentional use of the board to support student problem solving (TIMSS 1995 Video Study)
- Paying explicit attention to recording student thinking (Seago, Mumme, & Branca, 2004; Lucenta, Kelemanik, & Creighton, 2016)
- Using the board as a way to maintain continuity during whole-class collaborative inquiry (Staples, 2007)
- Public display of student work as a focal point for discussion based on the 5 Practices (Smith & Stein, 2011)
- Implicit use in examples to illustrate findings, for example: engaging with others' ideas supports student achievement (Webb et al., 2014)
- Importance of using different representations as a mathematical practice (NCTM, 2014)

Research on Use of Public Records to Support Equitable Mathematics Teaching

Public records can be useful for:

+ assigning competence (Cohen & Lotan, 1998) by providing an opportunity to name the smartnesses and mathematical contributions of multiple students (e.g., Featherstone et al., 2011; Gresalfi et al., 2009)
+ enhancing comprehension of what is being said for students with hearing difficulties by providing visual reinforcement (e.g. DeafTEC, n.d.)
+ helping emergent bilingual students to track on the big mathematical ideas by documenting those ideas (e.g., Moschkovich, 2013)
+ supporting students to develop positive math identities (e.g., Langer-Osuna & Avalos, 2015)

Core Principles Underlying Productive Use of Student Thinking

+ [Student] mathematics is at the forefront
+ Students are positioned as legitimate mathematical thinkers
+ Students are engaged in sense making
+ Students are working collaboratively

(Van Zoest, Peterson, Leatham & Stockero, 2016)

Session Purpose

+ Consider the different ways mathematics educators think about, use, and study public record to support student learning.
+ Consider how we can help teachers be purposeful in how they think about and use public record.

Some Questions to Consider

- We have noticed a couple of differences among our groups and pose two questions related to those differences that we would like you to consider as we share our work.
- These differences and any others you notice will inform our discussion at the end.

Public record of what?

What is the unit of analysis?

Using Public Records to Support Class Discussion

Using Public Records When Building on MOSTs

Blake Peterson, Keith Leatham, Shari Stockero, Laura Van Zoest

MOSTs

+ High-leverage instances of student thinking (Recognizing MOSTs Framework)
+ In-the-moment opportunity
+ Opportunity to make the MOST the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

...the teaching practice of building

Building on MOSTs

+ ***Establish*** the student mathematics of the MOST so that the object to be discussed is clear.
+ ***Grapple Toss*** that object to the class in a way that positions them to make sense of it.
+ ***Conduct*** a whole-class discussion that supports the students in making sense of the student mathematics of the MOST.
+ ***Make Explicit*** the important mathematical idea from the discussion.

Our Understanding of Public Record

- A public record is ...
  - a physical or visual representation of student mathematical thinking
    - *record of sense-making discussion around the MOST*
      - *initial student mathematical thinking that is the focus of a class discussion and*
      - *subsequent contributions related to the initial thinking*
  - publicly accessible to all participants within the classroom
    - *hold some degree of permanence*
- A public record is used ...
  - as an object of discussion during class discourse
  - to support collective problem solving

Broad Uses of the Public Record During Building

- Manipulating
  - Creating
  - Adding
  - Editing
- Referencing
  - Pointing gestures
  - Verbal Cues
  - A combination of gestures and verbal cues

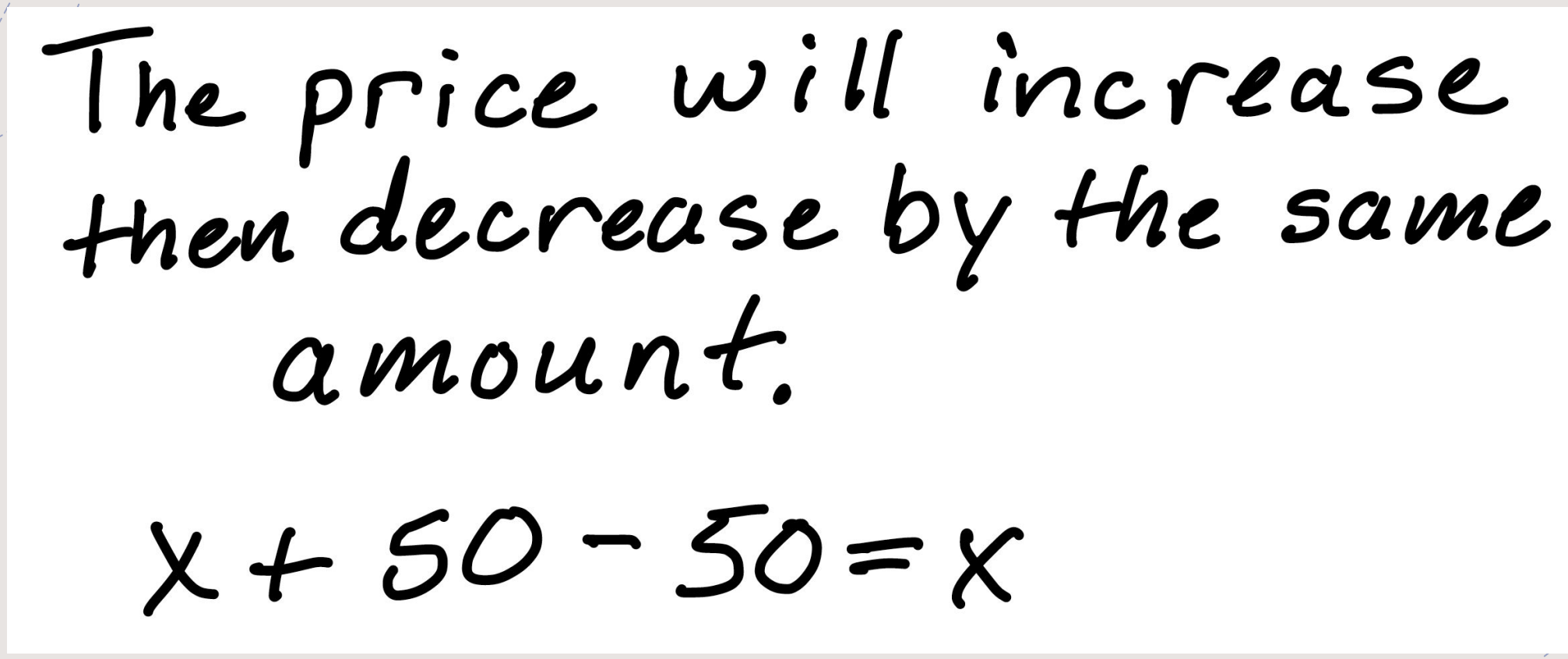
Manipulating in Establish

The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?

The price will increase then decrease by the same amount.

$x + 50 - 50 = x$

Referencing in Grapple Toss



How does this claim hold up mathematically?

Manipulating During Conduct

- Principles for a clearer Public Record during Conduct
  - Particularization
  - Placement
  - Parallelism

Manipulating During Conduct

The price will increase then decrease by the same amount.

$x + 50 - 50 = x$

necklace = $100

inc. by 50% = 50% of 100 = 50

100 + 50 = $150

50% of 150 = $75

necklace = $20 + 10 = 30

$30 - 10 = 20

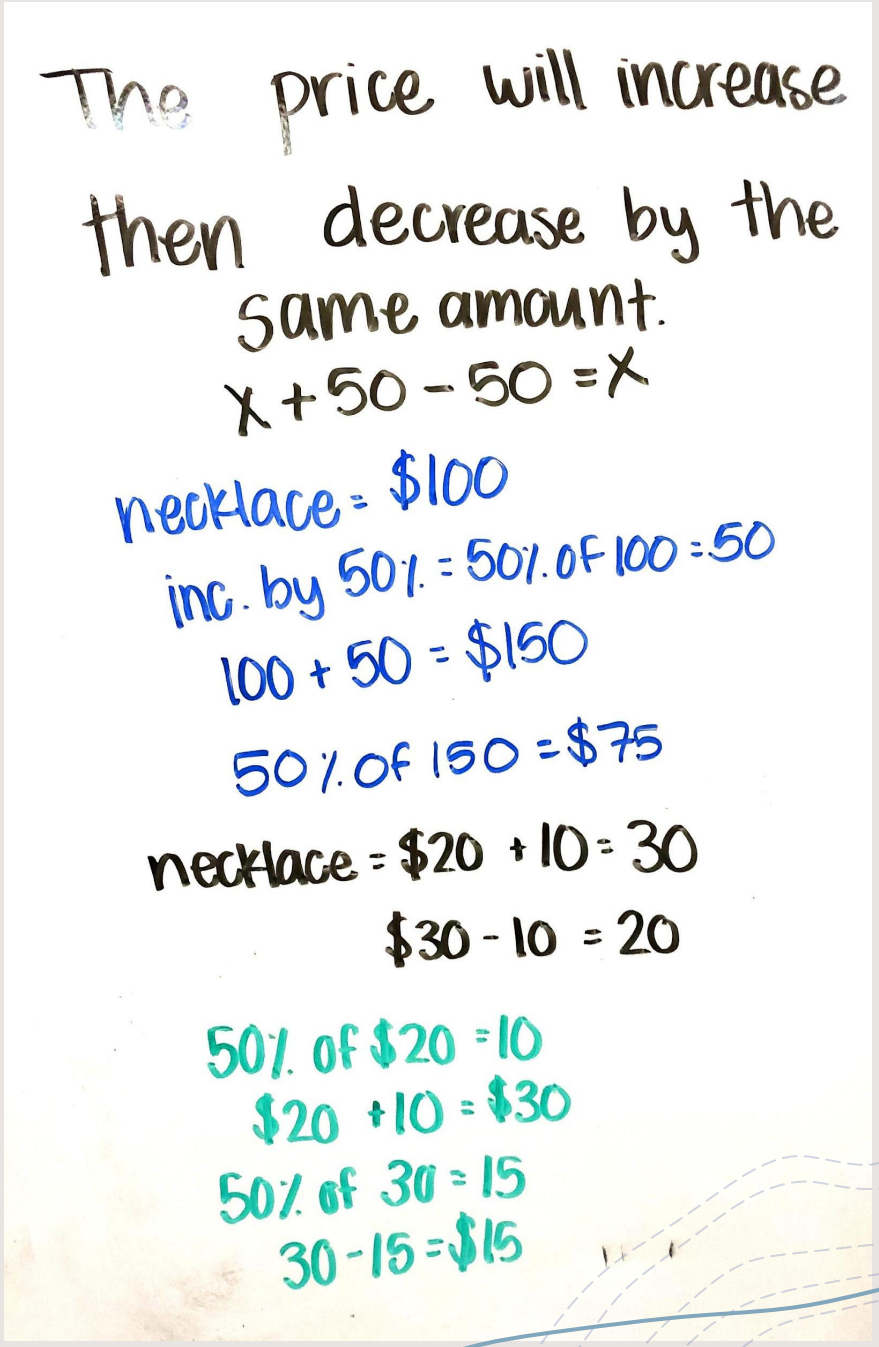
50% of $20 = 10

$20 + 10 = $30

50% of 30 = $15

30 - 15 = $15

Manipulating During Conduct - Particularization



Manipulating During Conduct – Placement

The price will increase then decrease by the same amount

$x + 50 - 50 = x$

necklace = $20 + 10 = 30

$30 - 10 = 20

necklace = $100

inc. by 50% = 50% of 100 = 50

100 + 50 = $150

50% of 150 = $75

50% of $20 = 10

$20 + 10 = $30

50% of 30 = $15

30 - 15 = $15

Manipulating During Conduct - Parallelism

Yes

The price will increase then decrease by the same amount

$x + 50 - 50 = x$

necklace = $20

50% of $20 is $10

inc. by 50% : $20 + $10 = $30

dec. by 50% : $30 - $10 = $20

No

necklace = $100

50% of $100 is $50

inc. by 50% : $100 + $50 = $150

50% of $150 is $75

dec. by 50% : $150 - $75 = $75

necklace = $20

50% of $20 is $10

inc. by 50% : $20 + $10 = $30

50% of $30 is $15

dec. by 50% : $30 - $15 = $15

Where we're going

- convinced of importance
- cognitive load
- lots more to learn

Using Public Records to Support Class Discussion

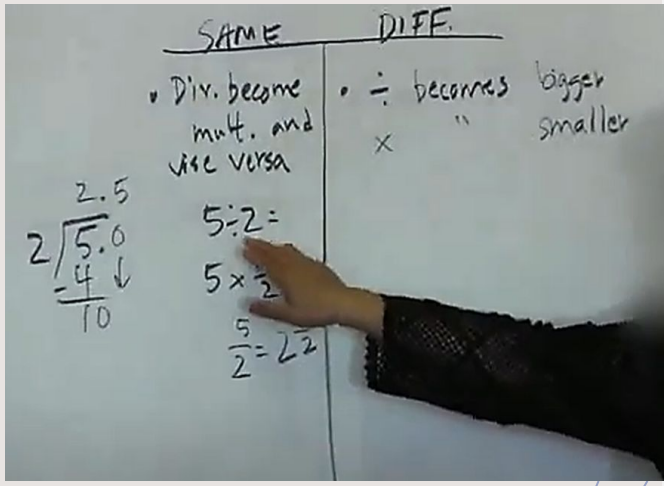
Working with Public Records of Students Mathematical Thinking Routine

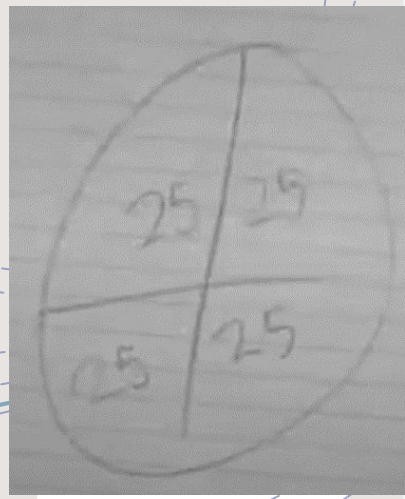
Christina Koehne, Eva Thanheiser, Kate Melhuish

