LeveragingMOSTs.org

Toward a Theory of Productive Use of Student Mathematical Thinking

Laura R. Van Zoest – Western Michigan University

Shari L. Stockero – Michigan ... University

Blake E. Peterson – ...

Keith R. Leatham – ...

Napthalin Atanga – ...

Lindsay Merrill – Brigham ...

Mary A. Ochieng – Western Michigan University

And…

Elizabeth Fraser (WMU)

Annick Rougee (U of M)

Rachel Gunn (BYU)

Supported by

Leveraging MOSTs: Developing a Theory of Productive Use of Student Mathematical Thinking

a 4-year collaborative research project funded by the National Science Foundation

(DRL-1220141, DRL-1220357, DRL-1220148)

Why do we need a *Theory of Productive Use of Student Mathematical Thinking*?

- Benefits of instructional practices that build on student thinking
  - Fennema, et al., 1996 [CGI]
  - Stein & Lane, 1996 [QUASAR]
  - NCTM, 2014
- Difficult to implement such practices
  - Ball & Cohen, 1999
  - Sherin, 2002
  - Silver, Chousseini, Gosen, Charalambous & Font Strawhun, 2005
  - PI's earlier work
- Need theory that articulates
  - What the *practice of productively using student mathematical thinking* (PUMT) looks like
  - How this practice typically develops
  - How that development can be facilitated

Developing the *Practice of Productively Using Student Mathematical Thinking* (PUMT)

Mathematical Opportunities in Student Thinking

Facilitating the development of PUMT

PUMT Hypothetical Learning Process

PUMT

Mathematically significant

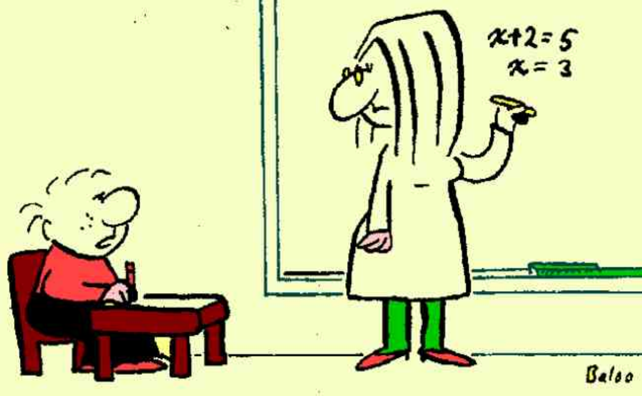
pedagogical **Opportunities**

to build on **in Student**

Thinking

"*Just a darn minute! Yesterday you said X equals* ***two!***"

MOST Characteristics



"Just a darn minute! Yesterday you said X equals two!"

Student Mathematical Thinking

Student Mathematics

Can the student mathematics be inferred?

Mathematical Point

Is there a mathematical point closely related to the student mathematics?

Mathematically Significant

Appropriate Mathematics

Is the mathematical point accessible to students with this level of mathematical experience, but not like to be already understood?

Central Mathematics

Is understanding the mathematical point a central goal for student learning in this classroom?

Pedagogical Opportunity

Opening

Does the expression of the student mathematics seem to create an intellectual need that, if met, will contribute to understanding the mathematical point of the instance?

Timing

Is now the right time to take advantage of the opening?

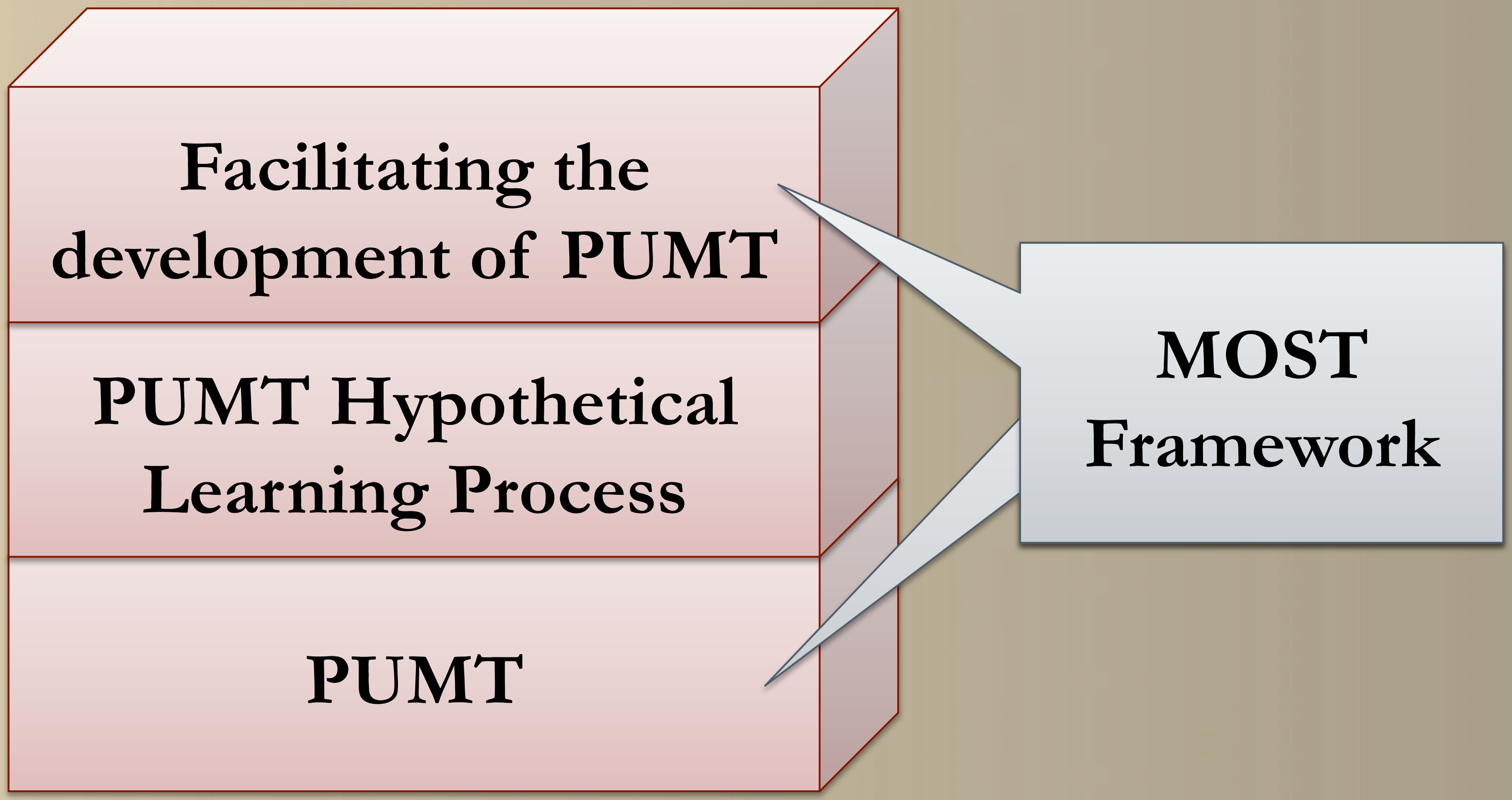
Leatham, K. R., Peterson, B. E., Stockero, S. L., & Van Zoest, L. R. (2015). Conceptualizing mathematically significant pedagogical opportunities to build on student thinking. *Journal for Research in Mathematics Education, 46*, 88-124.

MOSTs are opportunities…

…for the teacher to make student thinking the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

…to **build** on student thinking.

Developing the *Practice of Productively Using Student Mathematical Thinking* (PUMT)



Focus of Today’s Session

- Analyzing attributes of MOSTs (Presentation 1)

  What are the attributes of MOSTs available to teachers to build on in secondary mathematics instruction?

- Investigate teachers’ perceptions of productive use of student mathematical thinking (Presentation 2)

  What are teachers’ perceptions of productive use of student mathematical thinking?

- Analyze teachers’ responses to MOSTs (Presentation 3)

  To what extent are teachers able to recognize and productively respond to secondary school students’ mathematical thinking?

Focus of Today's Session

- Analyzing attributes of MOSTs (Presentation 1)

  What are the attributes of MOSTs available to teachers to build on in secondary mathematics instruction?

