Recognizing Opportunities for Productive Use of Student Thinking

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Shari L. Stockero – Michigan Technological University
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Blake E. Peterson – Brigham Young University

Mathematical Opportunities in Student Thinking

Leveraging MOSTs: Developing a Theory of Productive Use of Student Mathematical Thinking

- 4-year collaborative research project funded by the National Science Foundation
  (DRL-1220141, DRL-1220357, DRL-1220148)
- Co-Principal Investigators
  - Keith R. Leatham, Brigham Young University
  - Shari L. Stockero, Michigan Technological University
  - Blake L. Peterson, Brigham Young University
  - Laura R. Van Zoest, Western Michigan University
- Graduate Assistants
  - Lindsay Merrill, Brigham Young University
  - Mary A. Ochieng, Western Michigan University
  - Could it be you?

The Motivation for Our Work

- Graduates of our teacher education programs were successful in eliciting student thinking
- That student thinking was not being used to further students' mathematical understanding
- Saw "teachable moments" not get acted on
- Need to better understand these moments to prepare teachers to take advantage of them

From the literature…

- "critical moments in the classroom when students created a moment of choice or opportunity" (Jaworski, 1994, p. 527)
- "novel student idea[s] that prompt teachers to reflect on and rethink their instruction" (Schifter, 1996, p. 130)
- "potentially powerful learning opportunities" (Davis, 1997, p. 360)
- "significant mathematical instances" (Davies and Walker, 2005, p. 275)
- "[student's] comment provides the fodder for a content-related conversation" (Schoenfeld, 2008, p. 57)
- "crucial mathematic hinge moment[s]" (Thames and Ball, 2013, p. 31)

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Three Characteristics

- Students
- Mathematics
- Pedagogy

Three Characteristics

- Students—Student Thinking
- Mathematics
- Pedagogy

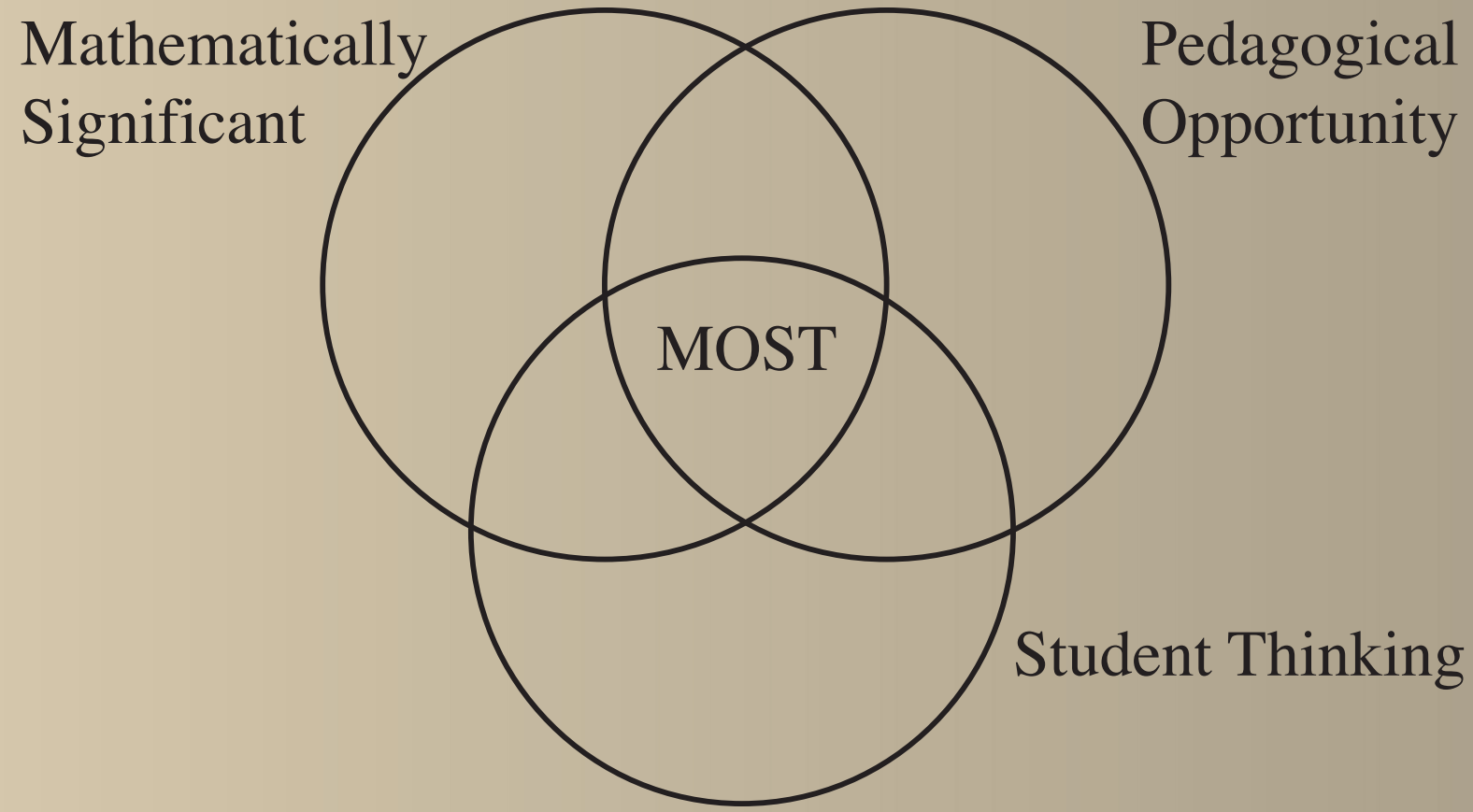
Three Characteristics

- Students—Student Thinking
- Mathematics—Mathematically Significant
- Pedagogy

Three Characteristics

- Students—Student Thinking
- Mathematics—Mathematically Significant
- Pedagogy—Pedagogical Opportunity