This work is supported by the National Science Foundation grant # DRL 1814114

Our Understanding of Public Record

- A public record is …
  - a physical or visual representation *of* ***student mathematical thinking***
    - *generated by teacher or student(s)*
    - *more involved than memorized facts or short answers*
  - publicly accessible to all participants within the classroom
    - *and remains visible (while students are engaging with it)*
  - *students must engage, or be prompted to engage, with the student thinking*
- A public record is used …
  - as an object of discussion during class discourse
  - to support collective problem solving
    - *students' habits of mind and interaction*





As a Teaching Routine

- A **teaching routine** can be operationalized as "recurring, patterned sequences of interaction teachers and students jointly enact to organize opportunities for student learning in classrooms" (DeBarger et al., 2011, p. 244).
- Time-stamped
  - ON: Record is visible and student begins verbally sharing their thinking (or class is prompted to engage with the thinking)
  - OFF: Record is removed and/or discussion has moved on
- Captures what is happening
  - Catalytic Teaching Habits
  - Other Simultaneous Teaching Routines
  - Students' Habits of Mind
  - Students' Habits of Interaction

Working with Public Records of Students' Thinking and Ties to Equitable Instruction

Positive ties to equitable instruction

- Provides opportunity to center and valorize students' mathematical thinking
- Provides a visual reinforcement that can increase access

Relevant elements of equitable instruction not captured

- Does not capture ***whose*** work is being made public.
- Does not capture ***how the mathematical thinking is positioned*** by the teacher and class

Public Records for Explanations

When we think public record, a first thought might be,

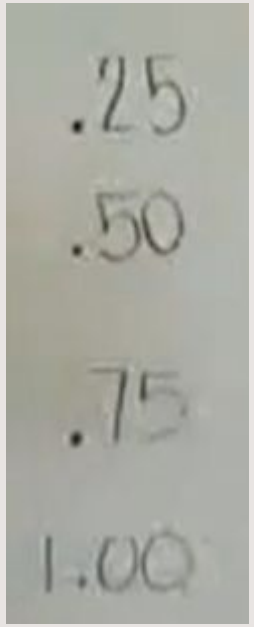
“This is a great way for students to **share** their thinking!”

However, **we go beyond sharing** and focus on only public records that are **prompted for engagement**.

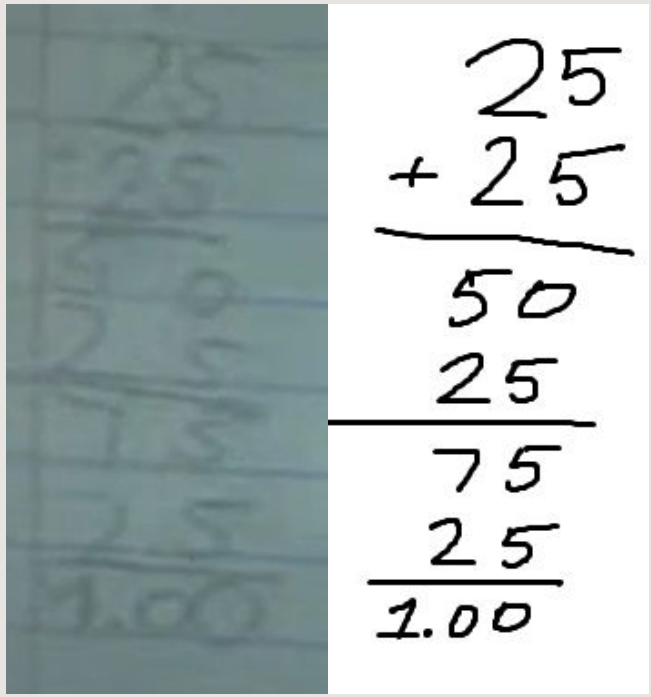
Additionally, **working** with public records can be utilized for an assortment of purposes -- reflected in different patterns of teaching habits, student habits, and routine overlaps.

Example 1:

What patterns can you see?



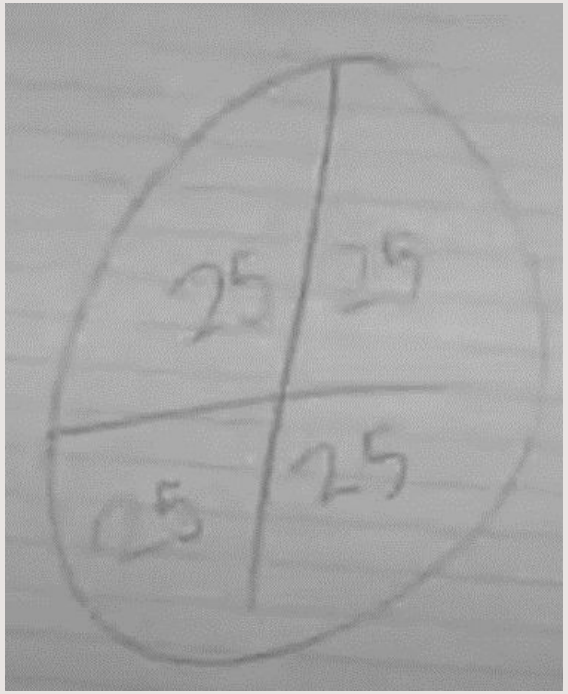
T: Working with public record of students' mathematical thinking

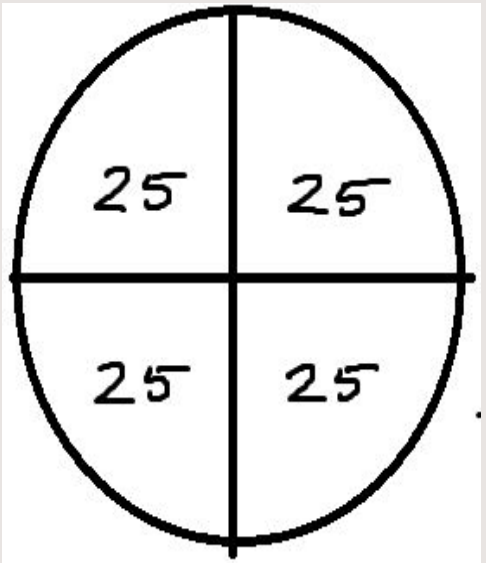


Example 1: Comparing, Connecting

T: Anybody think about this a different way?

S2: That is goes by 4th, if you were to take a hundred…if you were to take a circle, and it's a hundred, and you cut it up into 4 pieces, and you'd have 25, 25, 25, 25, is one.





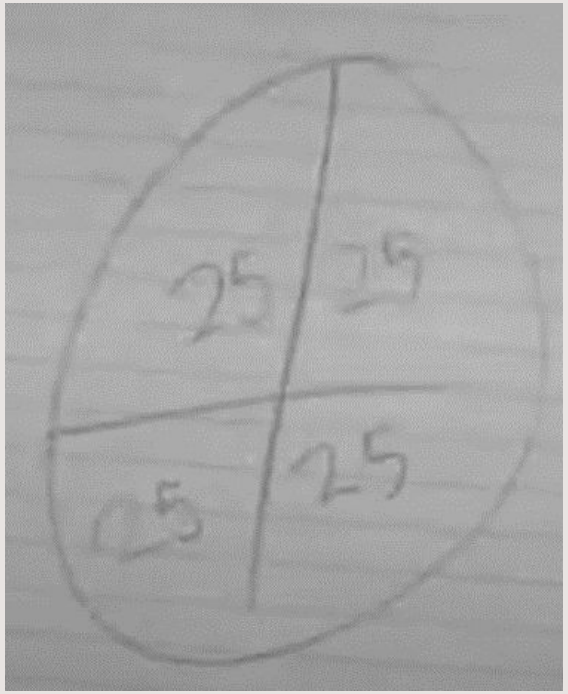
T: …isn't 25+25+25+25 100…

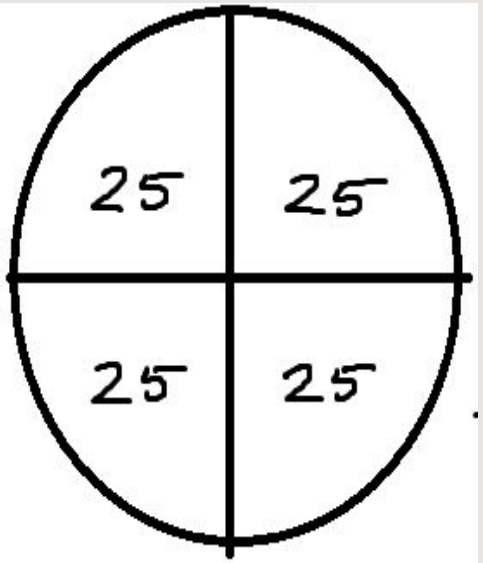
S: I thought of this in terms of money

Example 1: Comparing, Connecting

Teacher : Prompting students to compare or connect across students' reasoning
T: Anybody think about this a different way?

Student : Explore Multiple Pathways
Student: Connections
S2: That is goes by 4th, if you were to take a hundred…if you were to take a circle, and it's a hundred, and you cut it up into 4 pieces, and you'd have 25, 25, 25, 25, is one.



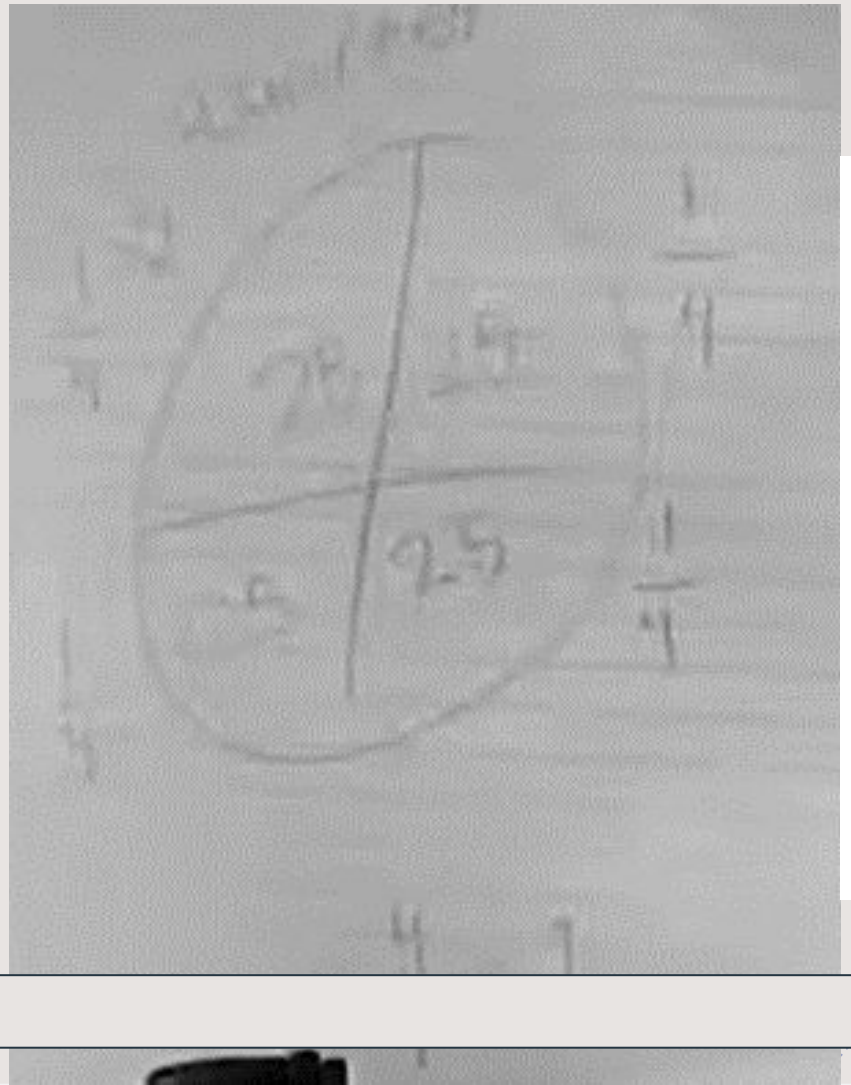


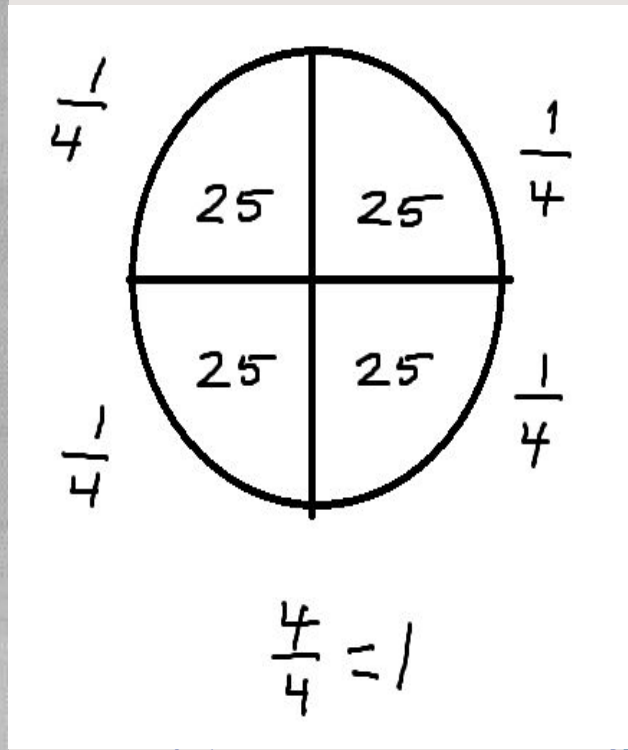
Teacher : Prompting students to analyze contradictions or misconceptions
T: …isn't 25+25+25+25 100…

Student: Explore Multiple Pathways
Student: Meaning of Tasks & Terms
Student: Connections
S: I thought of this in terms of money