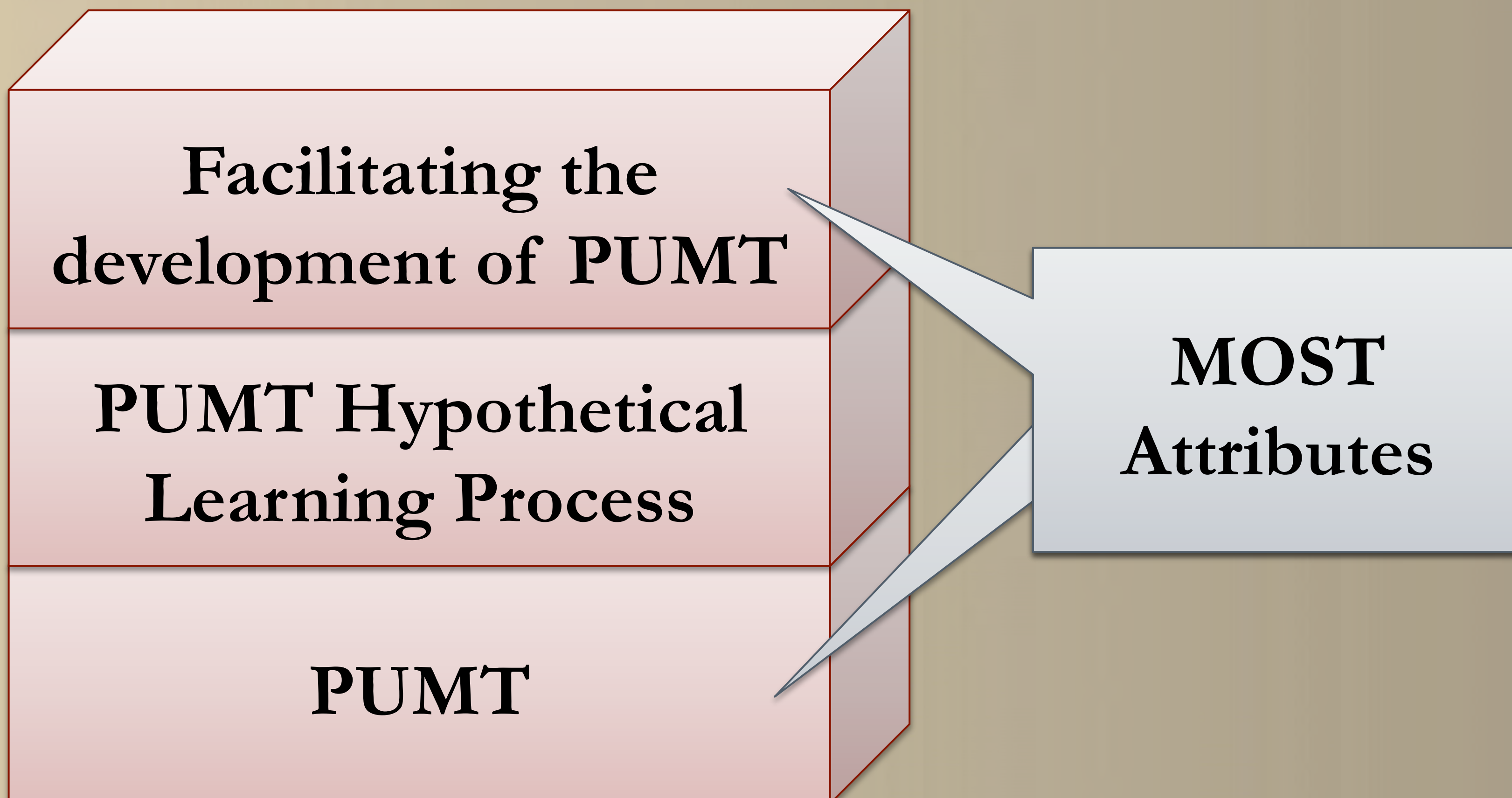
- Investigate teachers' perceptions of productive use of student mathematical thinking (Presentation 2)

  What are teachers' perceptions of productive use of student mathematical thinking?

- Analyze teachers' responses to MOSTs (Presentation 3)

  To what extent are teachers able to recognize and productively respond to secondary school students' mathematical thinking?

Developing the *Practice of Productively Using Student Mathematical Thinking* (PUMT)



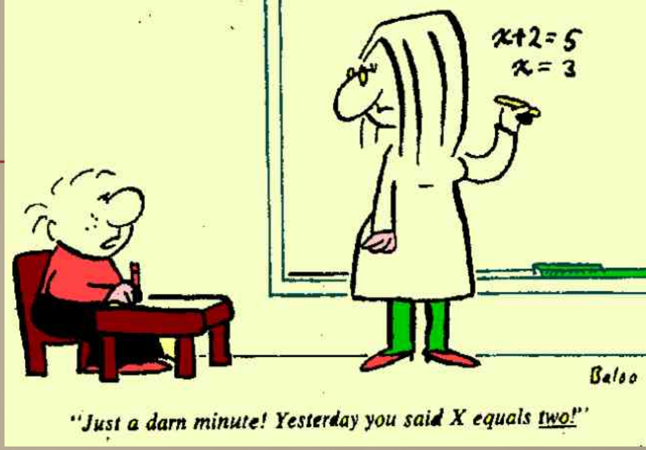
Methods

- 11 mathematics lessons
- 10 teachers
- California, Hawaii, Michigan, Mississippi, New Mexico & Utah
- Whole-class interactions (349 minutes)
- Unit of analysis was an instance of student thinking

MOST Analysis

- Pass 1 - Identified each instance of student mathematical thinking
  - 1651 instances in the 349 minutes of whole-class interaction
  - 4.7 instances per minute (range of 1.4 to 6.9 across 11 lessons)
- Pass 2 - Used the MOST Analytic Framework (Leatham et al., 2015) to identify MOSTs
  - 297 of the 1651 instances of student thinking were MOSTs
  - 0.9 MOSTs per minute (range of 0.1 to 1.6 across 11 lessons)
- Pass 3 – Investigated attributes of MOSTs (coding scheme expanded from Stockero & Van Zoest, 2013)
  - Seven attributes
  - Fall into two groups

MOST Attributes: Example



Student Mathematics:

Yesterday x equaled 2 and today x equals 3.

Mathematical Point:

A letter can be used to represent an unknown value in an equation and can represent different values for different equations.

Coding for Attributes

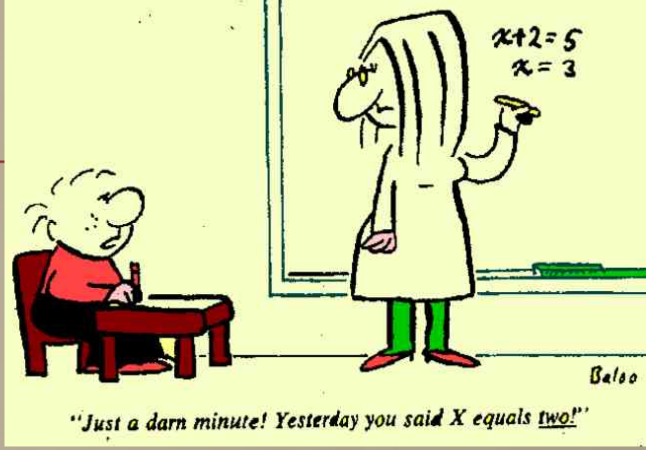
Locus		
Attribute	**Definition**	**Codes**
Prompt	The invitation or lack thereof that precipitated the MOST	• Spontaneous • Open Invitation Spontaneous • Open Invitation Selected • Targeted Invitation
Basis	Whether the student mathematics (SM) in the MOST is based on earlier work or in-the-moment thinking	• In-the-Moment • Pre-Thought
Math Goal	How far the mathematical idea captured in the MOST is from the day's lesson	• Lesson • Unit • Course • Math

Coding for Attributes

Cognition

Attribute	Definition	Codes
Form	The way in which the student thinking is expressed	• Question • Tentative Statement • Declarative Statement
Accuracy	The validity of the student mathematics (SM) of the MOST	• Correct • Incorrect • Incomplete • Combination • N/A
Intellectual Need	The extent to which the compelling nature of the MOST is transparent	• Obvious • Translucent • Hidden
Type	The nature of the SM that made the instance compelling	• Incorrect or Incomplete • Sense Making • Multiple Ideas or Solutions • Other