Relationship of the MOST Characteristics



Mathematically significant
pedagogical **O**penings
to build on **S**tudent
Thinking

Mathematical
Opportunities
in **S**tudent
Thinking

Instance of student thinking

Context: An 8th grade Algebra Class has been working on writing linear equations from tables.

Teacher: So, how do you determine the slope from any table of values?

Student: It is the amount between each number in the y-column.

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Mathematical Opportunities in Student Thinking

MOSTs Are “In-the-Moment Opportunities”

MOSTs Are "In-the-Moment Opportunities"

But opportunities for what?

Mathematical Opportunities in Student Thinking

MOSTs are opportunities for…

…the teacher to make student thinking the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

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Student: [The slope] is the amount between each number in the y-column.
Teacher (to the class): Is that always true, sometimes true, or never true?

Student: [The slope] is the amount between each number in the y-column.
Teacher (to the student): What do you mean by that?

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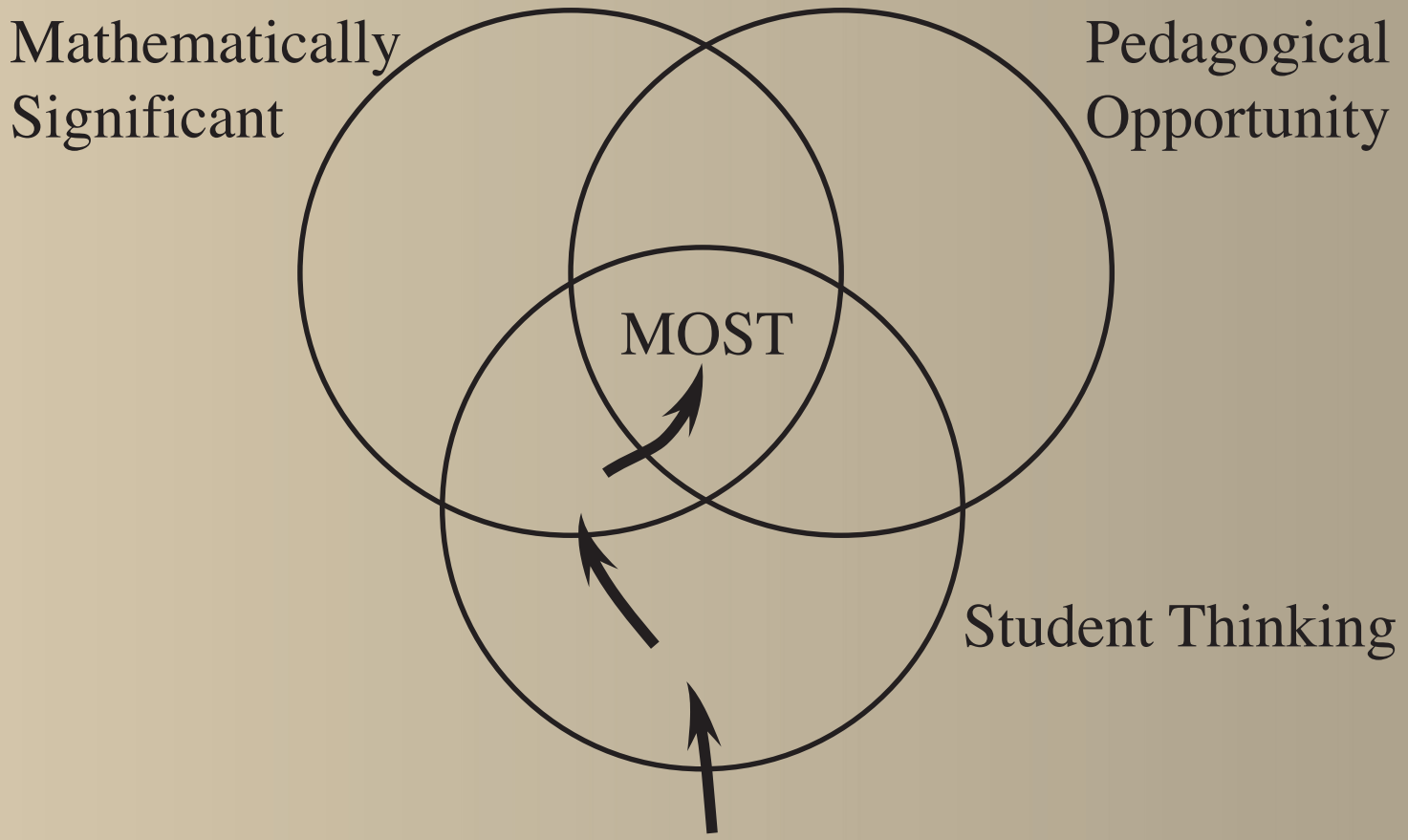
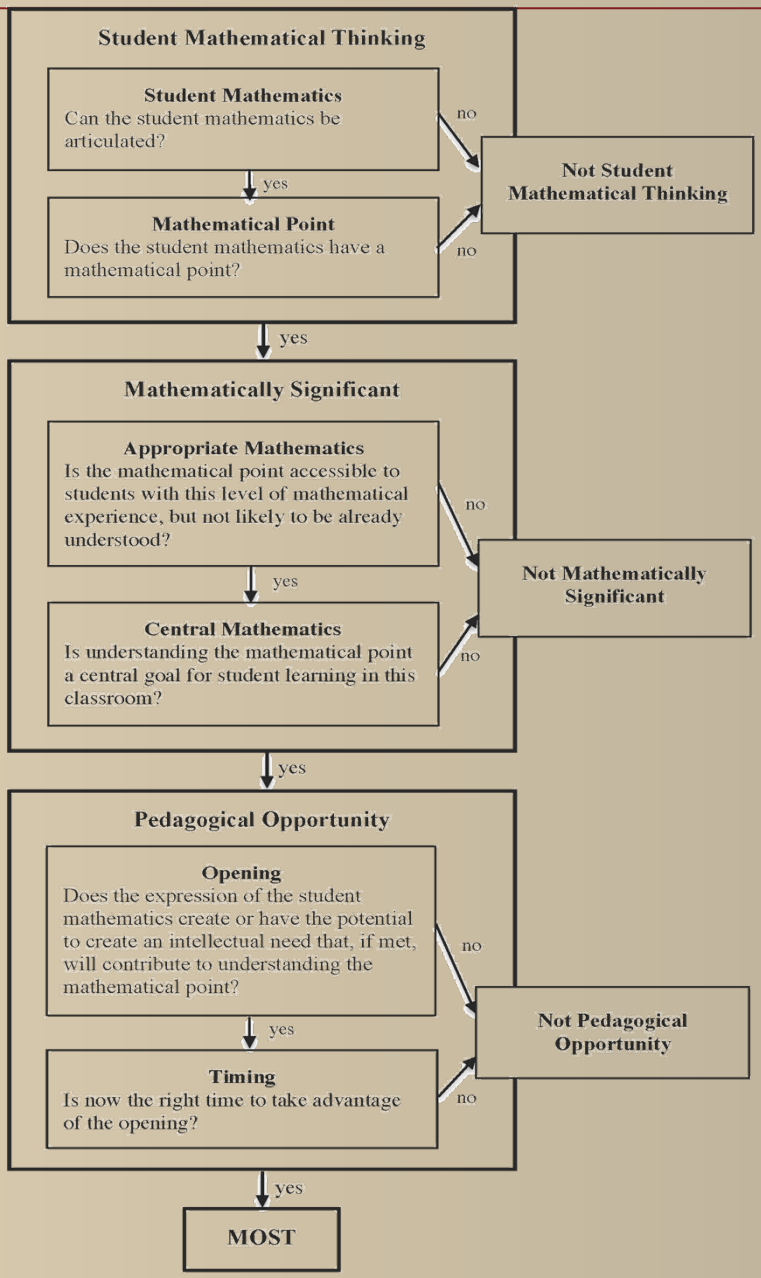
Is it a MOST?