Example 1: Comparing, Connecting,

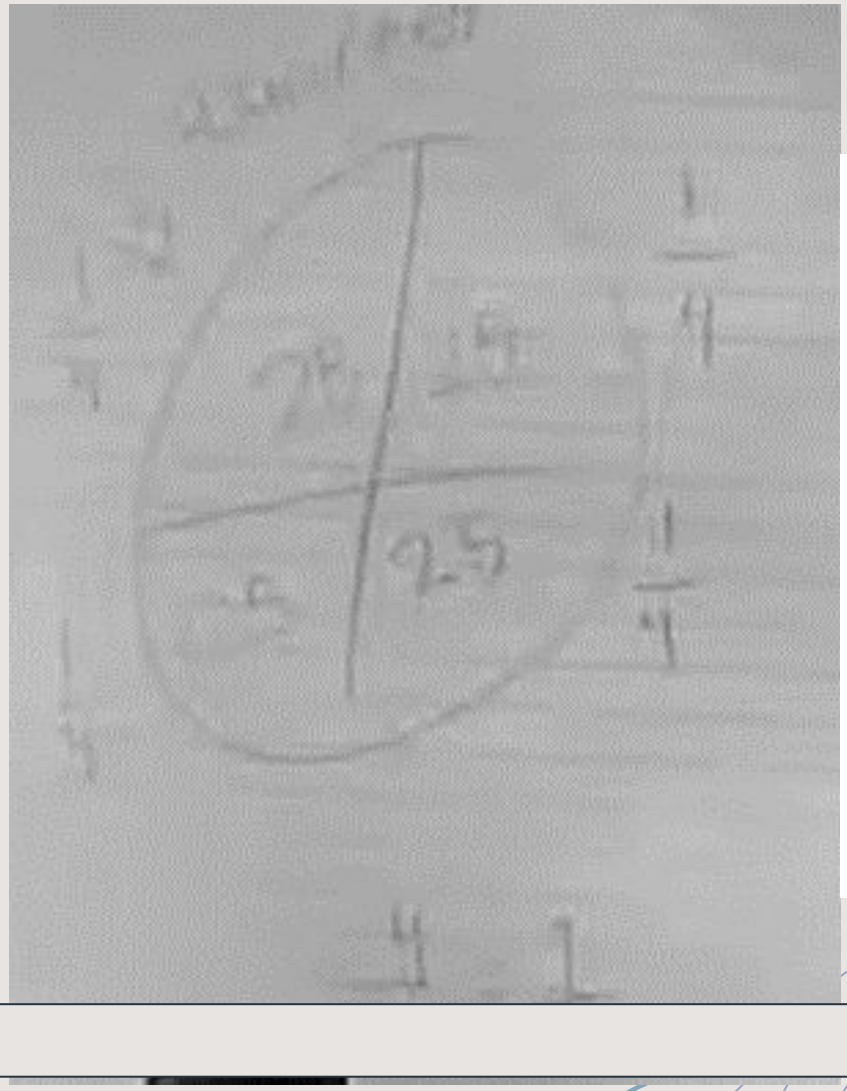
T: when we say ‘quarter’ what would that look like in terms of a fraction

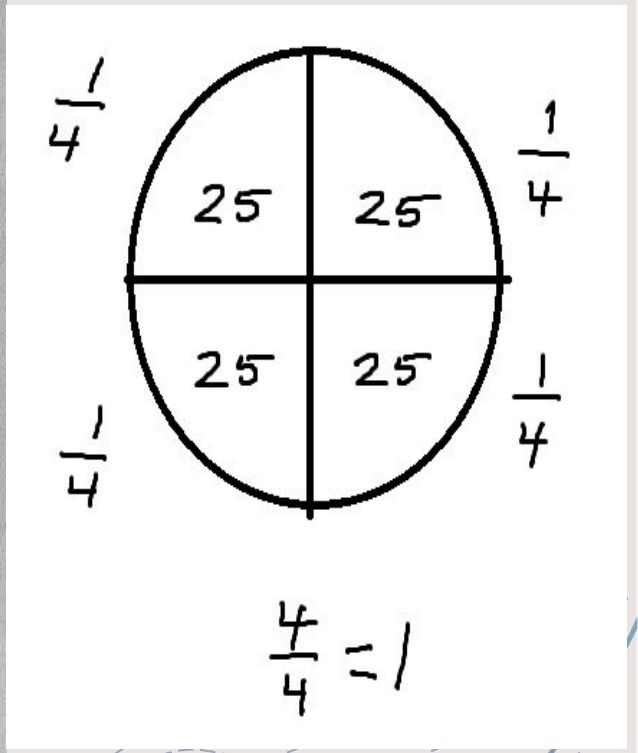




Example 1: Comparing, Connecting,

Teacher : Perceptions of the meanings of specific math concepts or properties
T: when we say ‘quarter’ what would that look like in terms of a fraction

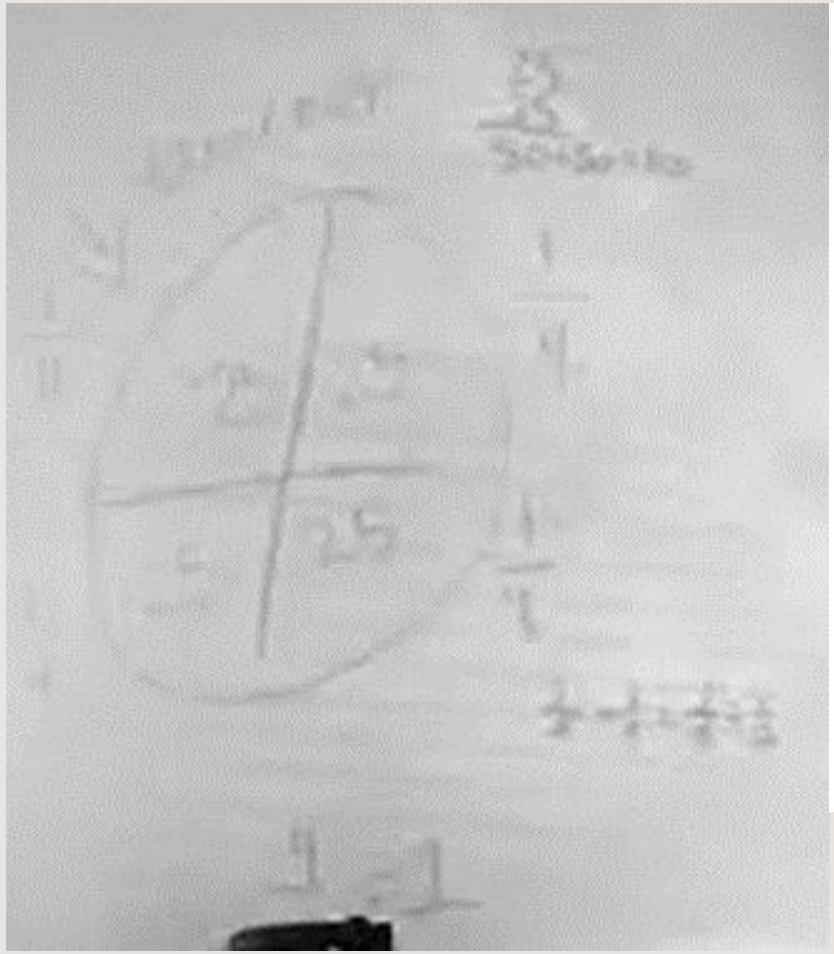


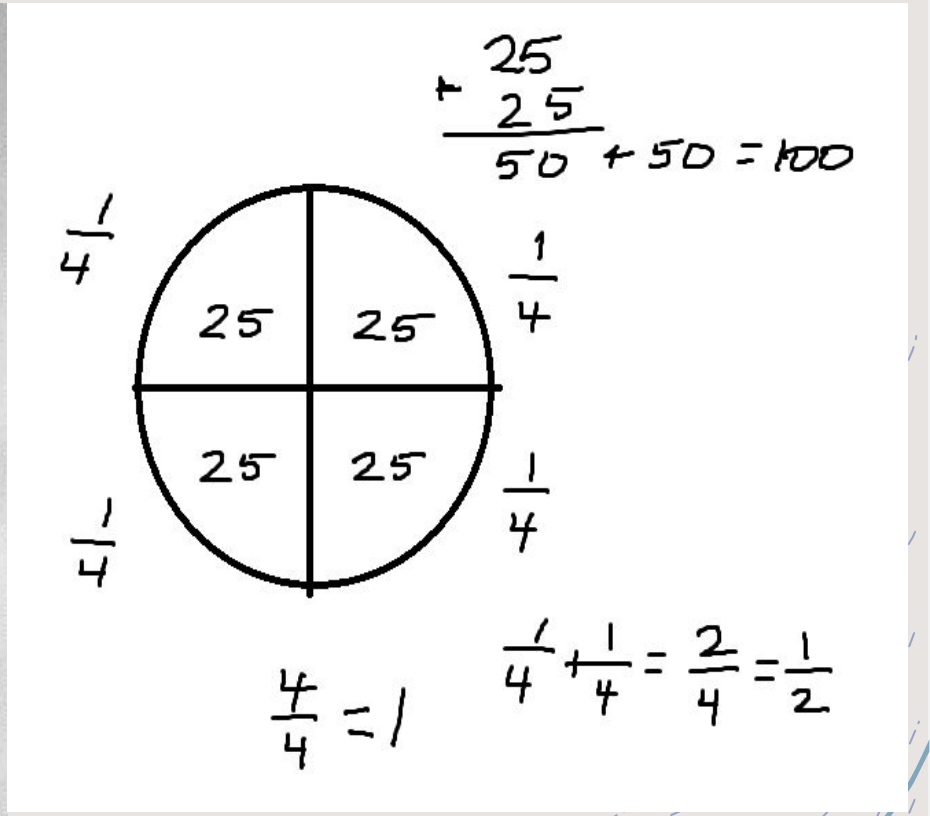


Example 1: Comparing, Connecting, & Justifying

T: Is that equivalent to adding one-fourth plus one-fourth? Can you show why or why not?

S4: I can show that it is equivalent because 25 plus 25 is half, is 50, so half of 100 …

