BEYOND THE "MOVE": A SCHEME FOR CODING TEACHERS' RESPONSES TO STUDENT MATHEMATICAL THINKING

Blake E. Peterson[1], Laura R. Van Zoest[2], Annick O. T. Rougée[3], Ben Freeburn[2], Shari L. Stockero[4] and Keith R. Leatham[1]

[1]Brigham Young University, [2]Western Michigan University, [3]University of Michigan, [4]Michigan Technological University

To contribute to the field's understanding of the teachers' role in using student thinking to shape classroom mathematical discourse, we developed the Teacher Response Coding Scheme (TRC). The TRC provides a means to analyze teachers' in-the-moment responses to student thinking during instruction. The TRC differs from existing schemes in that it disentangles the teacher move from the actor *(the person publically asked to consider the student thinking), the* recognition *(the extent to which the student recognizes their idea in the teacher move), and the* mathematics *(the alignment of the mathematics in the teacher move to the mathematics in the student thinking). This disentanglement makes the TRC less value-laden and more useful across a broad range of settings.*

Researchers (e.g., Fenemma et al., 1996) have found that teachers' use of student thinking during mathematics instruction supports student learning of mathematics. Both researchers (e.g., Franke, Kazemi, & Battey, 2007; Van Zoest, Peterson, Leatham, & Stockero, 2016) and recommendations for mathematics teaching (e.g., National Council of Teachers of Mathematics [NCTM], 2014) assert that teachers' use of student thinking undergirds features of effective mathematics instruction, such as classroom mathematical discourse. While the field benefits from research identifying how teachers may plan for and use written records of student work to facilitate whole-class mathematical discourse (Stein, Engle, Smith, & Hughes, 2008), less is known about how teachers respond in-the-moment to instances of students' mathematical thinking. We report here on a coding scheme designed to capture teachers' in-the-moment responses to instances of student mathematical thinking. Such a scheme could contribute to better understanding the role of the teacher in shaping meaningful mathematical discourse in their classrooms.

THEORETICAL PERSPECTIVES & RELATED LITERATURE

Current thinking about effective teaching and learning of mathematics as put forth by NCTM (2014) suggests fundamental ideas related to productive use of student mathematical thinking. As discussed elsewhere (Van Zoest et al., 2016), we see embedded in this document four core principles of quality mathematics instruction: (1) mathematics is at the forefront, (2) students are positioned as legitimate mathematical thinkers, (3) students are engaged in sense-making, and (4) students work

2017. In Kaur, B., Ho, W.K., Toh, T.L., & Choy, B.H. (Eds.). *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education,* Vol. 4, pp. 17-24. Singapore: PME.

collaboratively. These four principles form the basis of our theoretical perspective. As such they both provided a lens for examining existing research related to in-the-moment teacher responses to student mathematical thinking during whole-class interactions and informed the development of our coding scheme.

We found three themes in the literature related to teacher responses to student thinking: (1) student engagement in classroom communication, (2) responsiveness, and (3) attention to mathematics. These themes suggest important components to attend to in teacher responses, yet existing research seems to foreground only one of these components at a time. For example, Franke et al. (2009) foregrounded engagement by analyzing the way particular types of teacher questioning moves engaged students' in classroom communication. Bishop, Hardison, and Przybyla-Kuchek (2016) coded teachers' moves and student contributions to analyze teachers' responsiveness—the degree to which mathematical ideas in students' contributions were attended to by teachers' subsequent responses. Conner, Singletary, Smith, Wagner, and Francisco (2014) coded teachers' actions (moves) in support of collective argumentation, foregrounding the mathematics in the teacher responses. In general, existing research measures teacher responses against the particular component the researchers are foregrounding by incorporating that component into their definition of "move." In order to develop a more nuanced coding scheme, we disentangled these three components of a teacher's response from the teacher move. This disentanglement allows us to measure teacher responses against the core principles of our theoretical perspective, and provides a structure for other researchers to investigate teacher responses from their theoretical perspectives.

METHODOLOGY

Data for this paper come from a larger project (see LeveragingMOSTs.org) and included 278 instances of high-potential student mathematical thinking during whole-class interactions identified in 11 videotaped mathematics lessons from 6-12th grade classrooms that reflected the diversity of teachers, students, mathematics, and curricula present in US schools (Van Zoest et al., 2017). In addition, we analyzed 43 *Scenario Interviews* (Stockero et al., 2015) that involved teachers responding to a common set of eight instances of student thinking.

First, we articulated the student mathematics and mathematical point for each instance of student mathematical thinking. Student mathematics (SM) is defined as "a clearly articulated statement of an inference of what a student has expressed mathematically in the instance" (Van Zoest et al., 2017, p. 36). A mathematical point (MP) is "the *mathematical understanding* that (1) students could gain from considering a particular instance of student thinking and (2) is most closely related to the SM of the thinking" (Van Zoest et al., 2016, p. 326). We define a *mathematical understanding* (MU) to be *a well-specified statement of a mathematical truth*.