MOST Attributes: Example

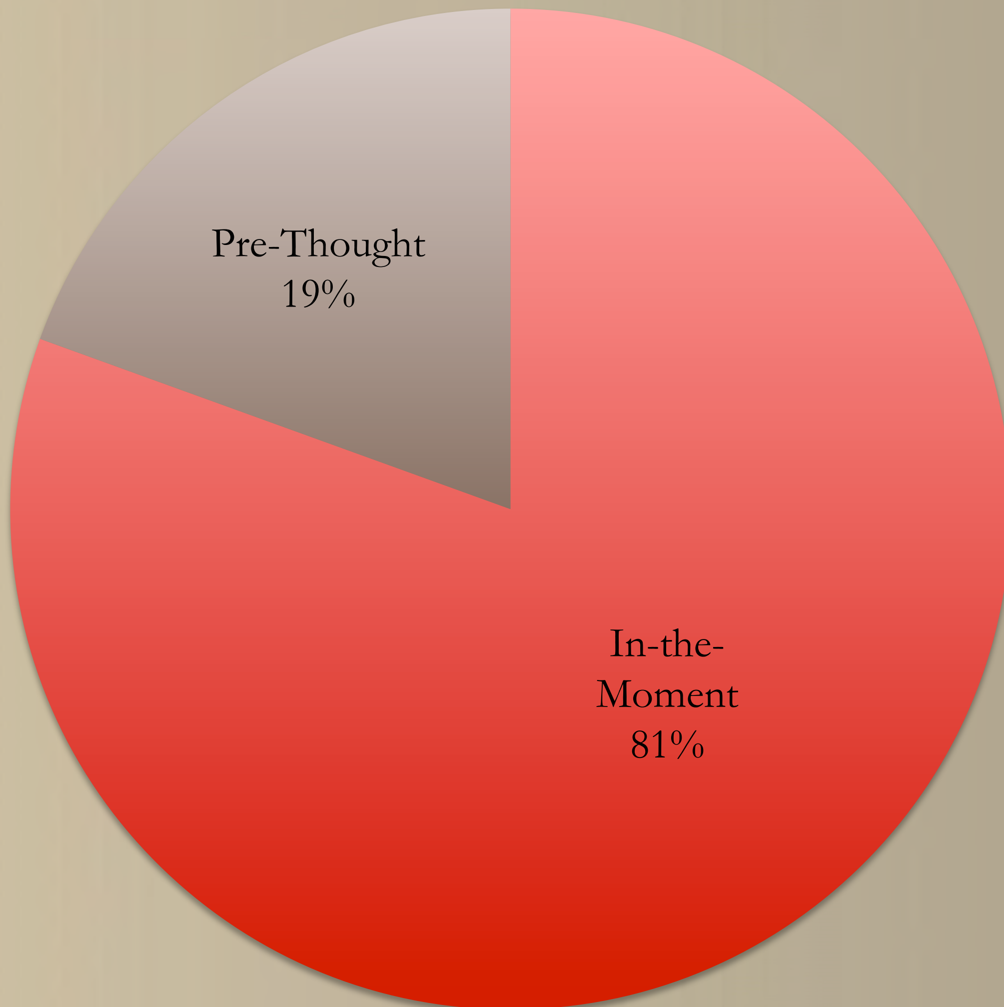


Student Mathematics: Yesterday x equaled 2 and today x equals 3.

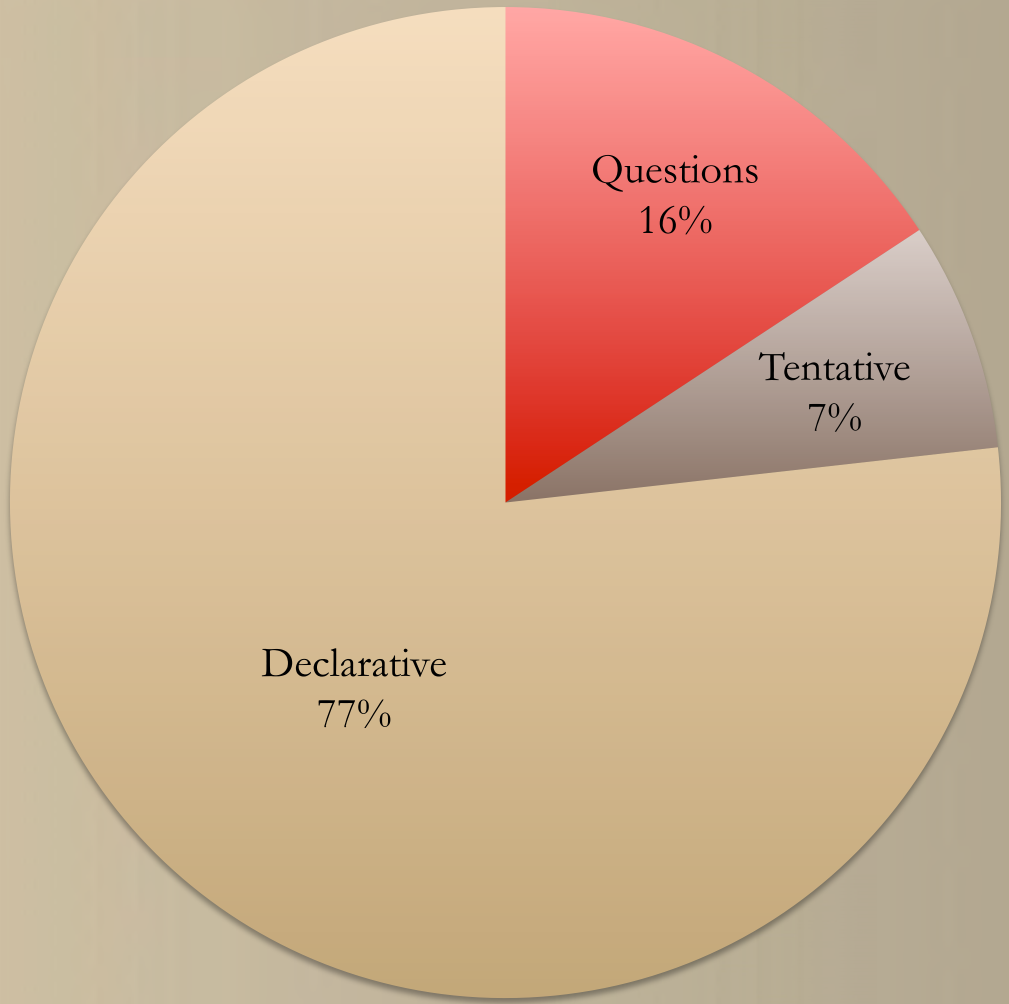
Mathematical Point: A letter can be used to represent an unknown value in an equation and can represent different values for different equations.

Prompt: Spontaneous
Basis: In-the-Moment
Math Goal: Course
Form: Declarative
Accuracy: N/A
Intellectual Need: Obvious
Type: Sense Making