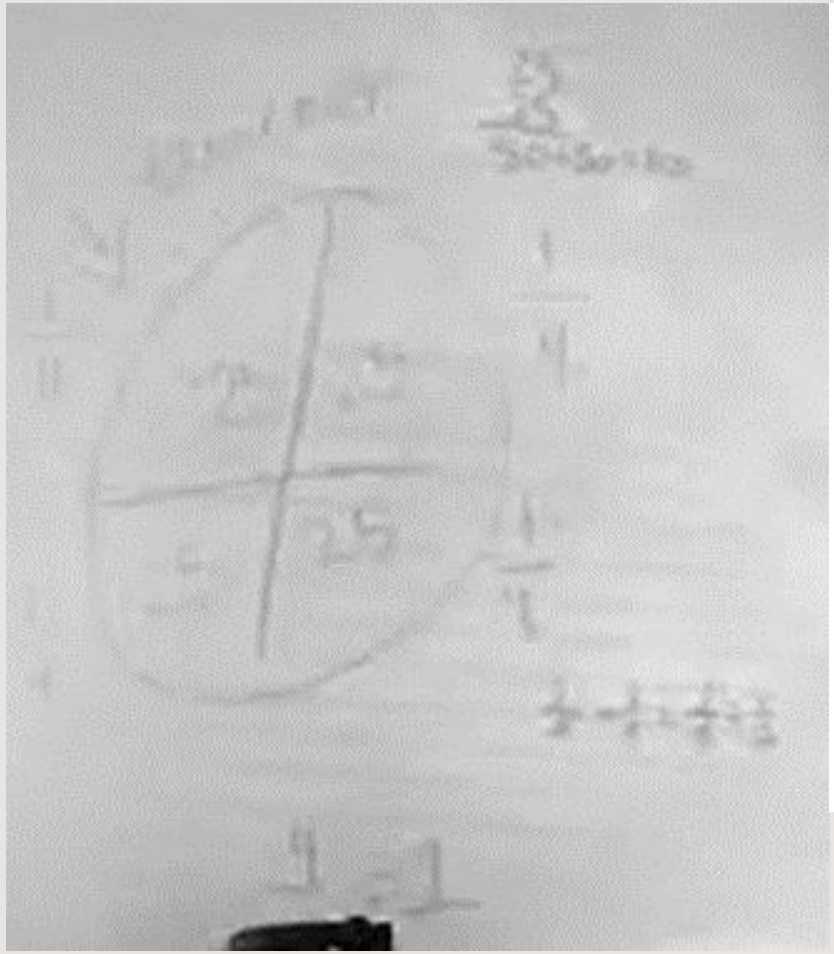


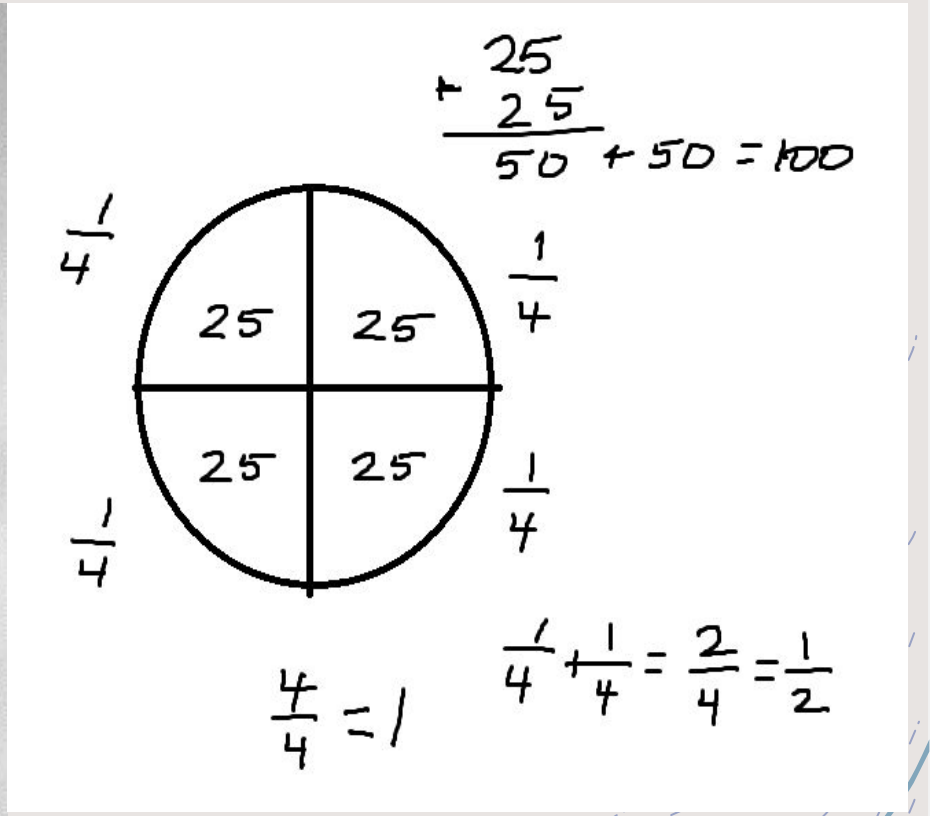


Example 1: Comparing, Connecting, & Justifying

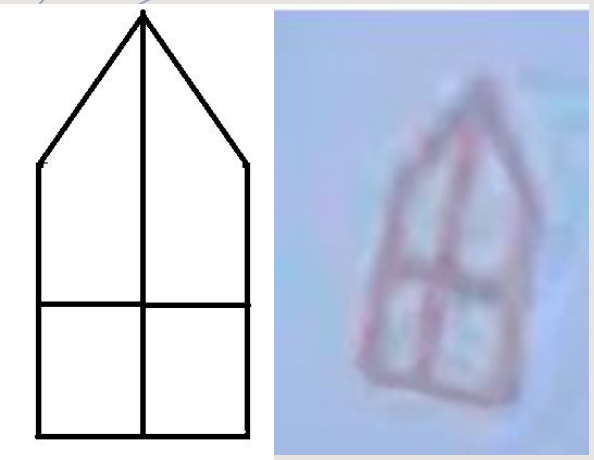
Teacher : Press for Justification
T: Is that equivalent to adding one-fourth plus one-fourth? Can you show why or why not?

Student: Justify
S4: I can show that it is equivalent because 25 plus 25 is half, is 50, so half of 100 …





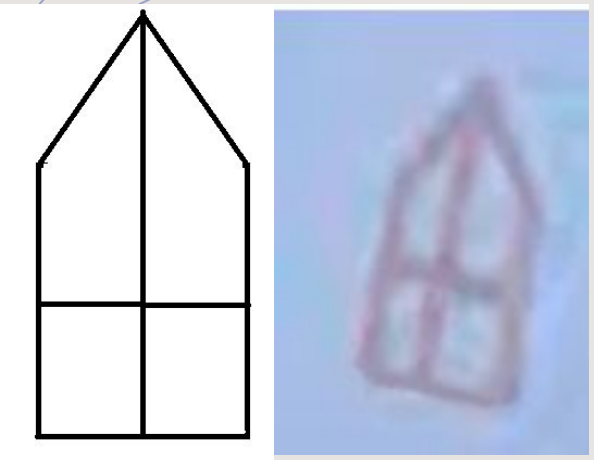
Example 2: Mistakes



T: Does [S1] have two lines of symmetry? Thumbs up if you think yes. Thumbs down if you think no.

S: [Thumbs down]

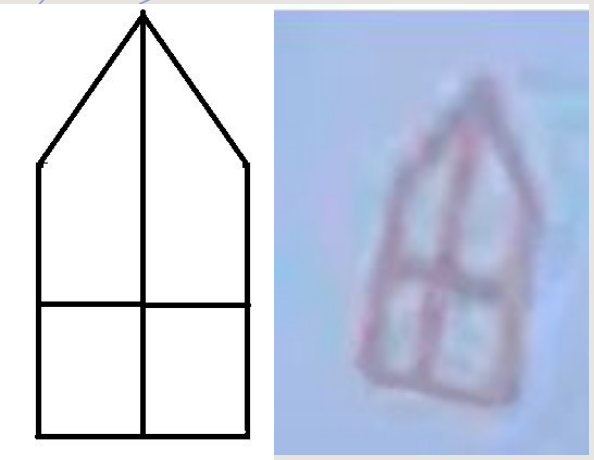
Example 2: Mistakes



T: Engaging in Visual Representation
T: Does [S1] have two lines of symmetry? Thumbs up if you think yes. Thumbs down if you think no.

S: [Thumbs down]

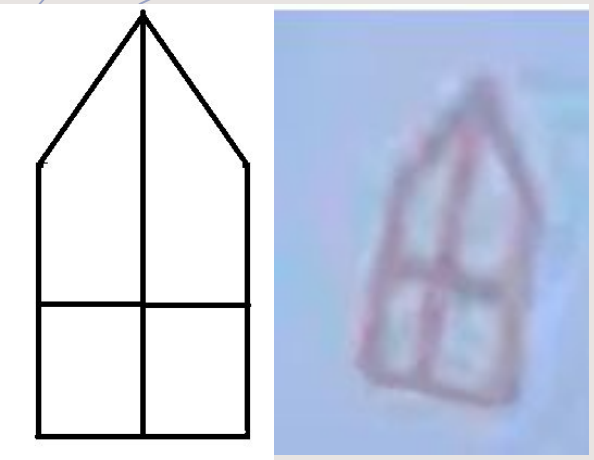
Example 2: Mistakes



T: Why not? ...could somebody give me some mathematical words for why it doesn't work

S: the bottom two are like square and the top two are triangles

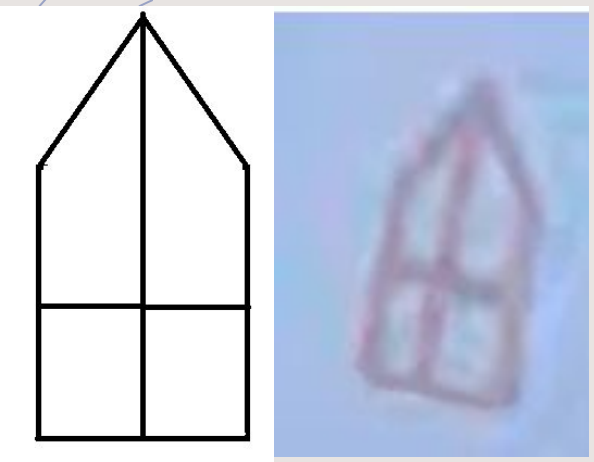
Example 2: Mistakes



T : Prompting students to analyze contradictions or misconceptions
T: Why not? ...could somebody give me some mathematical words for why it doesn't work

S: Critique & Debate
S:Representation
S: the bottom two are like square and the top two are triangles

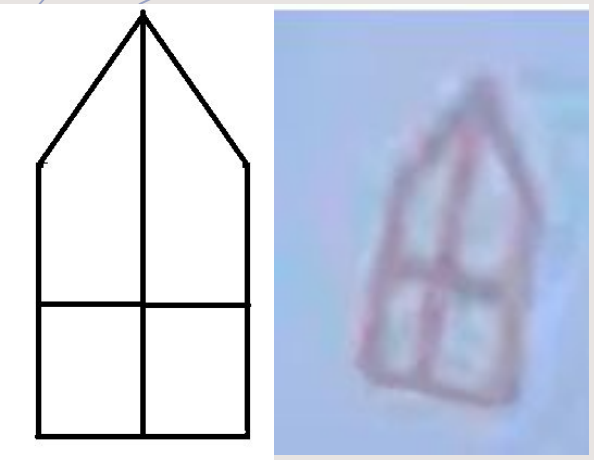
Example 2: Mistakes



T: but what could you have done to this shape?

S: put a triangle at the bottom

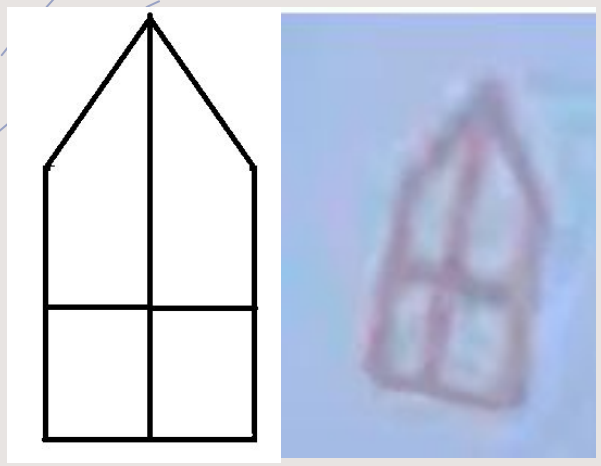
Example 2: Mistakes



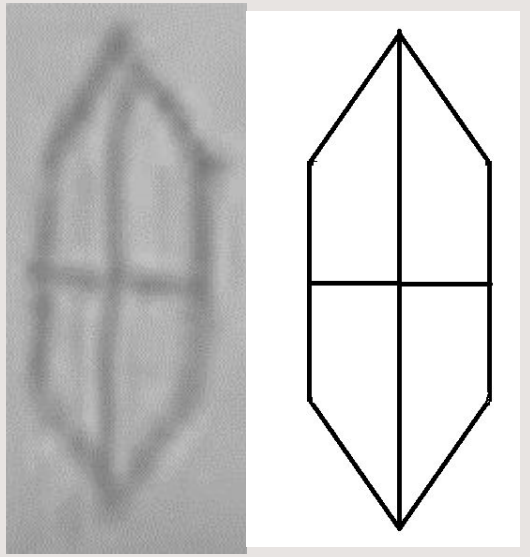
T: but what could you have done to this shape?

S: Representations
S: Explain
S: Mistakes & Stuck Points
S: put a triangle at the bottom

Example 2: Mistakes



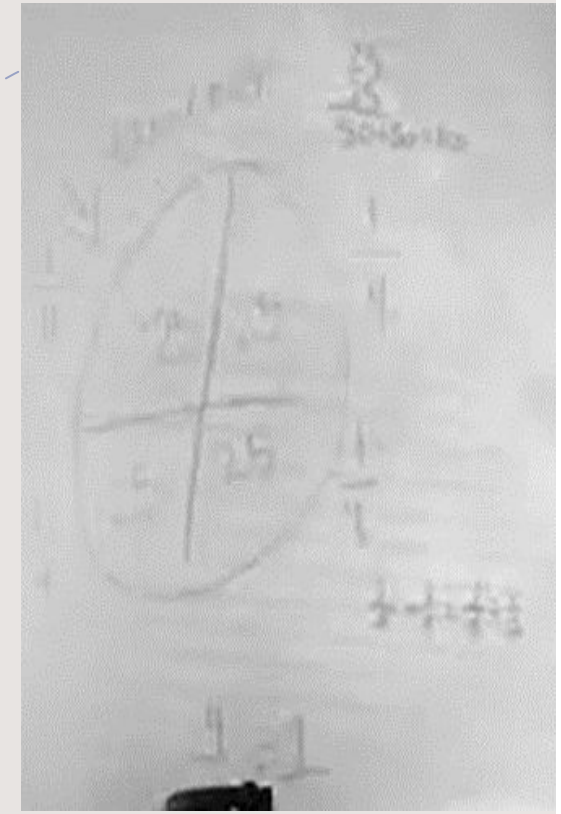
(New Public Record)

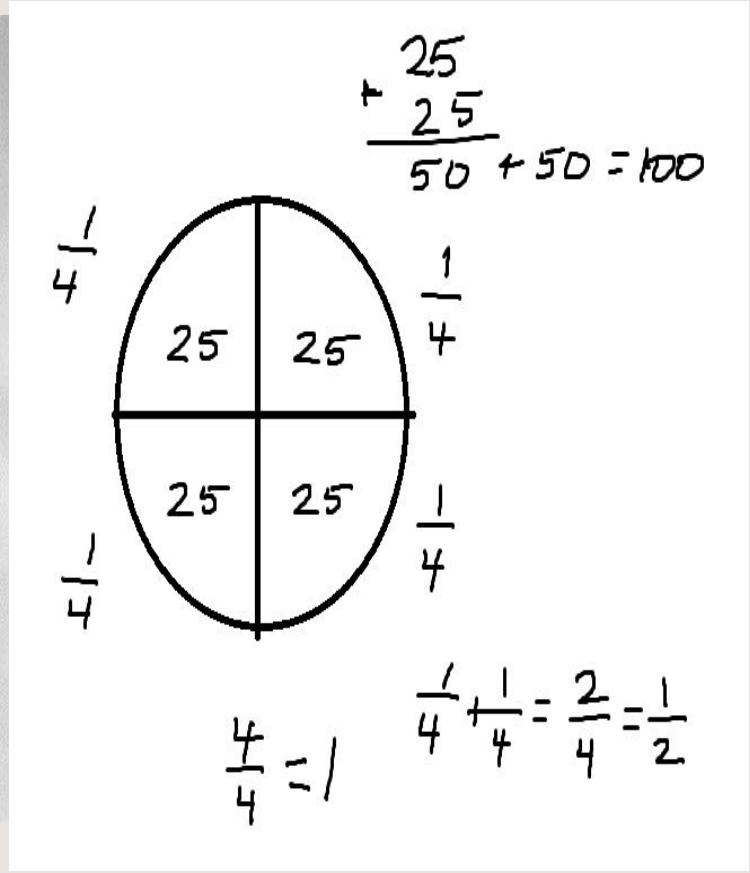


T: Does [S2] have two lines of symmetry? Thumbs up or thumbs down

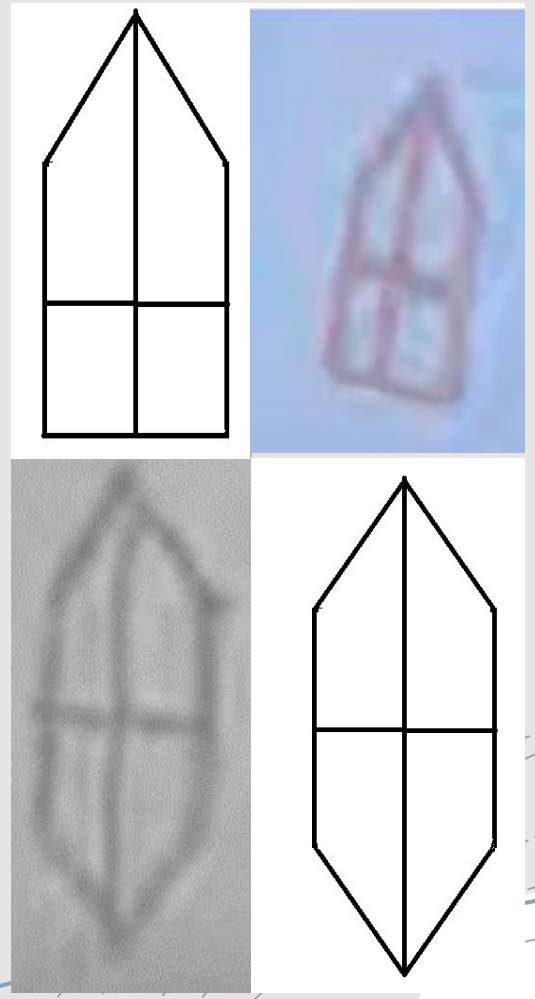
Working with Public Records of Students' Thinking

Example 1: Building a single public record where students are prompted to compare ideas





Example 2: A series of public records where students are prompted to evaluate and critique



Working with Public Records of Students' Thinking

- Focused on how (in terms of prompts) the teachers are **working with the records to engage students in class discussions**

- Working with the public record of thinking can provide opportunities for students to deepen their mathematical reasoning and engagement with one another's ideas.