Mathematically Significant
Pedagogical Opportunity
MOST
Student Thinking



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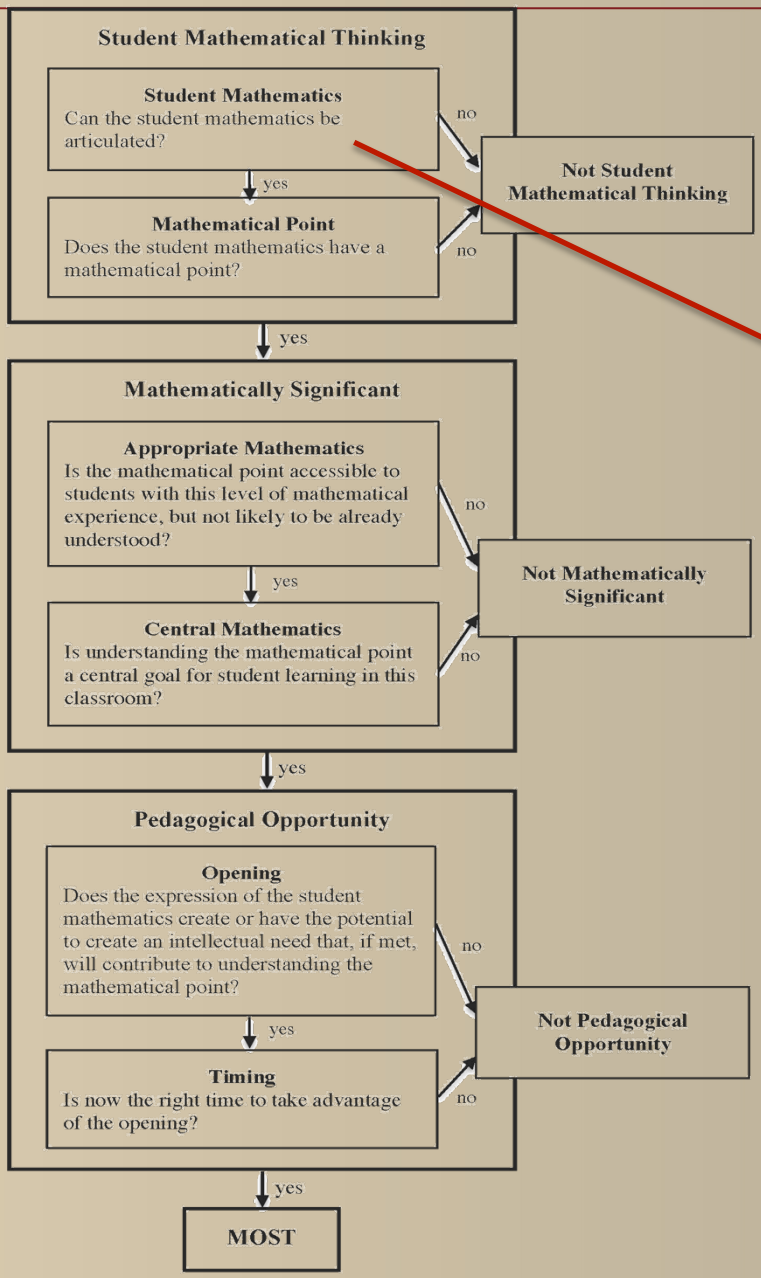
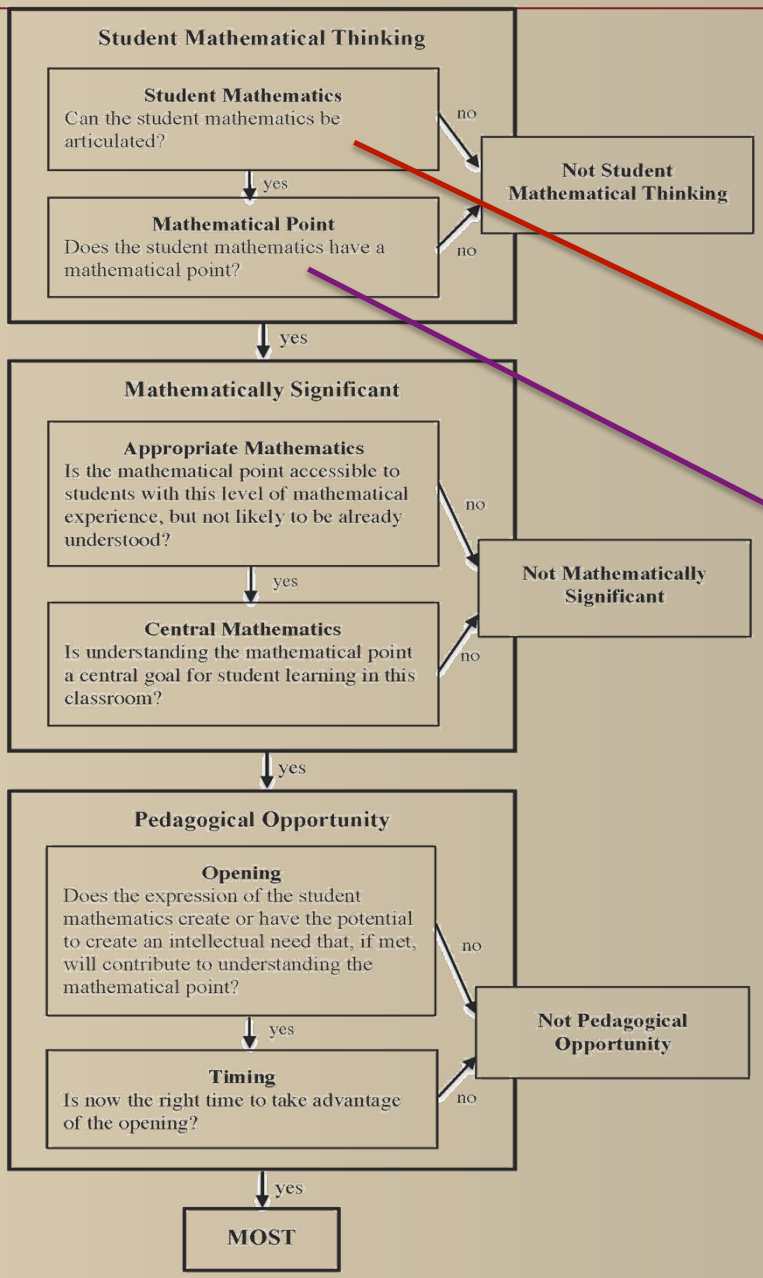


The teacher makes *student mathematical thinking* the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.