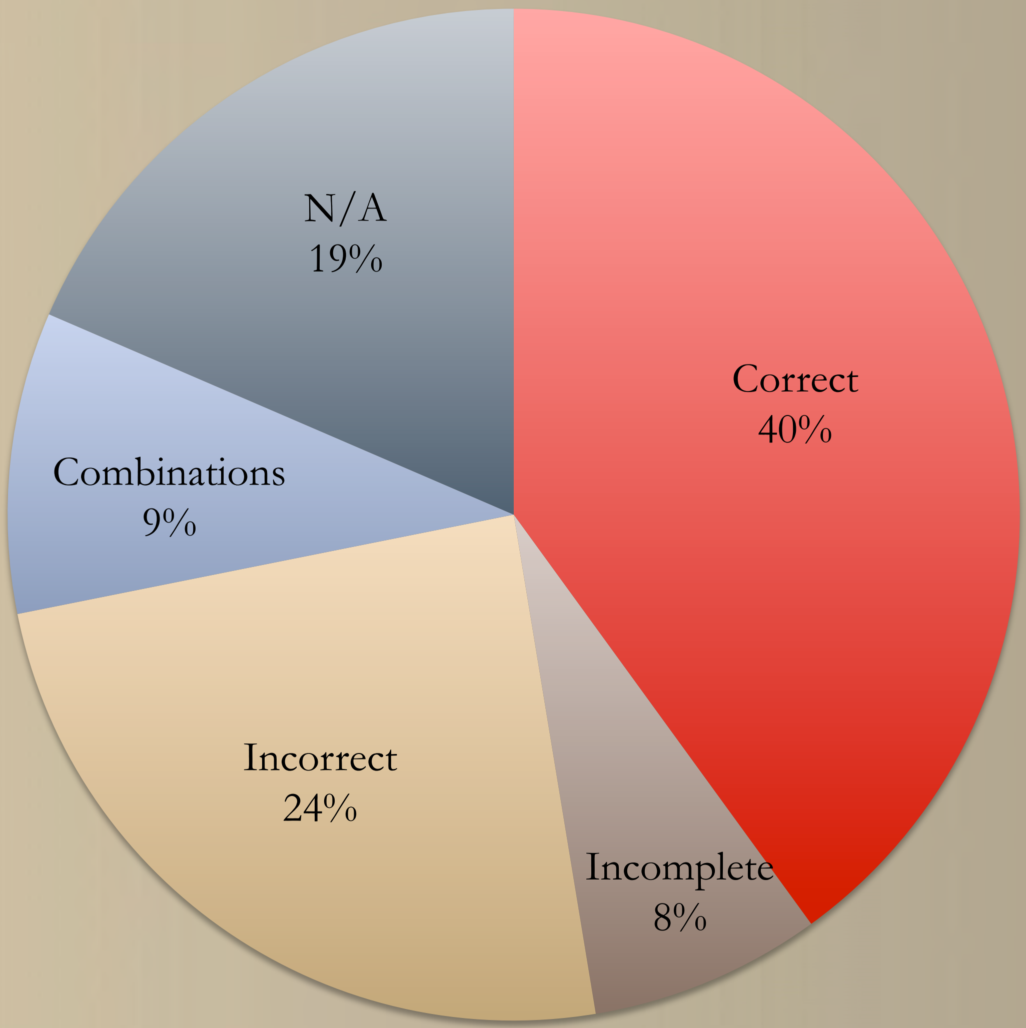
MOST Basis



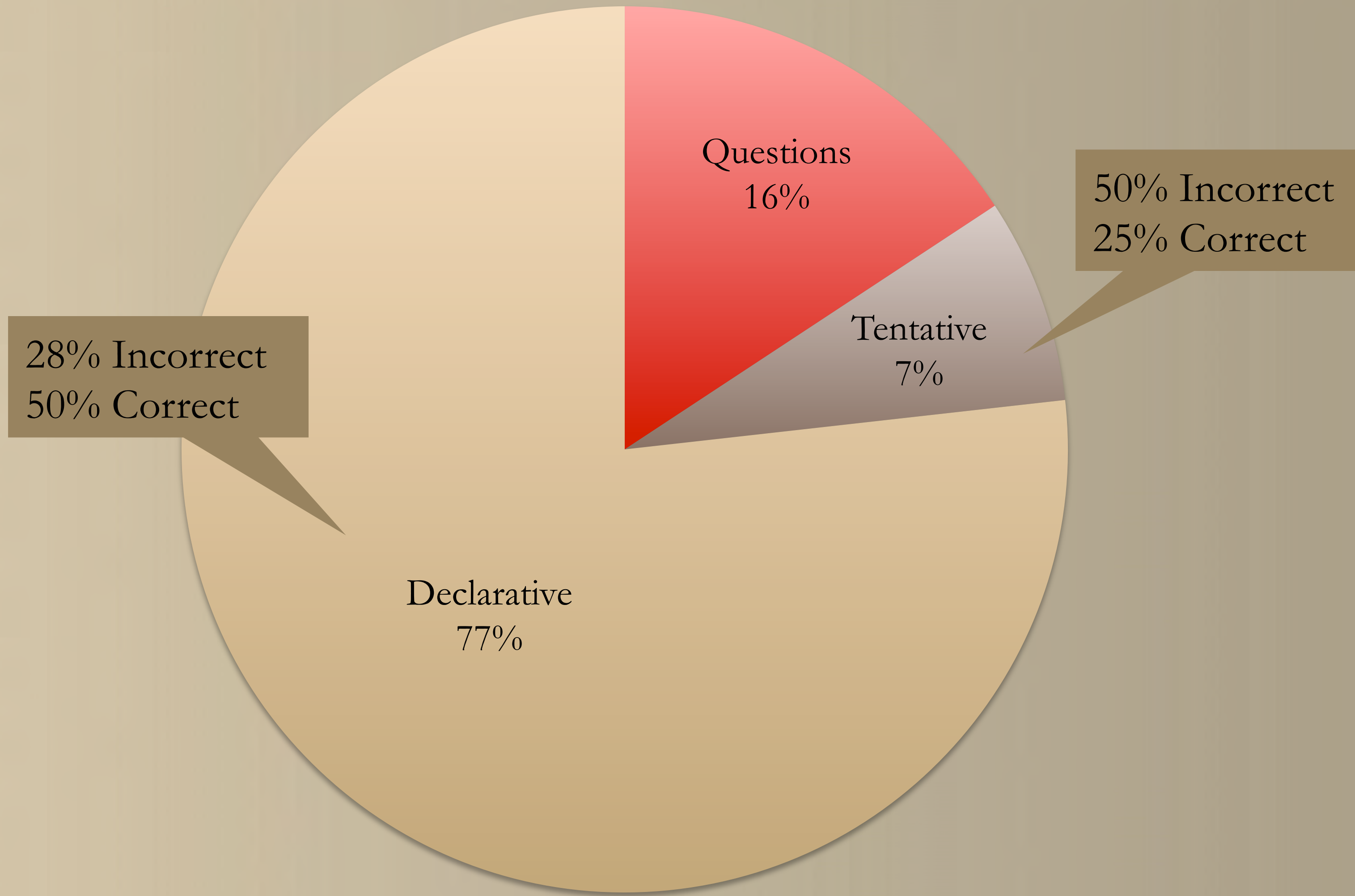
MOST Forms



MOST Accuracy



MOST Forms


Questions
16%
Tentative
7%
Declarative
77%
50% Incorrect
25% Correct
28% Incorrect
50% Correct

Focus of Today's Session

- Analyzing attributes of MOSTs (Presentation 1)

  What are the attributes of MOSTs available to teachers to build on in secondary mathematics instruction?

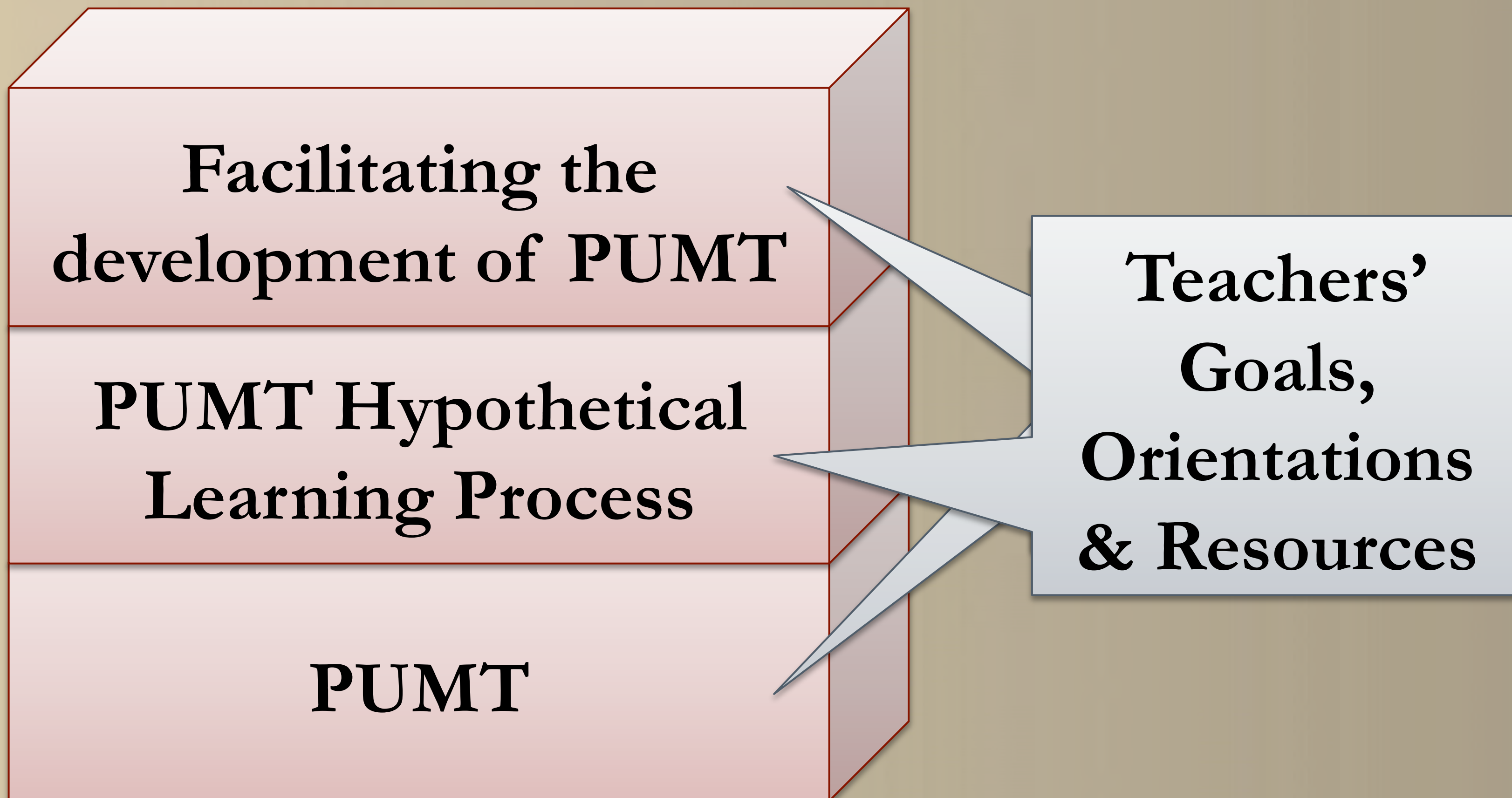
- Investigate teachers' perceptions of productive use of student mathematical thinking (Presentation 2)

  What are teachers' perceptions of productive use of student mathematical thinking?

- Analyze teachers' responses to MOSTs (Presentation 3)

  To what extent are teachers able to recognize and productively respond to secondary school students' mathematical thinking?

Developing the *Practice of Productively Using Student Mathematical Thinking* (PUMT)



The Scenario Interview

Purpose:

- To infer teachers' goals, orientations, and resources (GOR) in the context of using student thinking
- To capture how a teacher thinks about attending to student thinking during instruction
- To compare teachers' responses to a common set of instances of student thinking

Interview Format

- Interviewee presented with student statements that represent a range of thinking from two classroom lessons.
- Interviewee situated as the teacher.
- Contextual information is not initially provided; the teacher can ask for any contextual information they would like to know.
- Teacher is asked to describe what they might do next.
- A series of follow up questions probe the teacher's decision, rationale, possible assumptions, and use of contextual information.