Next, we identified the teacher response to each instance of student mathematical thinking in our data. We define a *teacher response* as *the collection of observable* *teacher actions that begins as a given instance of student mathematical thinking ends and ends when that instance of student mathematical thinking is no longer the focus*[1] *of the observable teacher actions*. For coding purposes, a teacher response may be subdivided into a series of teacher moves, each serving different instructional intents.

The resulting teacher responses in the videos and Scenario Interviews were the data for, and from, which our coding scheme was developed. We used constant comparative analysis (Glaser, 1965) to revisit and refine the codes until each response was authentically captured by the coding scheme.

RESULTS

Our disentanglement of the three themes in the literature from teacher moves led to the *Teacher Response Coding Scheme (TRC)*. Figure 1 lists the TRC coding categories and their relation to the literature themes. In Figure 2 we provide an illustrative instance of student thinking, the inferred student mathematics (SM) and the related mathematical point (MP) of the instance, and four possible teacher responses to this instance. In the following subsections we make connections between the TRC coding categories and literature themes and use the teacher responses in Figure 2 to illustrate these categories and their codes.

Category	**Coding Category Description**	**Literature theme**
Actor	Who is publically asked to consider the student thinking	Engagement
Recognition	The extent to which the student who contributed the thinking is likely to recognize their idea in what is being considered	Responsiveness
Mathematics	The extent to which the focus is on improving students' understanding of the MP of the instance of student thinking	Attention to mathematics
Move	What the actor is doing or being asked to do with respect to the instance of student thinking	

Figure 1: TRC coding categories and their connections to the literature.

Context: While working on a problem that related the amount of money accumulated by saving both a one-time gift and babysitting money that was earned weekly, a student said during class discussion, "I put the money on the bottom and weeks on the side."
Instance: "I put the money on the bottom and weeks on the side."
Student Mathematics (SM): I put the money on the x-axis and weeks on the y-axis.
Mathematical Point (MP): The placement of the variables on the axes of a graph is determined by what makes the most sense in the problem situation given the established convention of the x-axis representing the independent variable.

	Teacher Response	**Actor**	**Recognition: Actions**	**Recognition: Ideas**	**Math**	**Move**
1	"Remember, we always put the independent variable on the x-axes."	Teacher	Not	Peripheral	Peripheral	Correct
2	"Did anyone label the axes a different way?"	Whole class	Implicit	Core	Cannot Infer	Collect
3	[To same student] "Why is the amount of weeks dependent on the amount of money which you put on the bottom?"	Same student	Explicit	Peripheral	Core	Justify
4	[To another student] "And what do I like to do first when I make a graph?"	Other student	Not	Other	Cannot Infer	Literal

Figure 2: Coding for teacher responses to an instance of mathematical thinking.

Actor

To capture *who* is likely to be engaging in the intellectual work in response to student mathematical thinking, the Actor category answers the question, "Who is publicly invited or allowed to consider the instance of student thinking?" It does this with the following four codes: *teacher*, *same student(s), other student(s)*, and *whole class*. To illustrate distinctions among these codes, consider the sample teacher responses (TR) to the instance in Figure 2. In order to respond the teacher is likely to first privately consider the instance on some level. However, in TR1, "Remember we always put the independent variable on the x-axes," only the *teacher* engages in publicly considering the instance of student thinking. In contrast, TR2, "Did anyone label the axes a different way?" publically invites the *whole class* to consider the instance.

Recognition

To operationalize the *responsiveness* of teachers' responses to student thinking, we considered the extent to which the student who provides the instance would recognize their idea in the teacher's response. Through our iterative work in the data we noticed two distinct ways in which this recognition might occur in a teacher response: through attention to Student Action and attention to Student Ideas. The subcategory Student Action encompasses the exact, unique words a student has used (verbal), as well as any gestures or work provided by the student (non-verbal). The codes (*explicit*, *implicit*, or *not*) for student action capture the degree to which the teacher response uses the student action. To explore the subtle distinction between a response coded as *implicit* and one coded as *explicit*, consider TR2 and TR3. In TR3, the teacher uses language unique to that student instance ("on the bottom"). In contrast, in TR2 the teacher does not use this unique language, replacing "put," "money," and "weeks" with the verb "label," and replacing "on the bottom" and "in the side" with the term "axes." Hence, TR3 *explicitly* uses the student's actions while TR2 *implicitly* uses the student actions. Responses that do not use the student's unique actions or clear replacements (such as TR1 and TR4) are coded as *not*.