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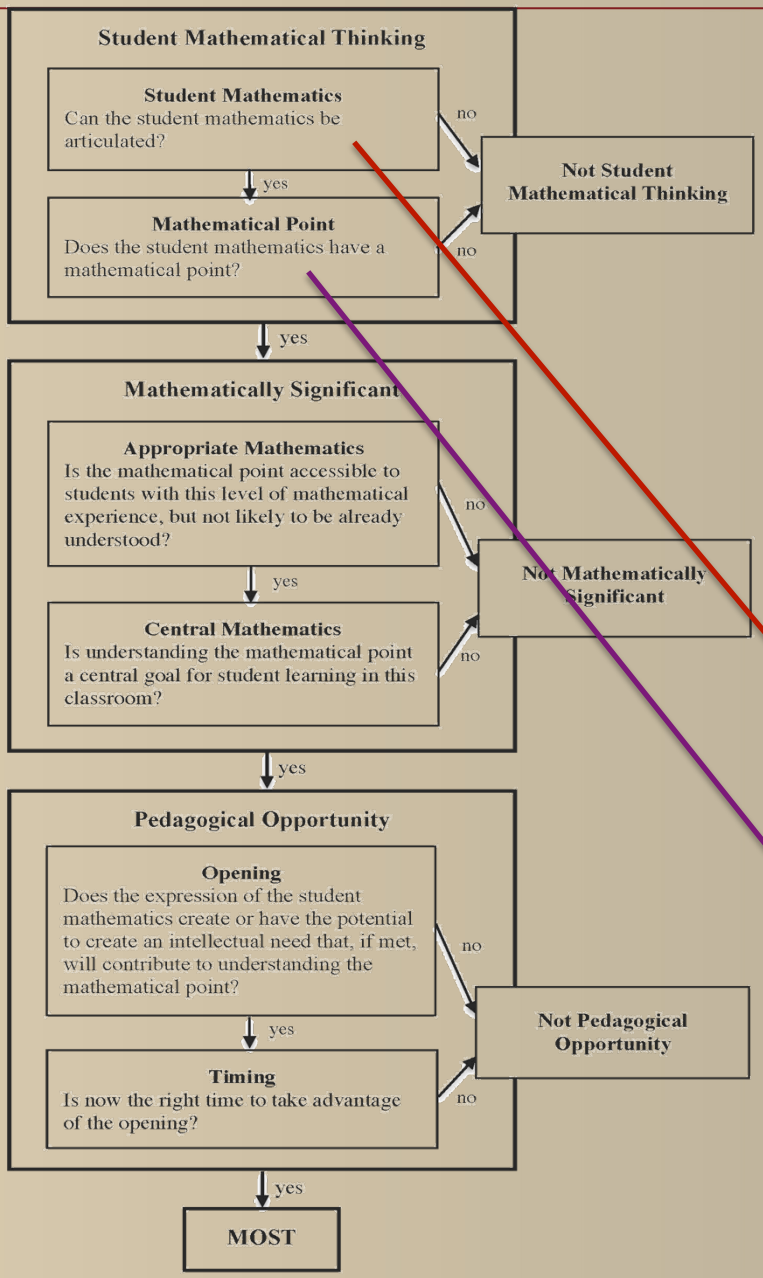
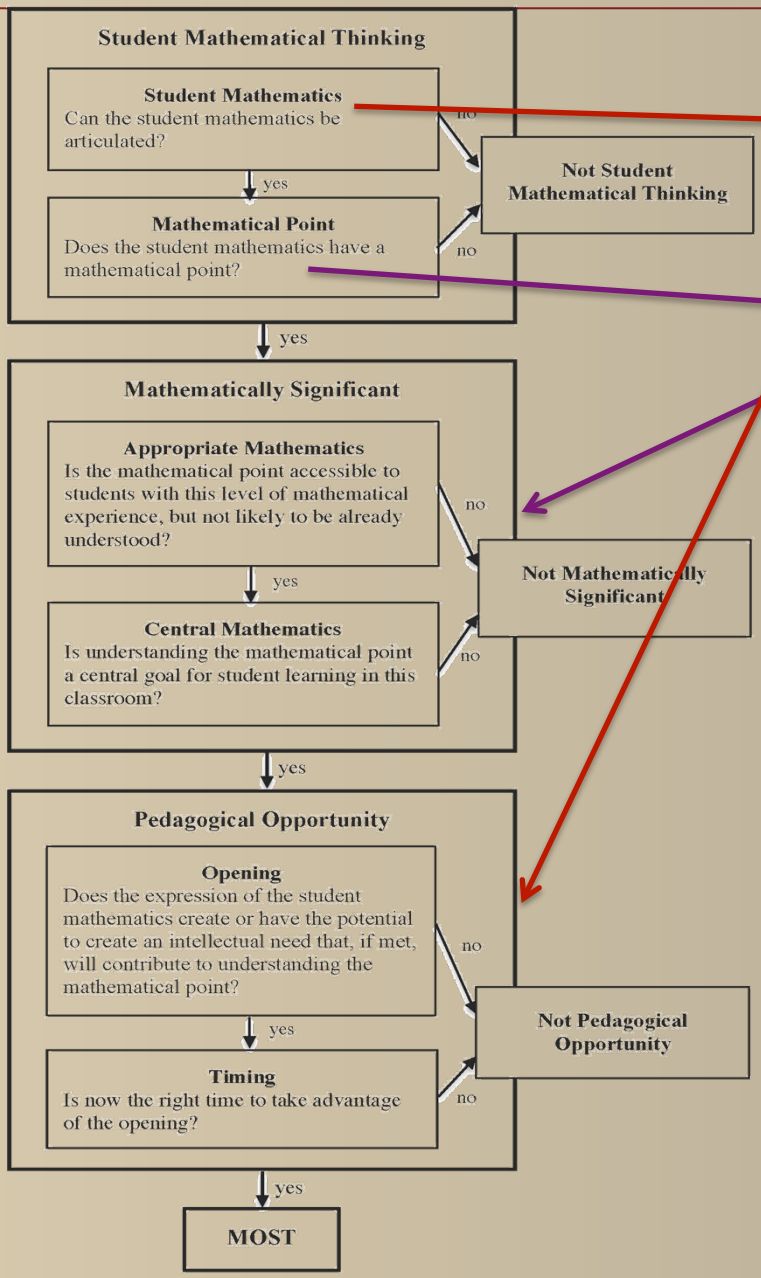