Sample Instance of Student Thinking

Jamie says, “I found the number in front of the x by subtracting the y-values in the table, 21 - 19, so that number is 2.”

x	y
0	15
2	19
3	21
5	25

Theoretical Framework

- Schoenfeld's (2011) theory of goal-oriented decision making
  - Goals: short or long-term; may relate to the learning of specific content, but may be broader outcomes for students or to teacher actions
  - Orientations: defined to include teachers' "dispositions, beliefs, values, tastes and preferences" (p. 29)
  - Resources: everything that a teacher could access to support instruction (e.g., physical materials; teachers' knowledge of mathematics content, teaching strategies, and typical student conceptions)

Analysis

- Unit of analysis: teacher statements
- Coded for goals, orientations, and resources
- Reconciled codes
- Grouped into themes
- Compared themes across teachers

Student Thinking as a Resource

Mathematical Opportunities in Student Thinking

Ms. Shaw	Mr. Mead	Ms. Dean
for making instructional decisions and helping students make sense of the mathematics in a lesson	to develop the mathematical ideas in a lesson and to tie ideas together	to accomplish what she wants to happen during a lesson
Example:	Example:	Example:
Responses to questions such as “Why are you using [2] as your radius?” (I5) as opportunities for students to “start to see yes it's a difference of 2 but it's not a circle of a radius of 2. I need to look at the difference of the [areas of the] circles versus just the difference of the radius." (I5)	After two students share different methods of solving a problem: “let’s verify whether or not the methods that you saw on the board today are correct or not, and that would give me something to go back to at the end as well.” (I54)	“If someone’s got a suggestion on how something works, hear them out because if they are correct, then you can use that as the stepping off place.....or no, she is not right and let's address why before it becomes a huge misconception, like that they think it works every time." (I21)

Student Thinking as a Resource

Mathematical Opportunities in Student Thinking

Ms. Shaw	Mr. Mead	Ms. Dean
for making instructional decisions and helping students make sense of the mathematics in a lesson	to develop the mathematical ideas in a lesson and to tie ideas together	to accomplish what she wants to happen during a lesson
Example:	Example:	Example:
Responses to questions such as "Why are you using [2] as your radius?" (I5) as opportunities for students to "start to see yes it's a difference of 2 but it's not a circle of a radius of 2. I need to look at the difference of the [areas of the] circles versus just the difference of the radius." (I5)	After two students share different methods of solving a problem: "let's verify whether or not the methods that you saw on the board today are correct or not, and that would give me something to go back to at the end as well." (I54)	"If someone's got a suggestion on how something works, hear them out because if they are correct, then you can use that as the stepping off place.....or no, she is not right and let's address why before it becomes a huge misconception, like that they think it works every time." (I21)

Student Thinking as a Resource

Mathematical Opportunities in Student Thinking

Ms. Shaw	Mr. Mead	Ms. Dean
for making instructional decisions and helping students make sense of the mathematics in a lesson	to develop the mathematical ideas in a lesson and to tie ideas together	to accomplish what she wants to happen during a lesson
Example:	Example:	Example:
Responses to questions such as “Why are you using [2] as your radius?” (I5) as opportunities for students to “start to see yes it's a difference of 2 but it's not a circle of a radius of 2. I need to look at the difference of the [areas of the] circles versus just the difference of the radius." (I5)	After two students share different methods of solving a problem: “let’s verify whether or not the methods that you saw on the board today are correct or not, and that would give me something to go back to at the end as well.” (I54)	“If someone’s got a suggestion on how something works, hear them out because if they are correct, then you can use that as the stepping off place.....or no, she is not right and let's address why before it becomes a huge misconception, like that they think it works every time." (I21)

Goals for Using Student Thinking

Ms. Shaw	Mr. Mead	Ms. Dean
to engage the student(s) in making sense of the mathematics behind the thinking	to position the students as thinkers by providing them with opportunities to share their thinking	to accurately evaluate students' thinking and use it to launch (if it is correct) or address misconceptions (if it is incorrect)
Example:	Example:	Example:
"What would my table look like [in this new situation]? What would my graph look like? How does [this modification] change those two representations?" (I42).	"I want to know more of that idea; I want to try to figure out what, yeah, what is the student actually thinking at that point" (I26). "I like to reward [students' thinking] and grab onto that thinking" (I31)	"[I]f they explain it, I can figure out where their thinking process is incorrect and I can nip that in the bud so that it doesn't become ingrained" (I67).

Goals for Using Student Thinking

Ms. Shaw	Mr. Mead	Ms. Dean
to engage the student(s) in making sense of the mathematics behind the thinking	to position the students as thinkers by providing them with opportunities to share their thinking	to accurately evaluate students' thinking and use it to launch (if it is correct) or address misconceptions (if it is incorrect)
Example:	Example:	Example:
"What would my table look like [in this new situation]? What would my graph look like? How does [this modification] change those two representations?" (I42).	"I want to know more of that idea; I want to try to figure out what, yeah, what is the student actually thinking at that point" (I26). "I like to reward [students' thinking] and grab onto that thinking" (I31)	"[I]f they explain it, I can figure out where their thinking process is incorrect and I can nip that in the bud so that it doesn't become ingrained" (I67).

Goals for Using Student Thinking

Mathematical Opportunities in Student Thinking

Ms. Shaw	Mr. Mead	Ms. Dean
to engage the student(s) in making sense of the mathematics behind the thinking	to position the students as thinkers by providing them with opportunities to share their thinking	to accurately evaluate students' thinking and use it to launch (if it is correct) or address misconceptions (if it is incorrect)
Example:	Example:	Example:
"What would my table look like [in this new situation]? What would my graph look like? How does [this modification] change those two representations?" (I42).	"I want to know more of that idea; I want to try to figure out what, yeah, what is the student actually thinking at that point" (I26). "I like to reward [students' thinking] and grab onto that thinking" (I31)	"[I]f they explain it, I can figure out where their thinking process is incorrect and I can nip that in the bud so that it doesn't become ingrained" (I67).

Orientations toward Student Thinking

Mathematical Opportunities in Student Thinking

Ms. Shaw	Mr. Mead	Ms. Dean
an important part of student learning is providing students ample opportunity to think about mathematical ideas	students can learn mathematics through mathematical exploration activities	she is responsible for explaining and demonstrating mathematical ideas to students
Example:	Example:	Example:
“I'm thinking I still have a lot of kids in the classroom who haven’t had a chance to resolve the dilemma between these two [different approaches]; I want every student in the classroom to have a chance to look through it and go, ‘How am I going to process this?’” (I23)	“I was assuming that this was where they’re developing their understanding through this problem. And so, they've had some situations before, or maybe not. Maybe they're just beginning to look at this situation and try[ing] to model it with some type of equation." (I13)	“[S]o I would explain that what you started with, that’s the y-intercept. That's the initial amount. And what you're adding each week is the slope. And then show how that would work on the equation.” (I6)

Orientations toward Student Thinking

Mathematical Opportunities in Student Thinking

Ms. Shaw	Mr. Mead	Ms. Dean
an important part of student learning is providing students ample opportunity to think about mathematical ideas	students can learn mathematics through mathematical exploration activities	she is responsible for explaining and demonstrating mathematical ideas to students
Example:	Example:	Example:
"I'm thinking I still have a lot of kids in the classroom who haven't had a chance to resolve the dilemma between these two [different approaches]; I want every student in the classroom to have a chance to look through it and go, 'How am I going to process this?'" (I23)	"I was assuming that this was where they're developing their understanding through this problem. And so, they've had some situations before, or maybe not. Maybe they're just beginning to look at this situation and try[ing] to model it with some type of equation." (I13)	"[S]o I would explain that what you started with, that's the y-intercept. That's the initial amount. And what you're adding each week is the slope. And then show how that would work on the equation." (I6)

Orientations toward Student Thinking

Mathematical Opportunities in Student Thinking

Ms. Shaw	Mr. Mead	Ms. Dean
an important part of student learning is providing students ample opportunity to think about mathematical ideas	students can learn mathematics through mathematical exploration activities	she is responsible for explaining and demonstrating mathematical ideas to students
Example:	Example:	Example:
"I'm thinking I still have a lot of kids in the classroom who haven't had a chance to resolve the dilemma between these two [different approaches]; I want every student in the classroom to have a chance to look through it and go, 'How am I going to process this?'" (I23)	"I was assuming that this was where they're developing their understanding through this problem. And so, they've had some situations before, or maybe not. Maybe they're just beginning to look at this situation and try[ing] to model it with some type of equation." (I13)	"[S]o I would explain that what you started with, that's the y-intercept. That's the initial amount. And what you're adding each week is the slope. And then show how that would work on the equation." (I6)

MOST Mathematical Opportunities in Student Thinking

Goals, Orientations and Resources

<table>
<tr><th></th><th>Student Thinking as a Resource</th><th>Goals for Using Student Thinking</th><th>Orientations toward Student Thinking</th></tr>
<tr><td>Ms. Dean</td><td>to accomplish what she wants to happen during a lesson</td><td>confirm correct answers and address misconceptions in incorrect answers</td><td>she is responsible for explaining and demonstrating mathematical ideas to students</td></tr>
<tr><td>Ms. Shaw</td><td rowspan="2">develop students' mathematical understanding</td><td>engage students in sense-making</td><td>students need ample time and opportunities to think about mathematical ideas</td></tr>
<tr><td>Mr. Mead</td><td>position students as mathematical thinkers</td><td>students can learn through mathematical exploration</td></tr>
</table>