Using Public Records to Support Class Discussion

Structuring Boardspace to Facilitate Repeated Reasoning

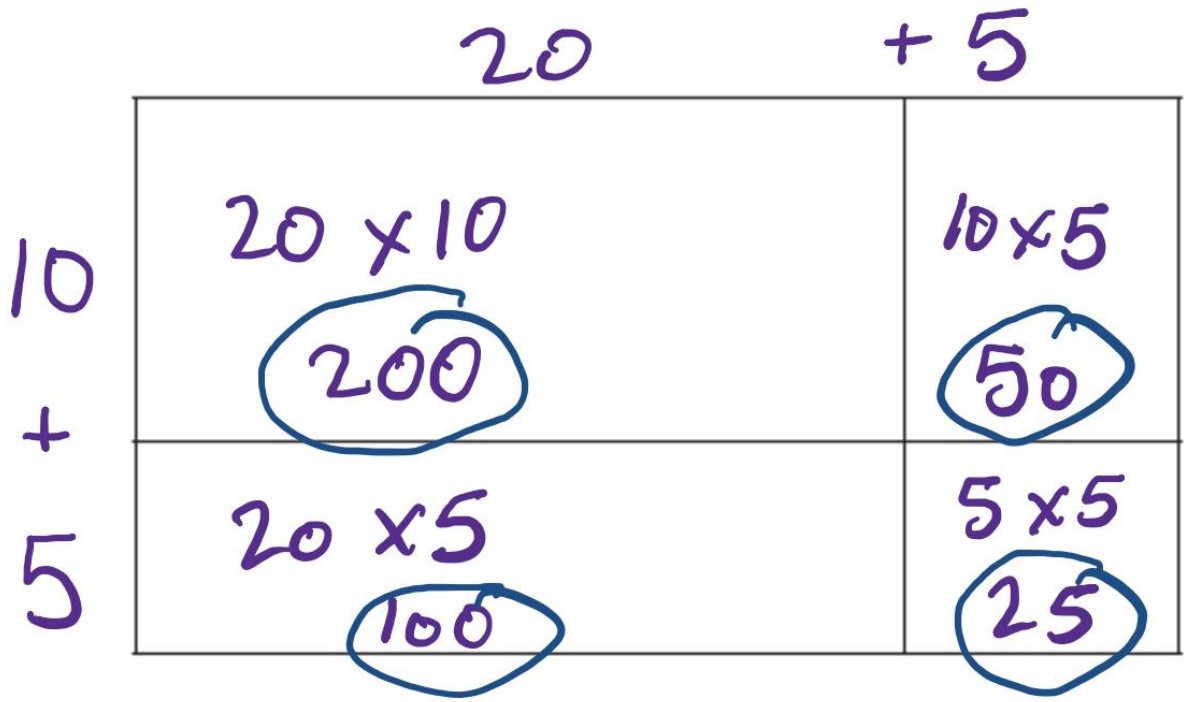
Bill DeLeeuw, Samuel Otten, Ruveyda Karaman Dundar

Our Understanding of Public Record

- A public record is ...
  - a physical and visual representation of student mathematical thinking
    - *(e.g., whiteboards, blackboards, SMARTboards™, poster paper, etc.)*
  - publicly accessible to all participants within the classroom
  - *an integral part of the planned lesson experience*
- A public record is used ...
  - as an object of discussion during class discourse
  - to support collective problem solving
  - *to provide a record of the complete mathematical story and experience of the lesson*
  - *to help students at risk of not being successful*
  - *to support the noticing of mathematical structure and regularity*

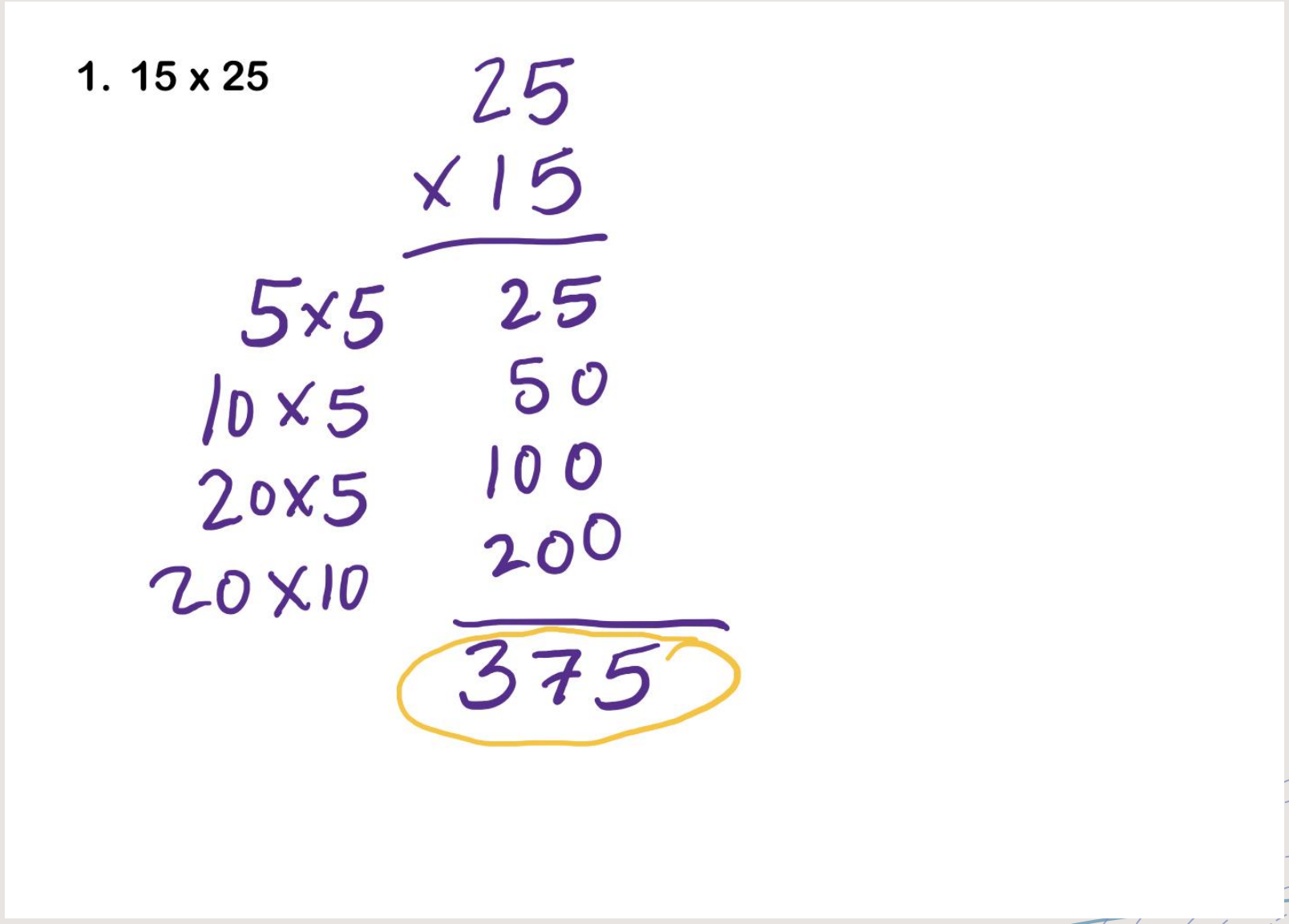
Missed Opportunities with Boardspace

1. 15 x 25



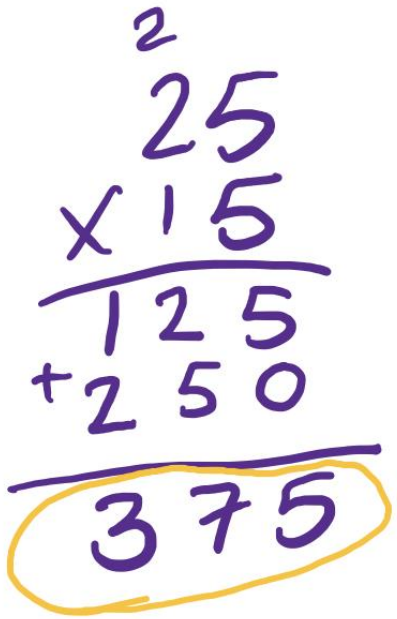
200 + 100 + 50 + 25 = 375

Missed Opportunities with Boardspace



Missed Opportunities with Boardspace

1. 15 x 25



Planful use of Boardspace

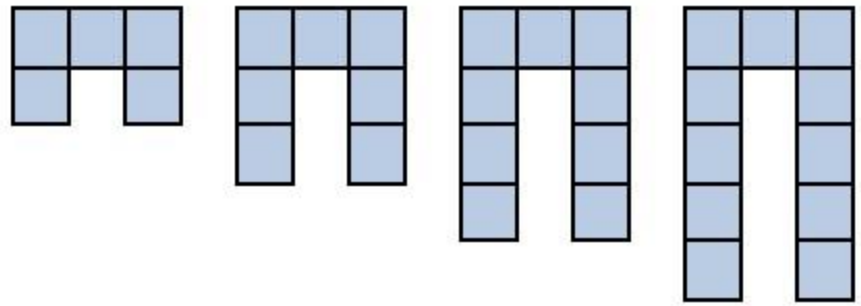
- Planning the lesson so that at the end of the lesson the board "Provide[s] a record of problems and solution methods and principles" (Stigler and Hiebert, 2009, p.95)
- A full board at the end presents students with more opportunities to make connections
- Provides a visual representation of the mathematics discussed that can be viewed as a whole journey and experience rather than a set of individual problems
  - Specifically, mathematical structure and regularity can be moved from the abstract realm to a visual representation
- Facilitates the implementation of tasks that promote reasoning and problem solving, the use and connection of mathematical representations, meaningful mathematical discourse, purposeful questioning, and the display of elicited student thinking (NCTM, 2014).

Structure and Repeated Reasoning

- Standards for Mathematical Practice (SMP) in the Common Core State Standards for Mathematics (CCSSM; NGA Center and CCSSO 2010),
  - *Look for and use structure* (SMP 7)
  - *Look for and express regularity in repeated reasoning* (SMP 8)

Look for Structure and Regularity: Example 1

The Growing Tower Sequence



Case 1 Case 2 Case 3 Case 4 Case 5 Case *n*

1a. Draw the 5th case in the growing pattern of blocks above.

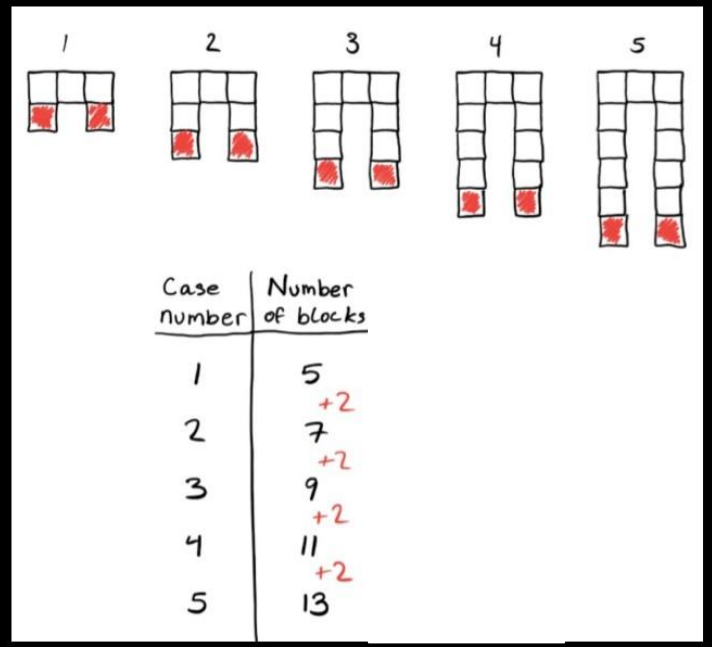
1b. Describe a pattern that you see in this sequence of cases.

1c. Write a rule that will tell you how many blocks are in any case.

