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The Complexity of Interpreting Student Thinking and Inferring its Potential to Foster Learning

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Background

Mathematical Opportunities in Student Thinking

- US reform documents have emphasized using student mathematical thinking (SMT) to inform instructional decisions (e.g., NCTM, 2014)
- Making sense of SMT has instructional value (Franke & Kazemi, 2001)
- Attending to and interpreting SMT are challenging (Peterson & Leatham, 2010; Stockero & Van Zoest, 2013)
- Attention to and interpretation of SMT are skills that teachers can develop (Jacobs, Lamb & Philipp, 2010)

Literature Review

Mathematical Opportunities in Student Thinking

- Some studies have provided insights about how teachers' views of SMT develop
  - teachers' views of SMT change from evaluative to thoughtful interpretations (Crespo, 2000)
  - teachers follow different pathways as they develop the skill of interpreting SMT (van Es & Sherin, 2008)

- Other studies have revealed factors that influence teachers' inferences of SMT
  - teachers' limited understanding of mathematical concepts (Maher & Davies, 1990)
  - teachers' orientations towards listening to students (Davis, 1996)

Research Questions

Mathematical Opportunities in Student Thinking

1. How does an exemplary teacher interpret SMT that emerges in-the-moment during whole class instruction?

2. What inferences does an exemplary teacher make about the potential of SMT (that emerges in-the-moment during whole class instruction) to foster learning of mathematical ideas?

Theoretical Framework

Mathematical Opportunities in Student Thinking

- **M**athematical **O**pportunities in **S**tudent **T**hinking (MOSTs)
  - Student Mathematical Thinking
    - Student Mathematics
    - Mathematical Point
  - Significant Mathematics
    - Appropriate
    - Central
  - Pedagogical Opportunity
    - Opening
    - Timing

Leatham, K. R., Peterson, B. E., Stockero, S. L., & Van Zoest, L. R. (2015). Conceptualizing Mathematically Significant Pedagogical Opportunities to Build on Student Thinking. Journal for Research in Mathematics Education, 46(1), 88-124.

Theoretical Framework

Mathematical Opportunities in Student Thinking

- A MOST is in-the-moment student thinking worth building on

- Building is making a MOST the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

Methodology

Mathematical Opportunities in Student Thinking

- Teacher-participant
  - chosen because he regularly incorporates student thinking into his lessons
  - recognized by his school district and university mathematics educators as an exemplary teacher
- Data
  - four videotaped math lessons from this teacher's classroom
  - corresponding follow-up conversations about instances of SMT

Data Analysis

Mathematical Opportunities in Student Thinking

- To understand teachers' in-the-moment responses to SMT, we looked simultaneously at the teacher's
  - interpretations of SMT relative to the MOST Analytic Framework
  - reflections on his responses to that thinking
- Four different units of analysis:
  - instances of SMT
  - the teacher's in-the-moment responses to those instances
  - the teacher's retrospective inference of the student mathematics
  - the teacher's retrospective reasoning for his responses
- We analyzed 34 instances which:
  - the interviewer thought were likely to be MOSTs
  - appeared to be treated as MOSTs
  - the teacher wished to discuss

Results

Mathematical Opportunities in Student Thinking

- For 18 of the 34 instances, the teacher's placement of the instances on the MOST Analytic Framework matched ours.
- For 14 of the 34 Instances the teacher's placement of the instances did not match ours.
- For two instances the teacher was able to infer SMT, while we were not able to make such an inference.
  - This could have been due to classroom norms, or the teacher's insights into his students, but it was not clear to us that there was a shared understanding of what the student said by the rest of the class.

Teachers' Placement Matched ours

- The teacher often spoke of the building potential that he saw in instances, and how this guided his in-the-moment responses

- The teachers' in-the-moment responses were meant to harness that building potential

Teachers' Placement Matched ours

Mathematical Opportunities in Student Thinking

- As an example, Students had been trying to come up with the formula of a specific geometric sequence. Students had shared three possibilities, and the teacher said:
  "Okay, we have three different equations now, which of them do you think is right?"

- Later in his reflection of this instance, the teacher said:
  "This was a good opportunity to compare and contrast the 3 equations, and decide if any of them were right."

Teachers' Placement did not Match ours

Mathematical Opportunities in Student Thinking

- The teacher's reasoning for this fell into the following categories
  - taking up only part of the SMT because:
    - the teacher did not understand all of the SMT
    - the teacher chose to focus on a specific aspect of the SMT rather than all of the SMT
  - considering additional context or thinking that was not part of the SMT
  - not seeing the importance in an instance of SMT

Example: Taking up part of the SMT

Mathematical Opportunities in Student Thinking

In this instance, the class was working with geometric sequences, and trying to understand the formula for a geometric sequence.

- A student said **"every time we plug in a number it gives us one term further than we wanted it to. So, if we subtract one then it puts us back one term every time."** The teacher interpreted the SMT as dealing with plugging values into an equation to check the validity of the equation

- Later, the teacher said that the student **"was probably saying that when I put in a two I wanted to get the second term, but it's giving me the third term"**. And the teacher realized that the student was working towards understanding that altering the exponent in the formula of a geometric sequence will yield a different term in the sequence

Example: Taking up part of the SMT

Mathematical Opportunities in Student Thinking

In this instance, the class was beginning to work with logarithmic equations

- A student said that **"If you add $log_2 2$, $log_2 9$, and $log_2 2$ you get the same thing as $log_2(9 \cdot 2 \cdot 2)$.", and went on to note that there was a relationship between this idea and the exponential product rule.**

- The teacher then focused only on the exponential product rule and asked the class **"when you multiply exponents you add them together. What does that mean?"** and after a student responded to this, the teacher dropped this instance of SMT, and moved on.