Goals, Orientations and Resources

<table>
<tr><th></th><th>Student Thinking as a Resource</th><th>Goals for Using Student Thinking</th><th>Orientations toward Student Thinking</th></tr>
<tr><td>Ms. Dean</td><td>to accomplish what she wants to happen during a lesson</td><td>confirm correct answers and address misconceptions in incorrect answers</td><td>she is responsible for explaining and demonstrating mathematical ideas to students</td></tr>
<tr><td>Ms. Shaw</td><td rowspan="2">develop students' mathematical understanding</td><td>engage students in sense-making</td><td>students need ample time and opportunities to think about mathematical ideas</td></tr>
<tr><td>Mr. Mead</td><td>position students as mathematical thinkers</td><td>students can learn through mathematical exploration</td></tr>
</table>

Goals, Orientations and Resources

	Student Thinking as a *Resource*	*Goals* for Using Student Thinking	*Orientations* toward Student Thinking
Ms. Dean	to accomplish what she wants to happen during a lesson	confirm correct answers and address misconceptions in incorrect answers	she is responsible for explaining and demonstrating mathematical ideas to students
Ms. Shaw	develop students' mathematical understanding	engage students in sense-making	students need ample time and opportunities to think about mathematical ideas
Mr. Mead		position students as mathematical thinkers	students can learn through mathematical exploration

Goals, Orientations and Resources

	Student Thinking as a *Resource*	*Goals* for Using Student Thinking	*Orientations* toward Student Thinking
Ms. Dean	to accomplish what she wants to happen during a lesson	confirm correct answers and address misconceptions in incorrect answers	she is responsible for explaining and demonstrating mathematical ideas to students
Ms. Shaw	develop students' mathematical understanding	engage students in sense-making	students need ample time and opportunities to think about mathematical ideas
Mr. Mead		position students as mathematical thinkers	students can learn through mathematical exploration

The Scenario Interview data allows us:

- To make inferences about what a teacher's practice might look like
- To understand the possible reasoning behind the teacher moves that we might observe during a lesson
- To understand why different uses of student thinking might make sense to different teachers based on the GOR that underlie their practice
- To perceive distinctions among teachers

Focus of Today's Session

- Analyzing attributes of MOSTs (Presentation 1)

  What are the attributes of MOSTs available to teachers to build on in secondary mathematics instruction?

- Investigate teachers' perceptions of productive use of student mathematical thinking (Presentation 2)

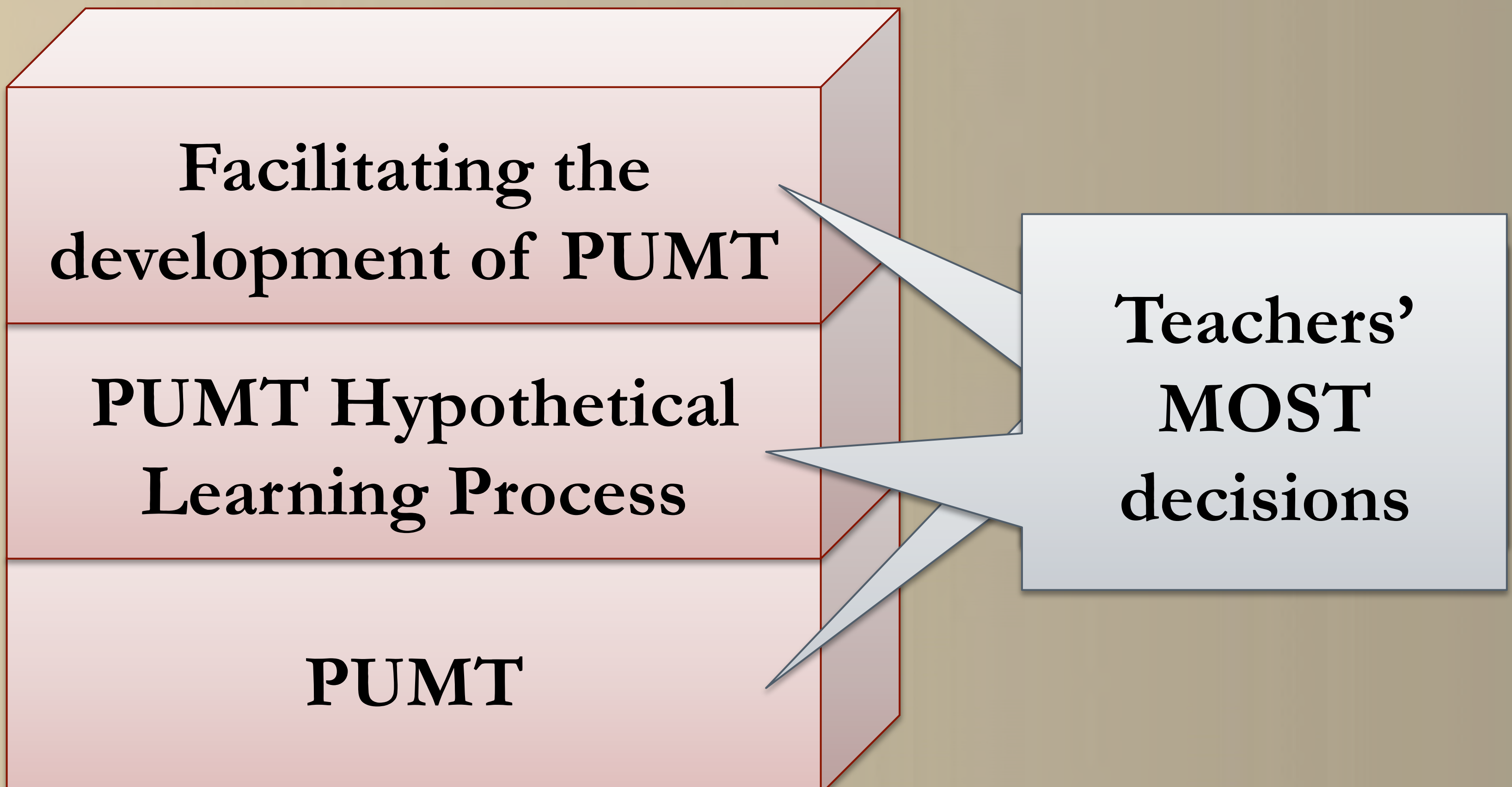
  What are teachers' perceptions of productive use of student mathematical thinking?

- Analyze teachers' responses to MOSTs (Presentation 3)

  To what extent are teachers able to recognize and productively respond to secondary school students' mathematical thinking?

Developing the *Practice of Productively Using Student Mathematical Thinking* (PUMT)

MOST Mathematical Opportunities in Student Thinking



Analyzing teachers' responses

Two data sources:

- Classroom videos which we can use to analyze teachers' responses to MOSTs we have identified
- Scenario interviews which we can use to analyze teachers' responses to a variety of student thinking

Analyzing teachers' responses

Two data sources:

– Classroom videos which we can use to analyze teachers' responses to MOSTs we have identified

– Scenario interviews which we can use to analyze teachers' responses to a variety of student thinking

Preliminary Coding Scheme

- *Actor* - Who was allowed or invited to participate in the MOST follow up?
- *Object of Consideration* – To what extent was the student thinking made the focus of discussion in the classroom?
  - Mathematical Point (MP)
  - Student Mathematics (SM) Words
  - Student Mathematics (SM) Idea
- *Move* – What action did the teacher take in response to the student statement?

Sample Instance of Student Thinking

Jamie says, "I found the number in front of the x by subtracting the y-values in the table, 21 - 19, so that number is 2."

Student Mathematics (SM): I found the m value in the equation $y = mx + b$ by subtracting the y-values in the table, 21-19, so that number is 2.

Mathematical Point (MP): To find the slope from a table of values, one must divide a change in the values of the dependent variable by the corresponding change in the values of the independent variable [$m = (y_2 - y_1)/(x_2 - x_1)$].

Ms. Dean

"So I would answer that by saying, 'You're on the right track.' I would probably go through and show the differences, you know 0 to 2 is a difference of 2, 2 to 3 is a difference of 1, 3 to 5 is a difference of 2. I'd probably write that up there, and show them the difference in corresponding y's. And then from there would be the lead in to how you find the slope as the change in y over the change in x. And I would say she was just lucky--he, Jamie, boy--um, because the change in x happened to be 1. And so they only had to do the change in y in that case. But if they had taken like 25 and 21, then the change in x is not 1 and you know--I probably would have done that example using her philosophy, Jamie's philosophy. And said 'see now the answer is different. How could that possibly be?' And explained you have to do change in y over change in x. And then I probably would have made a joke, and said 'lucky guess.'"