The teacher makes *student mathematical thinking* the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

Instance: [The slope] is the amount between each number in the y-column.

SM: If you are given any table, the slope is the amount between each number in the y-column.

MP: When computing slope from a table, the change in y must be divided by the change in x.



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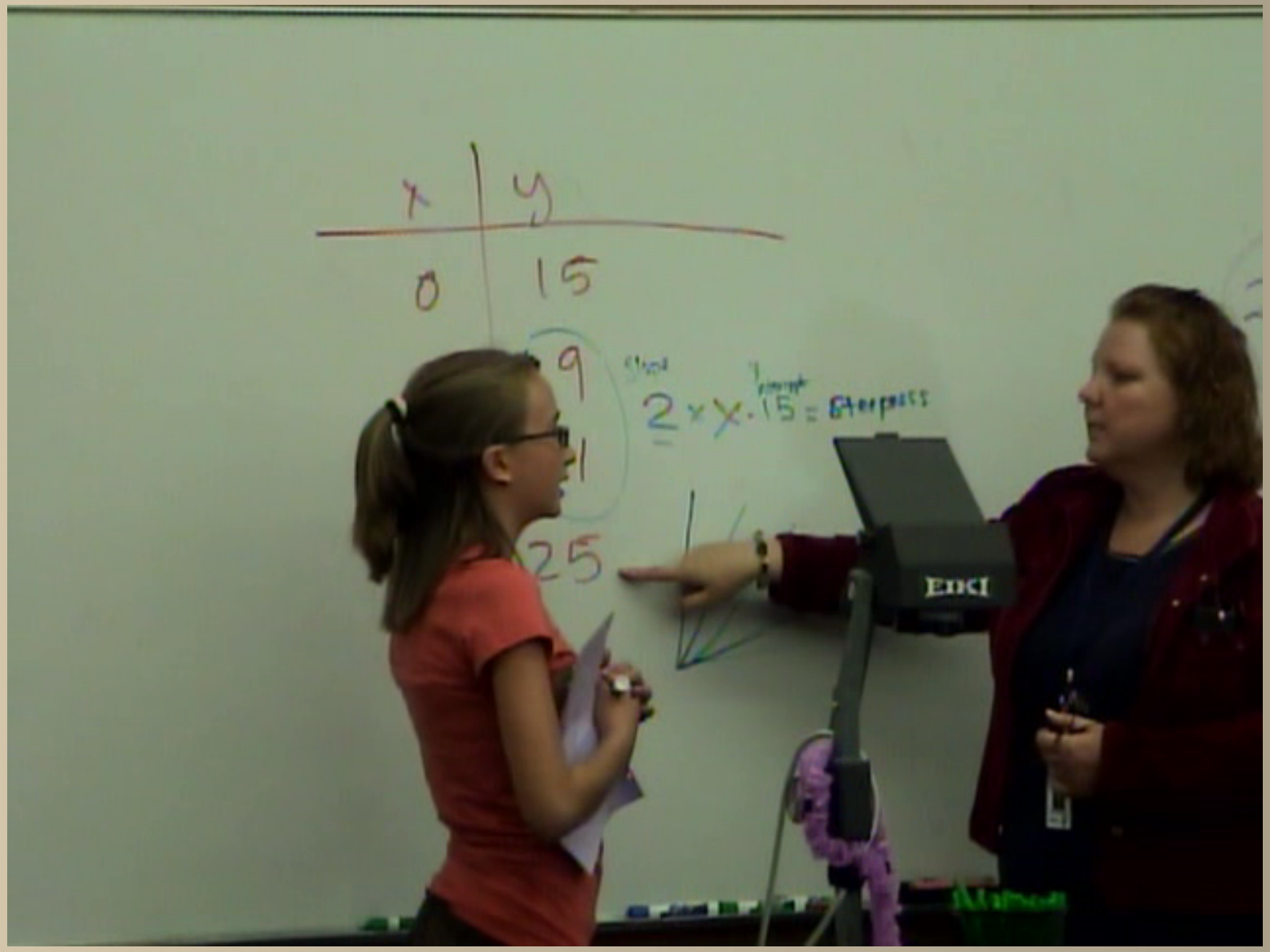
Instance: [The slope] is the amount between each number in the y-column.