The subcategory Student Ideas focuses on the mathematical idea(s) the student is putting forth in the instance. The codes (*core, peripheral, other, cannot infer*, and *not applicable*) for this category capture the extent to which the student is likely to recognize their idea in the teacher response. For example, TR2 focuses on the labelling of the axes, which is the *core* idea in the instance of student thinking. On the other hand, TR1 and TR3 start to veer from this main idea to a *peripheral* or related idea—the connection between the labels of a graph and the independent-dependent relationship between the variables. In contrast, TR4 focuses the actor on considering what the teacher likes to do first when they make a graph. This focus is not related, even peripherally, to the student's idea and hence, we code TR4 as going towards an idea *other* than the core idea in the instance of student thinking. Responses that do not engage with the instance of student thinking (e.g., "Let's not talk about that now.") are coded *not applicable*.

Mathematics

In order to gauge the extent to which a teacher response focuses on improving student understanding of the mathematical point (MP), the Mathematics category documents the alignment between the mathematical understanding (MU) that is the focus of the teacher response to an instance of student thinking and the MP of that instance. The codes are: *core, peripheral, other, cannot infer*, *non-mathematical* or *not applicable.* For example, TR2 and TR4 illustrate responses that are coded as *cannot infer*. In both of these responses, it is not yet evident what MU the teacher is pursuing. In contrast, in TR3 the MU seems to be the MP (see Figure 2) of the student thinking and hence the mathematics of the teacher response is *core.* TR1 focuses on the mathematical conventions of labelling axes, thus the MU may be articulated as, "By convention, the x-axis represents the independent variable and the y-axis represents the dependent variable." Though this MU is contained in the MP, it focuses on following the convention rather than on deciding which variable is independent and which is dependent. Thus this MU is *peripheral* to the MP. When a teacher response has an MU that is not even peripherally related to the MP of the instance, it is coded as *other.* Teacher responses that do not address mathematics (e.g., "Nice work David!") are coded as *non-mathematical*. An instance of student thinking for which an MP cannot be articulated (see Van Zoest et al., 2016) receives the code *not applicable*, because a match or alignment cannot be determined.

Move

We use *move* to capture what the actor is doing or being asked to do with respect to the instance of student thinking. We identified 14 moves (see Figure 3). Although many of these moves are recognizable from other literature, they differ in that their descriptions do not include the three components captured in our other categories.

Move	Description
Adjourn	The teacher either explicitly or implicitly indicates that the instance(s) will not be considered publicly at that time, but suggests the instance may be considered later.
Allow	The teacher invites or leaves space for students to respond to the instance.
Check-in	The teacher elicits students' self-assessment of their reaction to or understanding of the instance.
Clarify	The teacher provides an interpretation and asks for verification that it reflects what the student meant, or asks the student to say what they meant (about a specific piece of the instance) without asking for elaboration.
Collect	The teacher requests or provides additional ideas, methods, or solutions.
Connect	The teacher asks for or makes a connection between or among representations, methods/strategies, solutions, or ideas that includes the instance.
Correct	The teacher describes or asks for a correct way of approaching, or thinking about, the instance.
Develop	The teacher provides or asks for an expansion of the instance that goes beyond a simple clarification.

Dismiss	The teacher either explicitly or implicitly indicates that the instance(s) will not be considered publicly.
Evaluate	The teacher asks for or provides a determination of the correctness of the instance.
Justify	The teacher asks for or provides a justification of the instance.
Literal	The teacher asks for or provides brief factual information related to the instance.
Repeat	The teacher (verbally or in writing) repeats or rephrases the instance without changing the meaning or asks a student to repeat the instance.
Validate	The teacher says something about the instance to affirm its value and/or encourage student participation (e.g., thank you, good).

Figure 3: Teacher moves and their descriptions.

Figure 2 provides examples of four of these 14 moves. In TR1, the teacher *correct*s the student's labelling, reminding the class of labelling conventions. In TR2, the teacher is *collect*ing additional methods for labelling the axes from the class. In TR3, the student who generated the instance is asked to *justify* their choice of money as the independent variable and weeks as the dependent variable. TR4 asks a *literal* question to engage a different student in providing factual information about what the teacher likes to do first when they graph.

DISCUSSION & CONCLUSION

Our initial coding scheme was based on moves drawn from the literature, but as it was applied to the data, it was quickly seen that focusing only on those moves was insufficient to characterize teacher responses given our four principles of effective teaching. For example, in examining a teacher move such as *develop*, the nature of the move is very different if the teacher is expanding on what a student has said as compared to asking other students to expand on the instance of student thinking. This difference led to the development of the actor coding category, which describes who is being publicly invited to engage with the instance of student thinking. This category provides the means for measuring the degree to which students are being engaged in classroom communication across all moves.