Mathematical Opportunities in Student Thinking

Ms. Dean

Actor	Teacher
Object of Consideration	
Move	

"So I would answer that by saying, 'You're on the right track.' I would probably go through and show the differences, you know 0 to 2 is a difference of 2, 2 to 3 is a difference of 1, 3 to 5 is a difference of 2. I'd probably write that up there, and show them the difference in corresponding y's. And then from there would be the lead in to how you find the slope as the change in y over the change in x. And I would say she was just lucky--he, Jamie, boy--um, because the change in x happened to be 1. And so they only had to do the change in y in that case. But if they had taken like 25 and 21, then the change in x is not 1 and you know--I probably would have done that example using her philosophy, Jamie's philosophy. And said 'see now the answer is different. How could that possibly be?' And explained you have to do change in y over change in x. And then I probably would have made a joke, and said 'lucky guess.'"

Ms. Dean

Actor	Teacher
Object of Consideration	MP: Core
Move	

"So I would answer that by saying, 'You're on the right track.' I would probably go through and show the differences, you know 0 to 2 is a difference of 2, 2 to 3 is a difference of 1, 3 to 5 is a difference of 2. I'd probably write that up there, and show them the difference in corresponding y's. And then from there would be the lead in to how you find the slope as the change in y over the change in x. And I would say she was just lucky--he, Jamie, boy--um, because the change in x happened to be 1. And so they only had to do the change in y in that case. But if they had taken like 25 and 21, then the change in x is not 1 and you know--I probably would have done that example using her philosophy, Jamie's philosophy. And said 'see now the answer is different. How could that possibly be?' And explained you have to do change in y over change in x. And then I probably would have made a joke, and said 'lucky guess.'"

Ms. Dean

Actor	Teacher
Object of Consideration	MP: Core SM Words: Implicit
Move	

"So I would answer that by saying, 'You're on the right track.' I would probably go through and show the differences, you know 0 to 2 is a difference of 2, 2 to 3 is a difference of 1, 3 to 5 is a difference of 2. I'd probably write that up there, and show them the difference in corresponding y's. And then from there would be the lead in to how you find the slope as the change in y over the change in x. And I would say she was just lucky--he, Jamie, boy--um, because the change in x happened to be 1. And so they only had to do the change in y in that case. But if they had taken like 25 and 21, then the change in x is not 1 and you know--I probably would have done that example using her philosophy, Jamie's philosophy. And said 'see now the answer is different. How could that possibly be?' And explained you have to do change in y over change in x. And then I probably would have made a joke, and said 'lucky guess.'"

Ms. Dean

Actor	Teacher
Object of Consideration	MP: Core SM Words: Implicit; SM Ideas: Core
Move	

“So I would answer that by saying, ‘You're on the right track.' I would probably go through and show the differences, you know 0 to 2 is a difference of 2, 2 to 3 is a difference of 1, 3 to 5 is a difference of 2. I'd probably write that up there, and show them the difference in corresponding y's. And then from there would be the lead in to how you find the slope as the change in y over the change in x. And I would say she was just lucky--he, Jamie, boy--um, because the change in x happened to be 1. And so they only had to do the change in y in that case. But if they had taken like 25 and 21, then the change in x is not 1 and you know--I probably would have done that example using her philosophy, Jamie's philosophy. And said 'see now the answer is different. How could that possibly be?' And explained you have to do change in y over change in x. And then I probably would have made a joke, and said 'lucky guess.'"

Ms. Dean

Actor	Teacher
Object of Consideration	MP: Core SM Words: Implicit; SM Ideas: Core
Move	Correct

"So I would answer that by saying, 'You're on the right track.' I would probably go through and show the differences, you know 0 to 2 is a difference of 2, 2 to 3 is a difference of 1, 3 to 5 is a difference of 2. I'd probably write that up there, and show them the difference in corresponding y's. And then from there would be the lead in to how you find the slope as the change in y over the change in x. And I would say she was just lucky--he, Jamie, boy--um, because the change in x happened to be 1. And so they only had to do the change in y in that case. But if they had taken like 25 and 21, then the change in x is not 1 and you know--I probably would have done that example using her philosophy, Jamie's philosophy. And said 'see now the answer is different. How could that possibly be?' And explained you have to do change in y over change in x. And then I probably would have made a joke, and said 'lucky guess.'"

Mr. Mead

"Okay, so Jaime, why, why did you pick the numbers 2 and 3? Why did you pick where the x-values were 2 and 3? Why did you pick those two? [W]hat would happen, Jaime, if you used 15 and 19, what would be the rate of change then? Or 21 and 25? What would be the rate of change if you did that? Somehow to get them to think about that it is the change in y over the change in x and so, once again were they…did they have a reason? I'd be very interested 'cause those aren't the first, most of my students would pick the first two numbers. Right, so, this person didn't pick the first two numbers, they picked 2 and 3, which I'm intrigued by. Uhmm. Did the reason that she picked it or he picked it was it because they only when up by one? So they actually understood that and they wanted to pick those. Or, was it just kind of random that they picked those two numbers, so…I'd really have to dig into why they picked those two numbers, and if they picked them for the reason that that went up by one."

Mr. Mead

Actor	MOST Student
Object of Consideration	
Move	

"Okay, so Jaime, why, why did you pick the numbers 2 and 3? Why did you pick where the x-values were 2 and 3? Why did you pick those two? [W]hat would happen, Jaime, if you used 15 and 19, what would be the rate of change then? Or 21 and 25? What would be the rate of change if you did that? Somehow to get them to think about that it is the change in y over the change in x and so, once again were they…did they have a reason? I'd be very interested 'cause those aren't the first, most of my students would pick the first two numbers. Right, so, this person didn't pick the first two numbers, they picked 2 and 3, which I'm intrigued by. Uhmm. Did the reason that she picked it or he picked it was it because they only when up by one? So they actually understood that and they wanted to pick those. Or, was it just kind of random that they picked those two numbers, so…I'd really have to dig into why they picked those two numbers, and if they picked them for the reason that that went up by one."

Mathematical Opportunities in Student Thinking

Mr. Mead

Actor	MOST Student
Object of Consideration	MP: Core
Move	

"Okay, so Jaime, why, why did you pick the numbers 2 and 3? Why did you pick where the x-values were 2 and 3? Why did you pick those two? [W]hat would happen, Jaime, if you used 15 and 19, what would be the rate of change then? Or 21 and 25? What would be the rate of change if you did that? Somehow to get them to think about that it is the change in y over the change in x and so, once again were they…did they have a reason? I'd be very interested 'cause those aren't the first, most of my students would pick the first two numbers. Right, so, this person didn't pick the first two numbers, they picked 2 and 3, which I'm intrigued by. Uhmm. Did the reason that she picked it or he picked it was it because they only when up by one? So they actually understood that and they wanted to pick those. Or, was it just kind of random that they picked those two numbers, so…I'd really have to dig into why they picked those two numbers, and if they picked them for the reason that that went up by one."

Mr. Mead

Actor	MOST Student
Object of Consideration	MP: Core SM Words: Implicit
Move	

"Okay, so Jaime, why, why did you pick the numbers 2 and 3? Why did you pick where the x-values were 2 and 3? Why did you pick those two? [W]hat would happen, Jaime, if you used 15 and 19, what would be the rate of change then? Or 21 and 25? What would be the rate of change if you did that? Somehow to get them to think about that it is the change in y over the change in x and so, once again were they...did they have a reason? I'd be very interested 'cause those aren't the first, most of my students would pick the first two numbers. Right, so, this person didn't pick the first two numbers, they picked 2 and 3, which I'm intrigued by. Uhmm. Did the reason that she picked it or he picked it was it because they only when up by one? So they actually understood that and they wanted to pick those. Or, was it just kind of random that they picked those two numbers, so...I'd really have to dig into why they picked those two numbers, and if they picked them for the reason that that went up by one."

Mathematical Opportunities in Student Thinking

Mr. Mead

Actor	MOST Student
Object of Consideration	MP: Core SM Words: Implicit; SM Ideas: Core
Move	

"Okay, so Jaime, why, why did you pick the numbers 2 and 3? Why did you pick where the x-values were 2 and 3? Why did you pick those two? [W]hat would happen, Jaime, if you used 15 and 19, what would be the rate of change then? Or 21 and 25? What would be the rate of change if you did that? Somehow to get them to think about that it is the change in y over the change in x and so, once again were they…did they have a reason? I'd be very interested 'cause those aren't the first, most of my students would pick the first two numbers. Right, so, this person didn't pick the first two numbers, they picked 2 and 3, which I'm intrigued by. Uhmm. Did the reason that she picked it or he picked it was it because they only when up by one? So they actually understood that and they wanted to pick those. Or, was it just kind of random that they picked those two numbers, so…I'd really have to dig into why they picked those two numbers, and if they picked them for the reason that that went up by one."