2. Which Growing Tower case will have 21 squares?

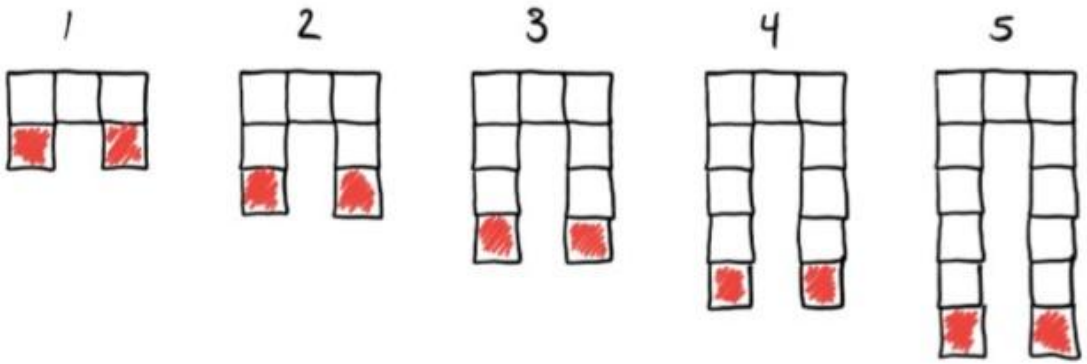
3. Is it possible for a case in the Growing Tower sequence to have 64 squares? Why or why not?

Look for Structure and Regularity: Example 1



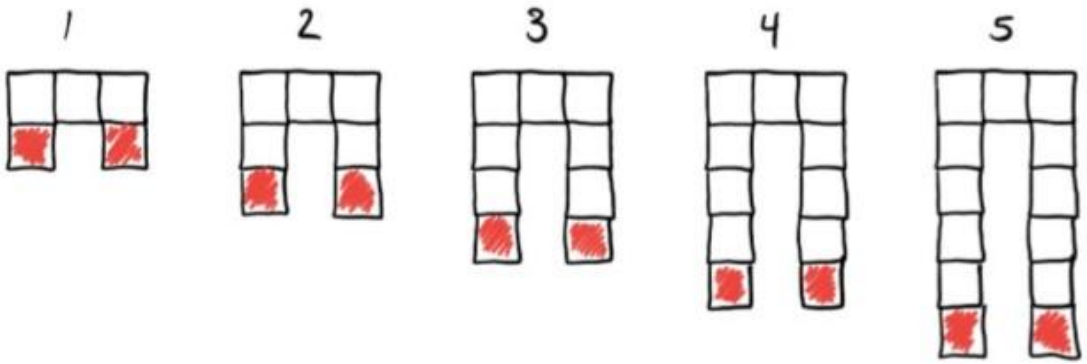
Case number	Number of blocks
1	5
	+2
2	7
	+2
3	9
	+2
4	11
	+2
5	13

Look for Structure and Regularity: Example 1



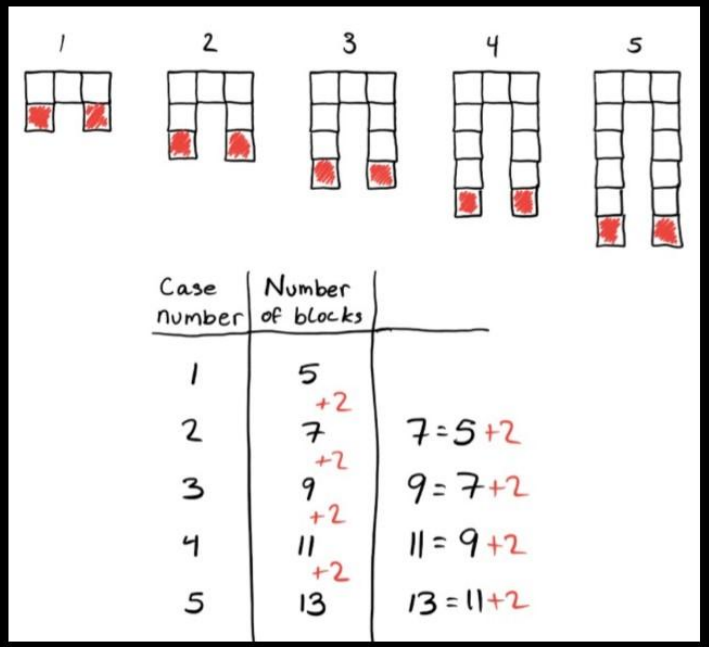
Case number	Number of blocks
1	5
	+2
2	7
	+2
3	9
	+2
4	11
	+2
5	13

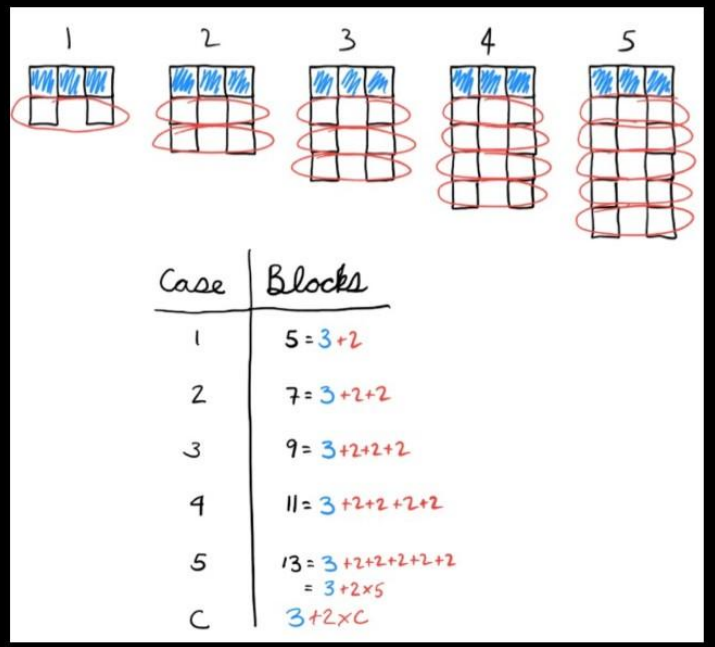
Look for Structure and Regularity: Example 1



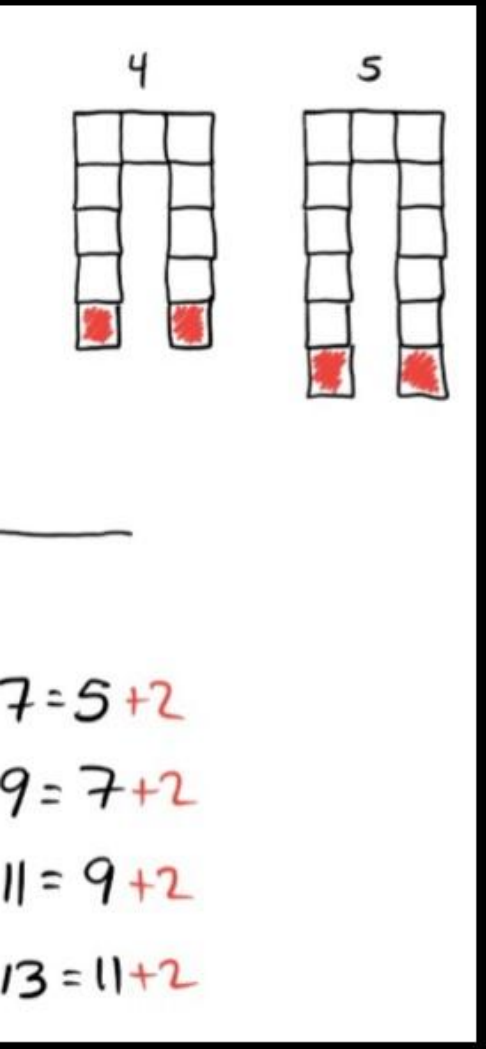
Case number	Number of blocks	
1	5	
	+2	
2	7	7 = 5 + 2
	+2	
3	9	9 = 7 + 2
	+2	
4	11	11 = 9 + 2
	+2	
5	13	13 = 11 + 2

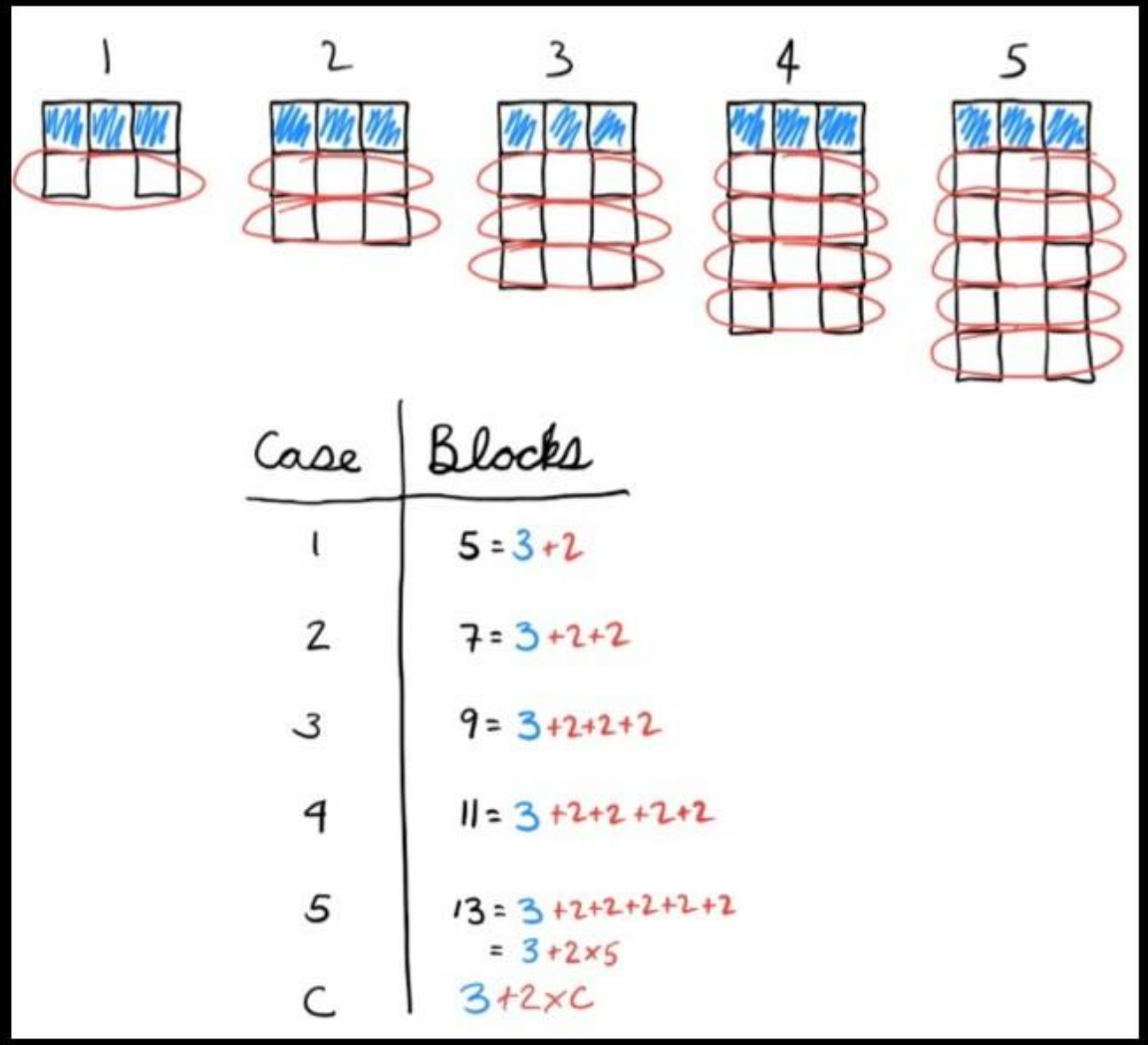
Look for Structure and Regularity: Example 1



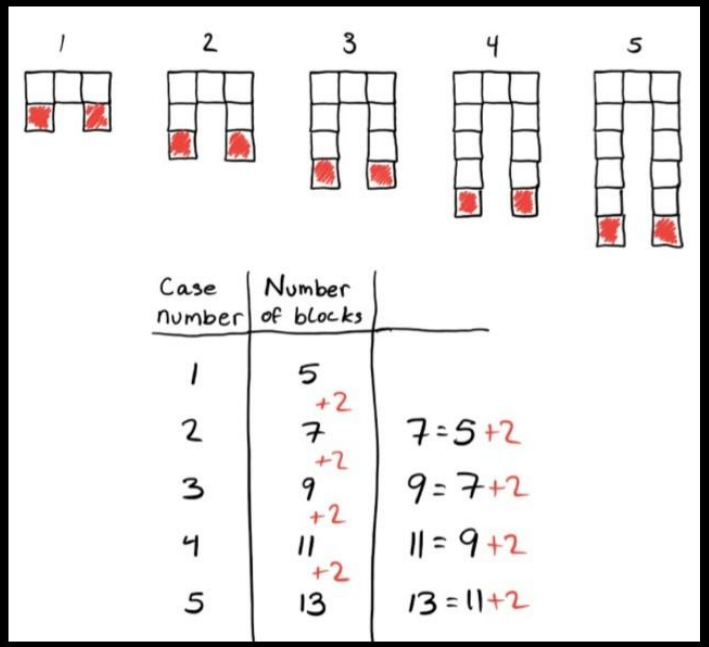


Look for Structure and Regularity: Example 1

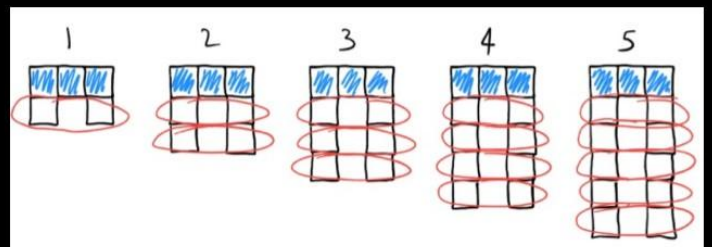




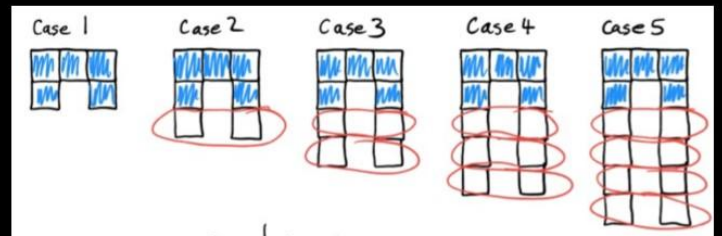
Look for Structure and Regularity: Example 1