Another important aspect of the coding scheme is the degree to which the teacher response honors the mathematical ideas in what the student has said. For example, when a teacher uses an instance of student mathematical thinking as a launching point to discuss what they feel the students need to hear, the student who contributed the thinking in the instance might wonder what their idea has to do with the line of reasoning the teacher is now pursuing and feel that their thinking was not valued. The Student Actions and Student Ideas subcategories measure the degree to which the teacher appears to view students as legitimate mathematical thinkers.

Franke, Kazemi and Battey (2007) suggested that a focus on student mathematics should be a critical element of mathematics classroom practice. The Mathematics category, by assessing the degree to which the MU of the teacher response is aligned with the MP of the instance of student thinking, provides a way to characterize teacher responses relative to this focus on student mathematics.

The TRC has several notable strengths. It is applicable across grade levels and mathematics content. It has descriptive power because it disentangles the teacher move from the actor, the degree to which the student thinking is honored, and the extent to which the response focuses on the mathematics of the student thinking. As a result, it paints a rich picture of the teacher response without being evaluative in nature. The researcher who applies this coding scheme decides which combination of codes might be more or less productive based on their own perspective. The flexibility of the TRC makes it useful for a broad range of researchers interested in better understanding the teacher role in shaping meaningful mathematical discourse in their classrooms.

Our long-term goal is to better understand the teaching practice of building on high-potential instances of student mathematical thinking in the moment they occur during whole-class interactions. Such complex teaching practices are difficult to study and often require the practice to be decomposed in order to "articulate, unpack and study" (Boerst, Sleep, Ball, & Bass, 2011, pg. 2859) it. We anticipate that decomposing and studying teacher responses using the TRC will provide insight into teachers' current responses to high-potential student thinking and contribute to better understanding the broader teaching practice of productively using student mathematical thinking.

Endnote

[1]Focus on an instance typically ends when the next instance of student thinking occurs. However, a teacher's response may end prior to the next instance of student thinking—that is, prior to the end of the teacher's conversational turn.

Acknowledgements

This work was funded in part by the U.S. National Science Foundation (NSF) under Grant Nos. 1220141, 1220357, and 1220148. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the NSF.

The authors would like to thank Rachel Bernard, Elizabeth Fraser, Mary Ochieng, and Kylie Palsky for their contributions to the development of the coding scheme.

References

Bishop, J. P., Hardison, H., & Przybyla-Kuchek, J. (2016). Profiles of responsiveness in middle grades mathematics classrooms. In M. B. Wood, E. E. Turner, M. Civil, & J. A. Eli (Eds.), *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1173-1180). Tucson: The University of Arizona.

Boerst, T. A., Sleep, L., Ball, D. L., & Bass, H. (2011). Preparing teachers to lead mathematics discussions. *Teachers College Record, 113*, 2844-2877.

Conner, A., Singletary, L. M., Smith, R. C., Wagner, P. A., & Francisco, R. T. (2014). Teacher support for collective argumentation: A framework for examining how teachers support students' engagement in mathematical activities. *Educational Studies in Mathematics, 86*, 401-429.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education, 27*, 403-434.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 225-256). Charlotte, NC: Information Age.

Franke, M. L., Webb, N. M., Chan, A. G., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education, 60*, 380-392.

Glaser, B. G. (1965). The constant comparative method of qualitative analysis. *Social Problems, 12*, 436-445.

National Council of Teachers of Mathematics. (2014). *Principles to Action: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*, 313-340.

Stockero, S. L., Van Zoest, L. R., Rougee, A., Fraser, E. H., Leatham, K. R., & Peterson, B. E. (2015). Uncovering teachers' goals, orientations, and resources related to the practice of using student thinking. In T. G. Bartell, K. N. Bieda, R. T. Putnam, K. Bradfield, & H. Dominguez (Eds.), *Proceedings of the 37th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1146-1149). East Lansing: Michigan State University.

Van Zoest, L. R., Stockero, S. L., Leatham, K. R., Peterson, B. E., Atanga, N. A., & Ochieng, M. A. (2017). Attributes of instances of student mathematical thinking that are worth building on in whole-class discussion. *Mathematical Thinking and Learning*, *19*, 33-54.

Van Zoest, L. R., Peterson, B. E., Leatham, K. R., & Stockero, S. L. (2016). Conceptualizing the teaching practice of building on student mathematical thinking. In M. B. Wood, E. E. Turner, M. Civil, & J. A. Eli (Eds.), *Proceedings of the 38th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1281–1288). Tucson: University of Arizona.

Van Zoest, L. R., Stockero, S. L., Leatham, K. R., & Peterson, B. E. (2016). Theorizing the mathematical point of building on student mathematical thinking. In C. Csíkos, A. Rausch, & J. Szitányi (Eds.), *Proceedings of the 40th conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 323-330). Szeged, Hungary: PME.