MOST — Mathematical Opportunities in Student Thinking

Mr. Mead

Actor	MOST Student
Object of Consideration	MP: Core SM Words: Implicit; SM Ideas: Core
Move	Elaborate

"Okay, so Jaime, why, why did you pick the numbers 2 and 3? Why did you pick where the x-values were 2 and 3? Why did you pick those two? [W]hat would happen, Jaime, if you used 15 and 19, what would be the rate of change then? Or 21 and 25? What would be the rate of change if you did that? Somehow to get them to think about that it is the change in y over the change in x and so, once again were they…did they have a reason? I'd be very interested 'cause those aren't the first, most of my students would pick the first two numbers. Right, so, this person didn't pick the first two numbers, they picked 2 and 3, which I'm intrigued by. Uhmm. Did the reason that she picked it or he picked it was it because they only when up by one? So they actually understood that and they wanted to pick those. Or, was it just kind of random that they picked those two numbers, so…I'd really have to dig into why they picked those two numbers, and if they picked them for the reason that that went up by one."

Ms. Shaw

“At this point, I think what I want Jamie to show is how that representation shows up on the graph as well. And the reason I want to do that is because if they’re looking at--well, here’s an assumption that I’m making. When they graphed it, they graphed using a scale of one for the weeks. Because what I want Jamie to think about is ‘Yes, that two shows up because you’re going over every single one on the graph. What would happen if I took out week two and I had week one and week three? Then what am I looking at and how would that show up? So I want to start out looking at the graph first. Why does that show up in the graph? How do you resolve that? And then what happens if it’s not every week? What happens if it’s every other week? And see, pose that question to them and see if they could justify and resolve that. So, you know if I did twenty-one minus seventeen now I have four, why is it four instead of two? And be able to bring that back to the graph and to the table."

Ms. Shaw

Actor	MOST Student
Object of Consideration	
Move	

“At this point, I think what I want Jamie to show is how that representation shows up on the graph as well. And the reason I want to do that is because if they’re looking at--well, here’s an assumption that I’m making. When they graphed it, they graphed using a scale of one for the weeks. Because what I want Jamie to think about is ‘Yes, that two shows up because you’re going over every single one on the graph. What would happen if I took out week two and I had week one and week three? Then what am I looking at and how would that show up? So I want to start out looking at the graph first. Why does that show up in the graph? How do you resolve that? And then what happens if it’s not every week? What happens if it’s every other week? And see, pose that question to them and see if they could justify and resolve that. So, you know if I did twenty-one minus seventeen now I have four, why is it four instead of two? And be able to bring that back to the graph and to the table."

Ms. Shaw

Actor	MOST Student
Object of Consideration	MP: Core
Move	

"At this point, I think what I want Jamie to show is how that representation shows up on the graph as well. And the reason I want to do that is because if they're looking at--well, here's an assumption that I'm making. When they graphed it, they graphed using a scale of one for the weeks. Because what I want Jamie to think about is 'Yes, that two shows up because you're going over every single one on the graph. What would happen if I took out week two and I had week one and week three? Then what am I looking at and how would that show up? So I want to start out looking at the graph first. Why does that show up in the graph? How do you resolve that? And then what happens if it's not every week? What happens if it's every other week? And see, pose that question to them and see if they could justify and resolve that. So, you know if I did twenty-one minus seventeen now I have four, why is it four instead of two? And be able to bring that back to the graph and to the table."

Ms. Shaw

Actor	MOST Student
Object of Consideration	MP: Core SM Words: Implicit
Move	

"At this point, I think what I want Jamie to show is how that representation shows up on the graph as well. And the reason I want to do that is because if they're looking at--well, here's an assumption that I'm making. When they graphed it, they graphed using a scale of one for the weeks. Because what I want Jamie to think about is 'Yes, that two shows up because you're going over every single one on the graph. What would happen if I took out week two and I had week one and week three? Then what am I looking at and how would that show up? So I want to start out looking at the graph first. Why does that show up in the graph? How do you resolve that? And then what happens if it's not every week? What happens if it's every other week? And see, pose that question to them and see if they could justify and resolve that. So, you know if I did twenty-one minus seventeen now I have four, why is it four instead of two? And be able to bring that back to the graph and to the table."

Mathematical Opportunities in Student Thinking

Ms. Shaw

Actor	MOST Student
Object of Consideration	MP: Core SM Words: Implicit; SM Ideas: Peripheral
Move	

"At this point, I think what I want Jamie to show is how that representation shows up on the graph as well. And the reason I want to do that is because if they're looking at--well, here's an assumption that I'm making. When they graphed it, they graphed using a scale of one for the weeks. Because what I want Jamie to think about is 'Yes, that two shows up because you're going over every single one on the graph. What would happen if I took out week two and I had week one and week three? Then what am I looking at and how would that show up? So I want to start out looking at the graph first. Why does that show up in the graph? How do you resolve that? And then what happens if it's not every week? What happens if it's every other week? And see, pose that question to them and see if they could justify and resolve that. So, you know if I did twenty-one minus seventeen now I have four, why is it four instead of two? And be able to bring that back to the graph and to the table."

Ms. Shaw

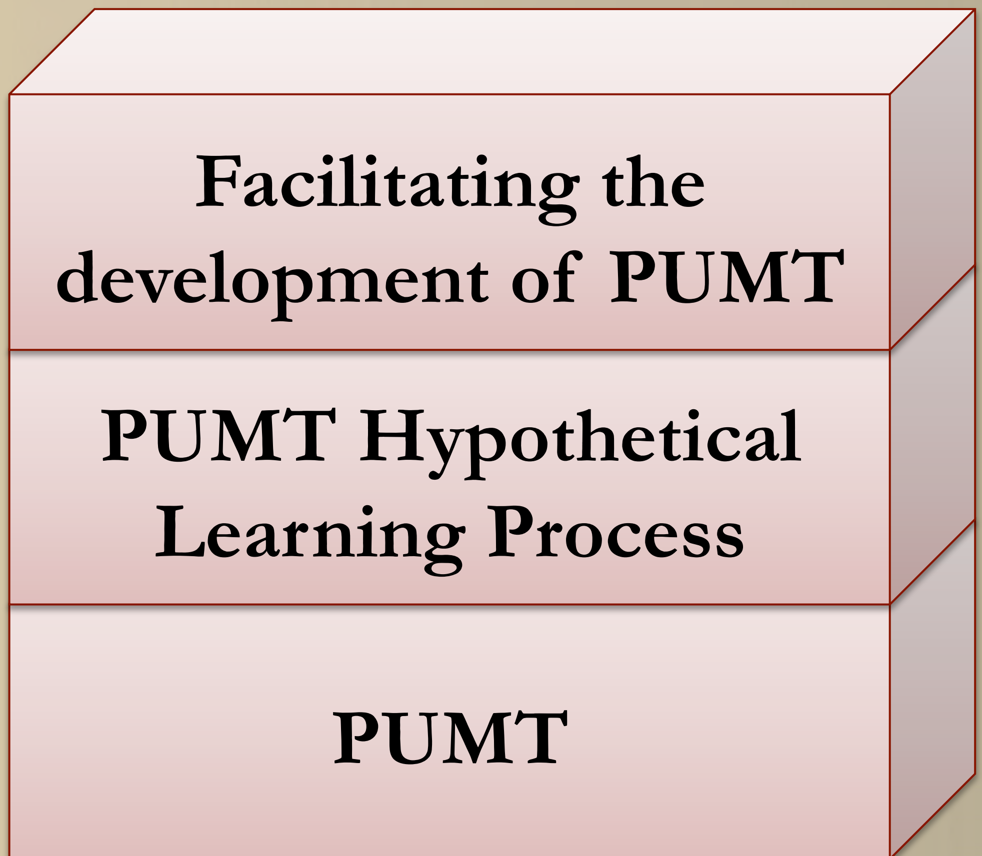
Actor	MOST Student
Object of Consideration	MP: Core SM Words: Implicit; SM Ideas: Peripheral
Move	Connect

"At this point, I think what I want Jamie to show is how that representation shows up on the graph as well. And the reason I want to do that is because if they're looking at--well, here's an assumption that I'm making. When they graphed it, they graphed using a scale of one for the weeks. Because what I want Jamie to think about is 'Yes, that two shows up because you're going over every single one on the graph. What would happen if I took out week two and I had week one and week three? Then what am I looking at and how would that show up? So I want to start out looking at the graph first. Why does that show up in the graph? How do you resolve that? And then what happens if it's not every week? What happens if it's every other week? And see, pose that question to them and see if they could justify and resolve that. So, you know if I did twenty-one minus seventeen now I have four, why is it four instead of two? And be able to bring that back to the graph and to the table."

Teacher Decisions

	Ms. Dean	Mr. Mead	Ms. Shaw
Actor	Teacher	MOST Student	MOST Student
MP	Core	Core	Core
SM Words	Implicit	Implicit	Implicit
SM Ideas	Core	Core	Peripheral
Move	Correct	Elaborate	Connect

Developing the *Practice of Productively Using Student Mathematical Thinking* (PUMT)



Discussion Questions

- What else needs to be known to understand the *Practice of Productively Using Student Mathematical Thinking*?
  - MOST Framework
  - MOST Attributes
  - Goals, Orientations and Resources
  - Teacher Responses to MOSTs
- How can the things identified in the first bullet be researched?