Case number	Number of blocks	
1	5	
2	7 (+2)	7 = 5 + 2
3	9 (+2)	9 = 7 + 2
4	11 (+2)	11 = 9 + 2
5	13	13 = 11 + 2

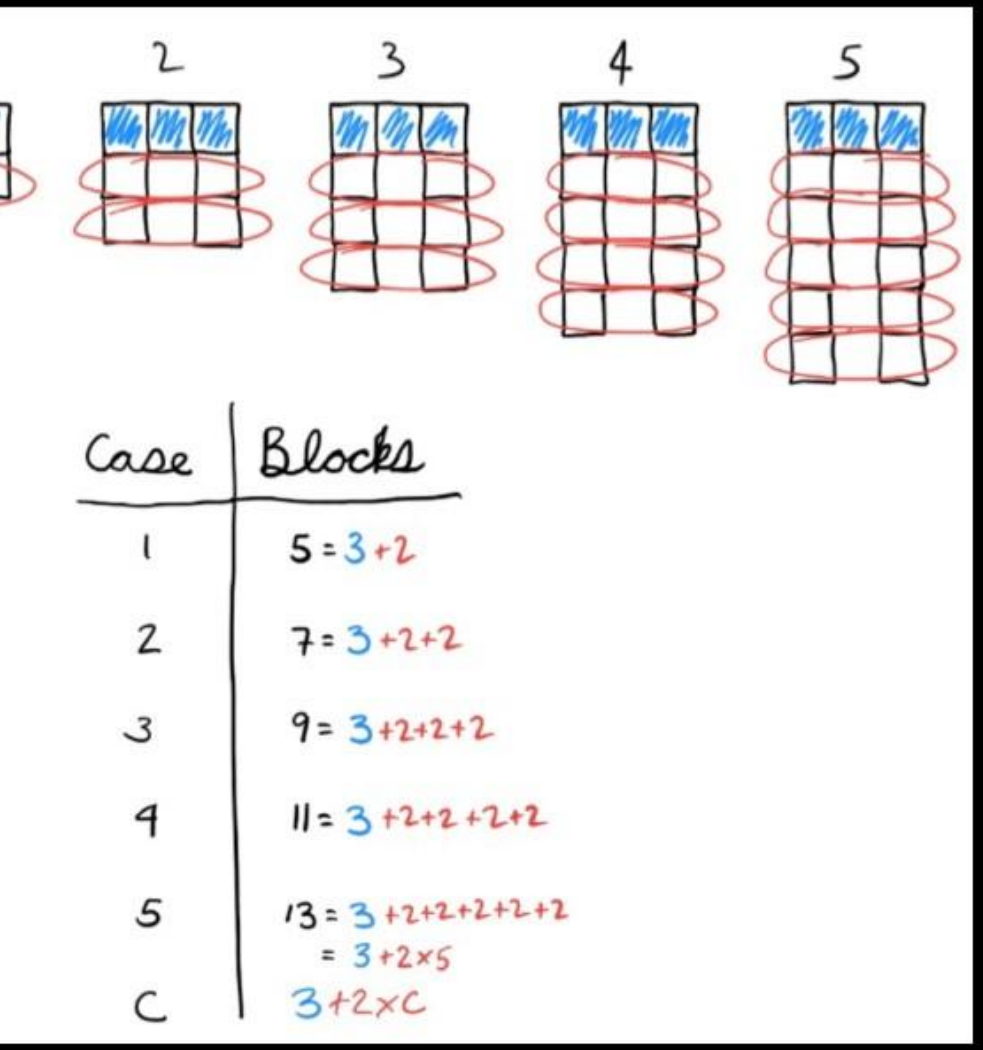


Case	Blocks
1	$5 = 3 + 2$
2	$7 = 3 + 2 + 2$
3	$9 = 3 + 2 + 2 + 2$
4	$11 = 3 + 2 + 2 + 2 + 2$
5	$13 = 3 + 2 + 2 + 2 + 2 + 2$ $= 3 + 2 \times 5$
C	$3 + 2 \times C$

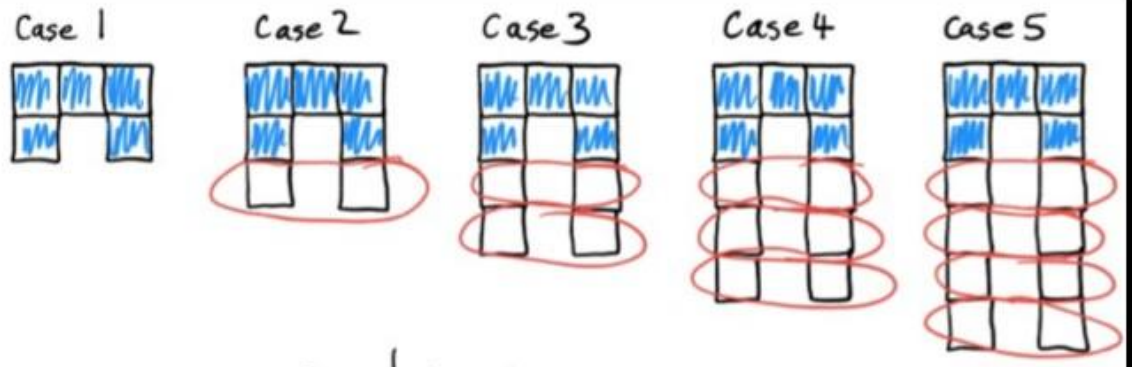


n	total
1	5
2	$7 = 5 + 2$
3	$9 = 5 + 2 + 2$
4	$11 = 5 + 2 + 2 + 2$
5	$13 = 5 + 2 + 2 + 2 + 2$ $= 5 + 2 \times$
n	$5 + 2 \times (n - 1)$

Look for Structure and Regularity: Example 1

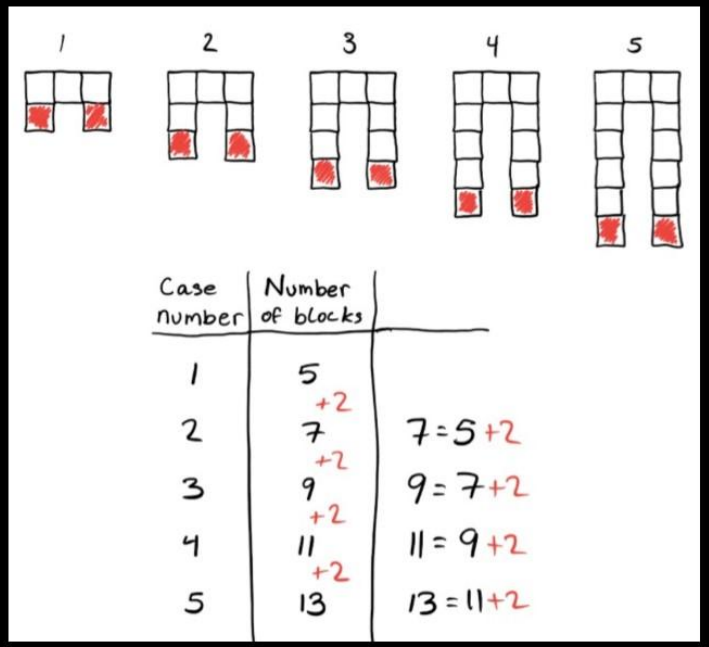


Case	Blocks
1	5 = 3 + 2
2	7 = 3 + 2 + 2
3	9 = 3 + 2 + 2 + 2
4	11 = 3 + 2 + 2 + 2 + 2
5	13 = 3 + 2 + 2 + 2 + 2 + 2 = 3 + 2 × 5
C	3 + 2 × C

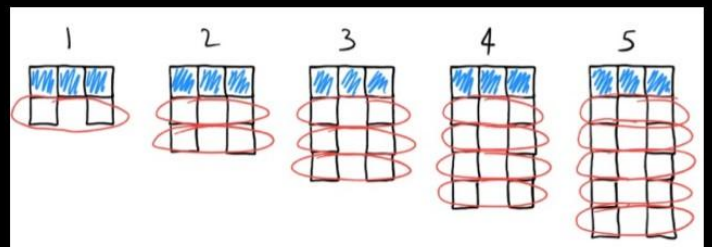


n	total
1	5
2	7 = 5 + 2
3	9 = 5 + 2 + 2
4	11 = 5 + 2 + 2 + 2
5	13 = 5 + 2 + 2 + 2 + 2 = 5 + 2 ×
n	5 + 2 × (n − 1)

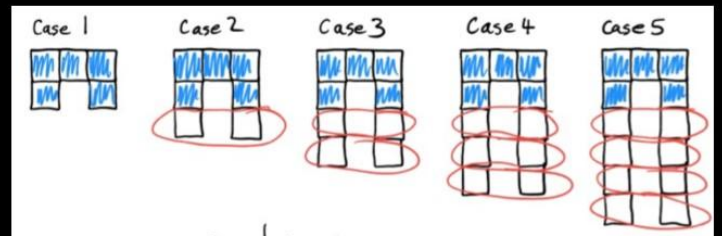
Look for Structure and Regularity: Example 1



Case number	Number of blocks	
1	5	
2	7 (+2)	7 = 5 + 2
3	9 (+2)	9 = 7 + 2
4	11 (+2)	11 = 9 + 2
5	13	13 = 11 + 2



Case	Blocks
1	$5 = 3 + 2$
2	$7 = 3 + 2 + 2$
3	$9 = 3 + 2 + 2 + 2$
4	$11 = 3 + 2 + 2 + 2 + 2$
5	$13 = 3 + 2 + 2 + 2 + 2 + 2$ $= 3 + 2 \times 5$
C	$3 + 2 \times C$

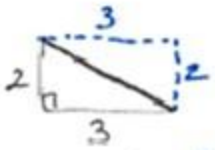


n	total
1	5
2	$7 = 5 + 2$
3	$9 = 5 + 2 + 2$
4	$11 = 5 + 2 + 2 + 2$
5	$13 = 5 + 2 + 2 + 2 + 2$ $= 5 + 2 \times$
n	$5 + 2 \times (n - 1)$

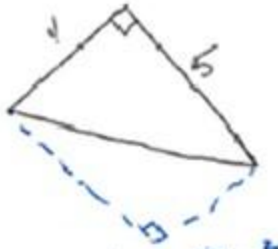
Look for Structure and Regularity: Example 2

Triangle Area $= \frac{1}{2}$ base · height

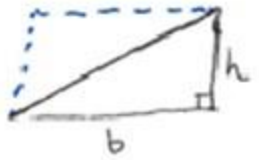
CASE 1: RIGHT



Rectangle = 6 un²
So Triangle = $\frac{1}{2}(6) = 3$ un²



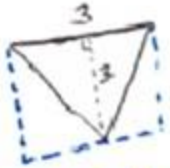
Rectangle = 20 un²
Triangle = $\frac{1}{2}(20) = 10$ un²



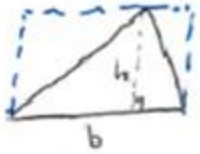
Area of Triangle = $\frac{1}{2}bh$

CASE 2: ACUTE

Big Rectangle = 30 un²
Small Δ's half of small □'s
So Big Triangle = $\frac{1}{2}$ Big Rectangle = 15 un²

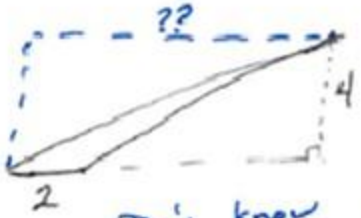


Big Rectangle = 9 un²
Big Triangle = $\frac{9}{2}$ un²



Area of Triangle = $\frac{1}{2}bh$

CASE 3: OBTUSE



Don't know the Rectangle area.

Area of Triangle = $\frac{1}{2}bh$

Key Elements of Structuring Boardspace to Facilitate Repeated Reasoning

+ *Moves the abstract structure and repeated reasoning into the visual realm.*
+ *Provides a picture of the lesson as a whole.*
+ *Planning boardspace use facilitates effective incorporation of student thinking.*

What is next for Structuring Boardspace to Facilitate Repeated Reasoning

- How do we help teachers develop the skill of planning their use of boardspace with specific goals in mind (e.g. structure and repeated reasoning)?
- How much and in what ways do students benefit when teachers planfully use boardspace?