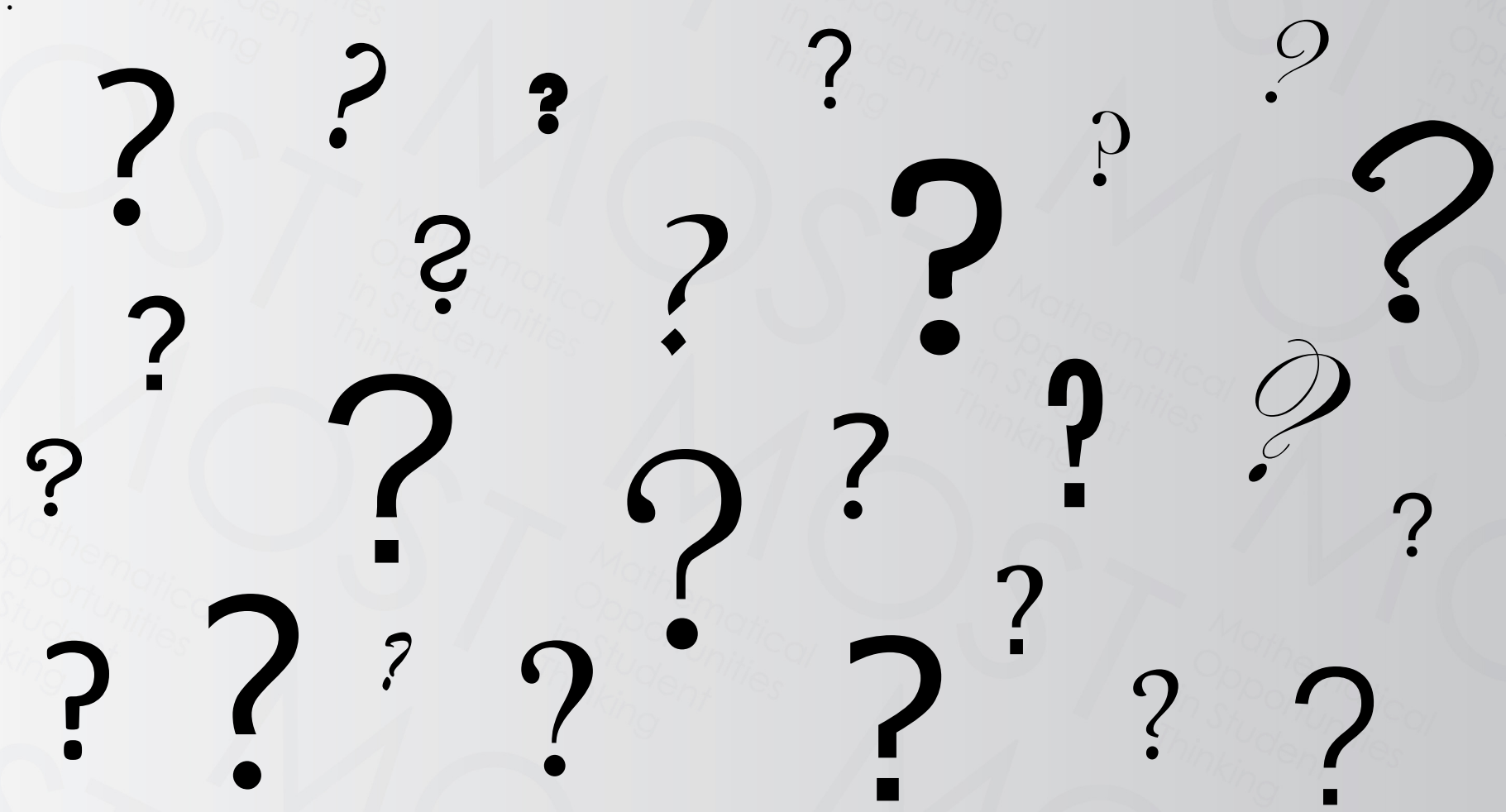
Conclusions

Mathematical Opportunities in Student Thinking

- Incorporating SMT in ways that foster learning requires that teachers correctly identify and interpret SMT
- Even exemplary teachers find incorporating SMT to be challenging
- Professional development supporting teachers making sense of in-the-moment SMT may be necessary
- Future studies could look at developing teachers' ability to accurately interpret SMT and its underlying potential to foster learning

Questions

Mathematical Opportunities in Student Thinking



References

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *The Elementary School Journal, 93*, 371–397.

Crespo, S. (2000). Seeing more than right and wrong answers: Prospective teachers' interpretations of students' mathematical work. *Journal of Mathematics Teacher Education, 3*, 155-181.

Davis, B. (1996). *Teaching mathematics: Toward a sound alternative*. New York: Garland Publishing.

Franke, M. L., & Kazemi, E. (2001). Learning to teach mathematics: Focus on student thinking. *Theory Into Practice, 40*(2), 102-109.

Jacobs, V. R., Lamb, L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169-202.

Maher, C. A., & Davies, R. (1990). Teacher's learning: Building representations of childrens' meanings. Journal for Research in Mathematics Education (Monograph No 4), 79-90.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

van Es, E. A. & Sherin, M. G. (2008). Mathematics teachers' "learning to notice" in the context of a video club. Teaching and Teacher Education, 24, 244-276.