Using Public Records to Support Class Discussion

Representing

Michael D. Hicks, Virginia Tech

Christina Koehne, SUNY New Paltz

Mai Bui, Texas State University

Jessica Bishop, Texas State University

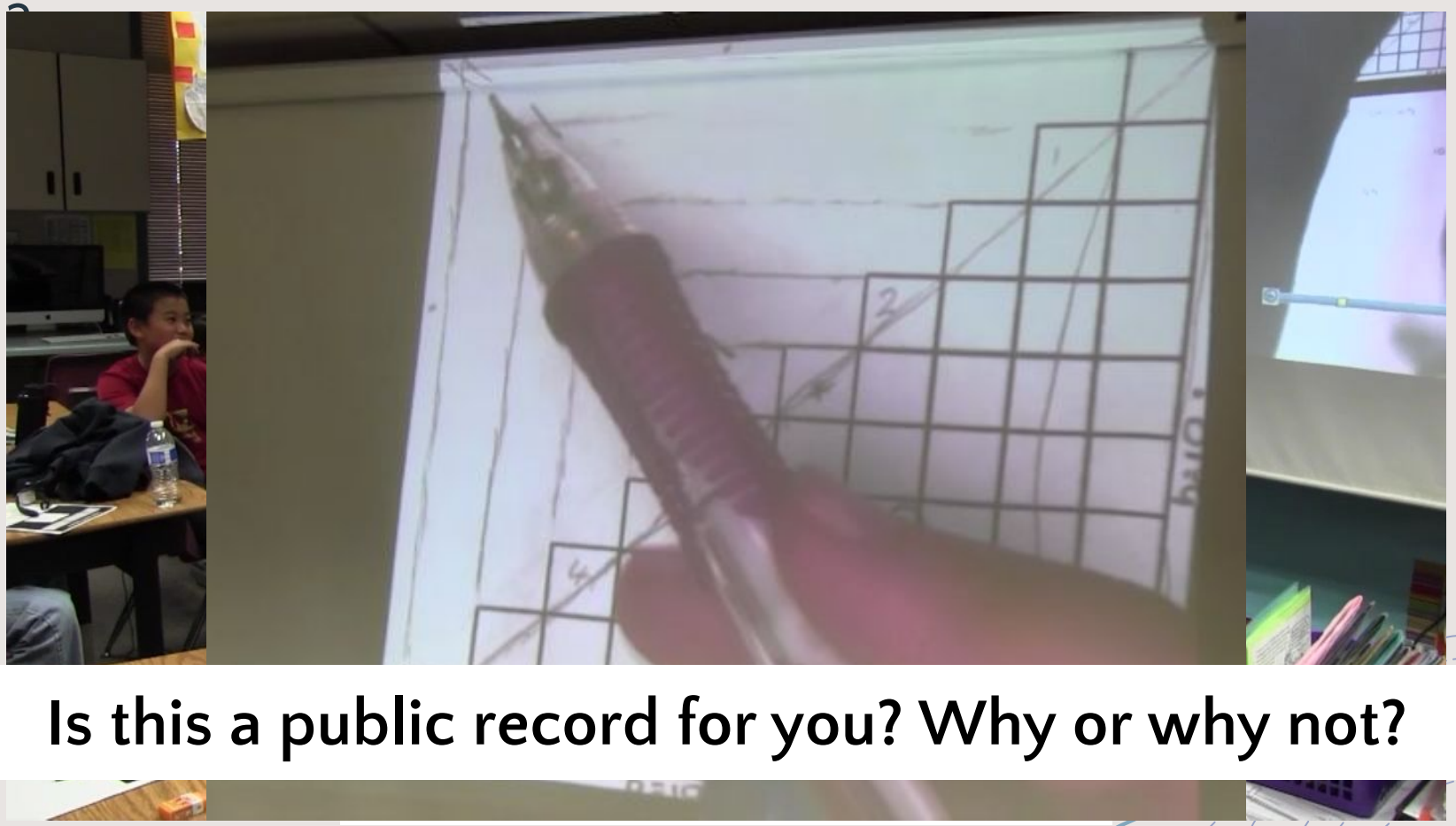
This work is supported by the National Science Foundation grant # DRL 1649979

Our Understanding of Public Record

- Our focus is not on public records, per se, but mathematical authority (Benne, 1970).
- *Who* has opportunities to represent public records is a significant marker of authority.
- We use the term *Representing* instead of *Public Record.*

An Example of Representing a Public Record

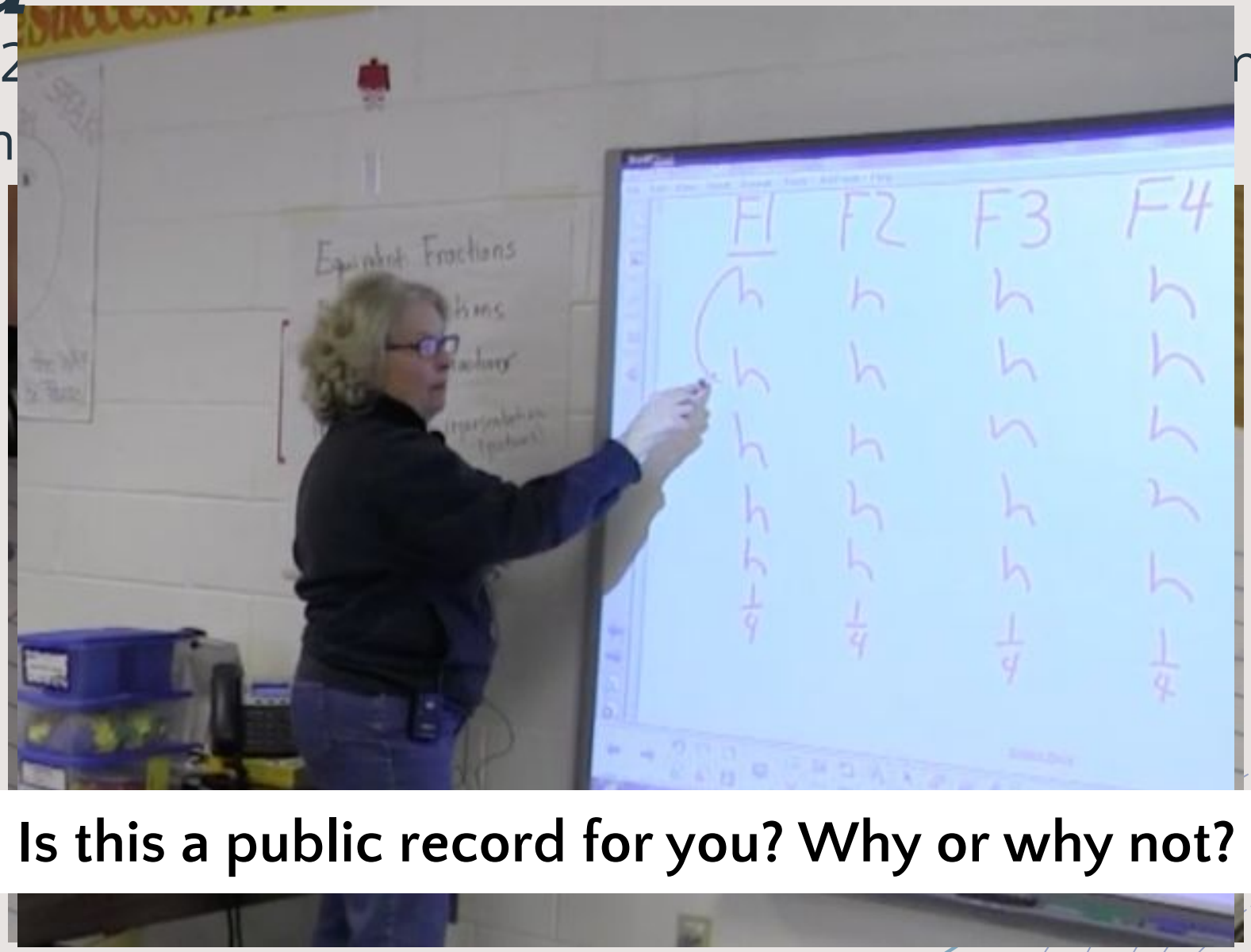
Example 1: As a contractor you specialize in outdoor brick stairwells. How many bricks will you need to build a 10-brick-high stairwell?



Is this a public record for you? Why or why not?

An Example of Representing a Public Record

Example 2 nuch will each frien

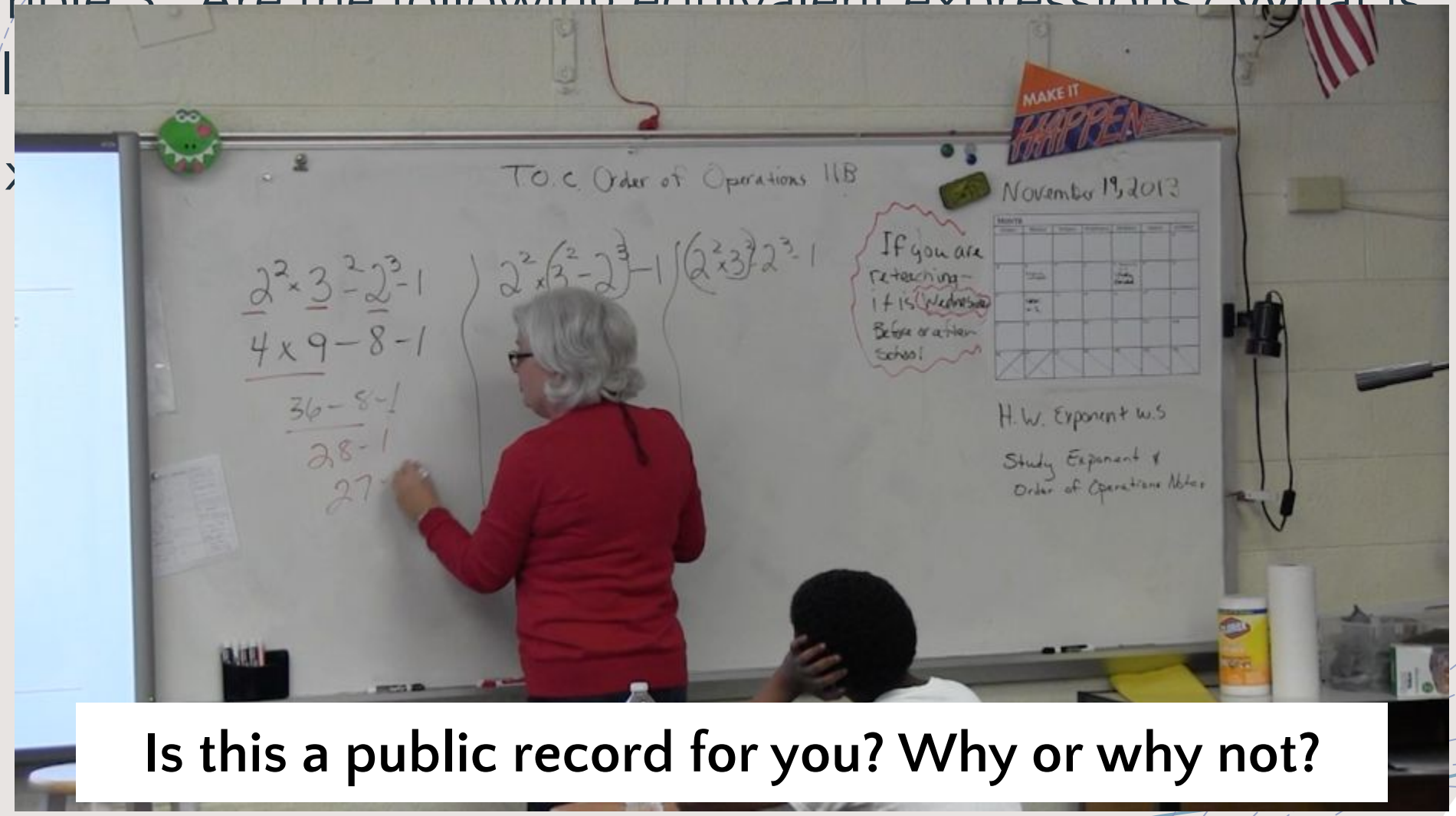


Is this a public record for you? Why or why not?

An Example of Representing a Public Record

Exam
simil

$2^2 \times$



Is this a public record for you? Why or why not?

Identifying Instances of Public Records

- Turn and talk (1 minute)
- Consider the following questions
  - Which of these examples is a public record for you?
  - What are your criteria for a public record?

Revisiting Our Understanding of Public Record

- We focus on the *activity of representing* rather than the product (i.e., the public record itself)
- We do not restrict our focus only to student thinking. Instead, we account *for all public records,* regardless of whose idea is represented and who generated the public record.

- Authoring: The creation of a mathematical idea.
- Representing: A form of communication that is visually observable, publicly accessible, and mathematically meaningful.

Differentiating the Examples

Example #	Author Code	Representer Code
1	Student	Student

Differentiating the Examples

Example #	Author Code	Representer Code
1	Student	Student
2	Student	Teacher

Differentiating the Examples

Example #	Author Code	Representer Code
1	Student	Student
2	Student	Teacher
3	Teacher	Teacher

Looking Across a Broader Data Set

- We coded for author and representer during whole-class instruction (WCI) in 57 algebra lessons across 11 teachers.
- How often do you think representing occurs in a lesson?
  - Representing occurs during ***86%*** of WCI.
- How often do you think student ideas are represented?
  - Student ideas were represented during ***40%*** of WCI
- How often do you think students play a role in representing their own ideas?
  - Students played a role in representing their own ideas during ***19%*** of WCI.
- One extension of this work is to disaggregate the collective 'student' category into specific individuals to track who has authority for what and how those opportunities are distributed across ethnicity, gender, race, etc.

Shared Authority for Authoring and Representing: Another Example

- Students need opportunities to author and represent mathematics.

- However, teachers can productively share authority for authorship and representation with students.

- Example 4: What is – – x?
- In other words, what is negative negative x?

Shared Authority for Authoring and Representing: Another Example

https://txst-my.sharepoint.com/:v:/g/personal/jp1139_txstate_edu/EX5dIHbpf_BCjX9i3mAmeB0B07JUmfmQtn2WBFH-283jyA?e=VEz6xe

Analyzing the Last Example

Example #	Author Code	Representer Code
4	Both	Both

Using Public Records to Support Class Discussion

Using Public Records When Building on MOSTs

Blake Peterson, Keith Leatham, Shari Stockero, Laura Van Zoest

Working with Public Records of Students Mathematical Thinking Routine

Christina Koehne, Eva Thanheiser, Kate Melhuish

Structuring Boardspace to Facilitate Repeated Reasoning

Bill DeLeeuw, Samuel Otten, Ruveyda Karaman Dundar

Representing

Michael Hicks, Christina Koehne, Mai Bui, Jessica Bishop

Discussion Question

- How do the different ways public records are created, organized, and used influence the learning environment?

- How do these ways of seeing public records support student learning?
  - public records of student thinking
  - public records of mathematical representations
  - public records that highlight structure
  - public record as a measure of authority