Is it a MOST?

YES!

Context

- 8th grade Algebra class
- Discussion about writing a linear expression to model a problem in context


x
y
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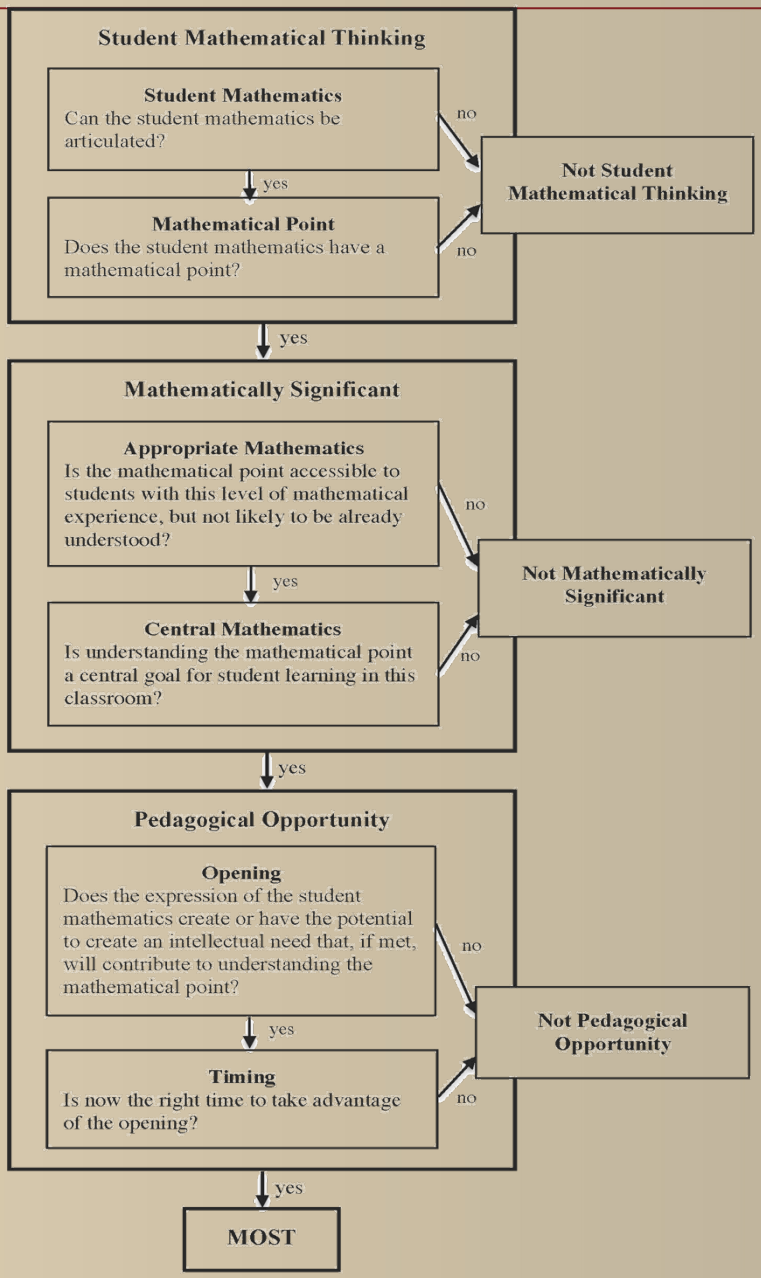
25
2 x x .15 =
EDCI

Example 1



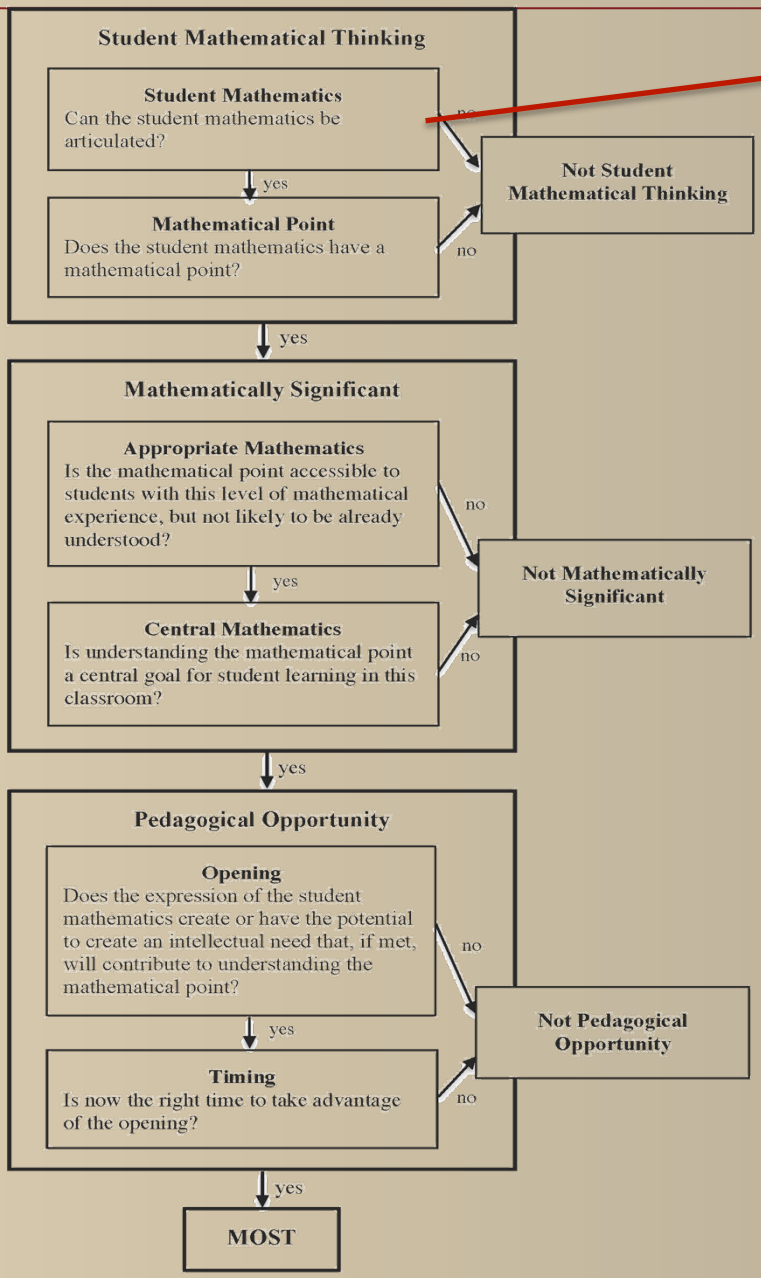
SM:

MP:

Teacher: If I take this x and multiply it by 2 and I add 15, say x is 5, what am I going to get out?

Student: Um, 25.

Is this instance a MOST?



SM:

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Problem: If I take this x and multiply it by 2 and I add 15, say x is 5, what am I going to get out?

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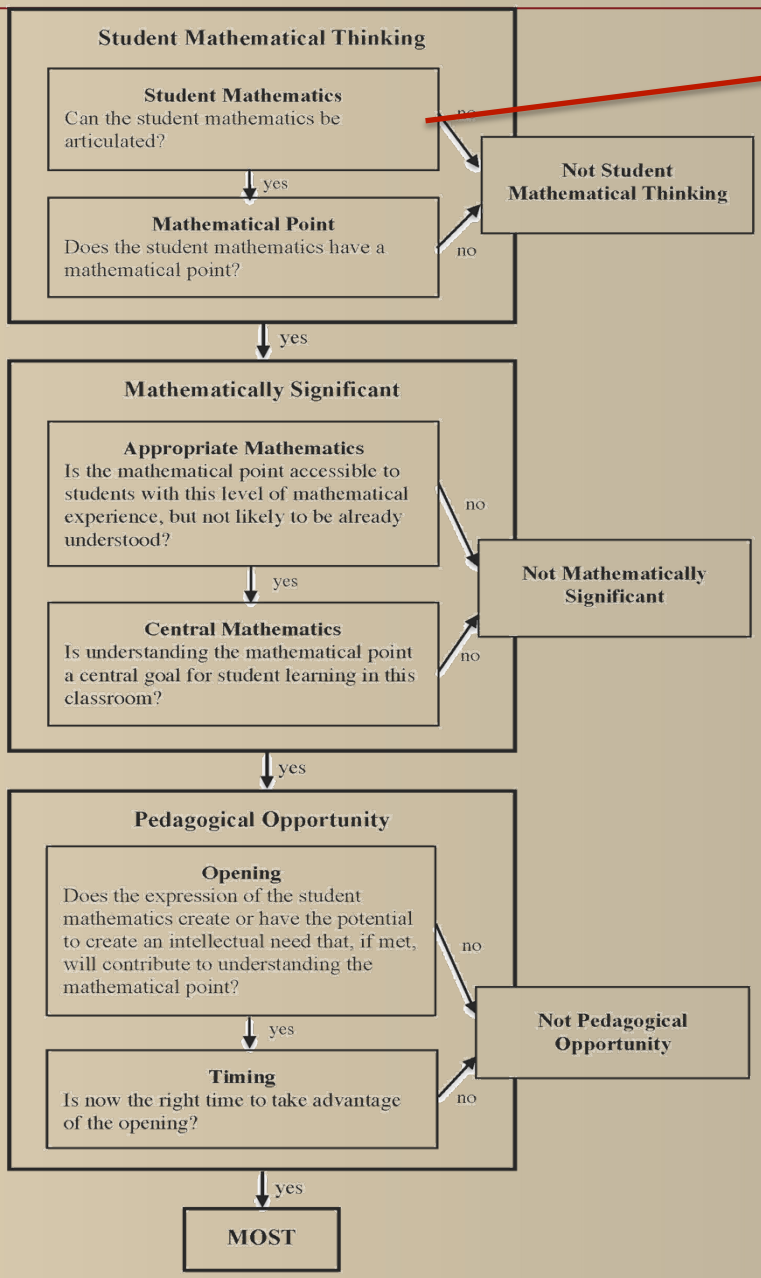

SM: If x = 5, then 2x + 15 = 25

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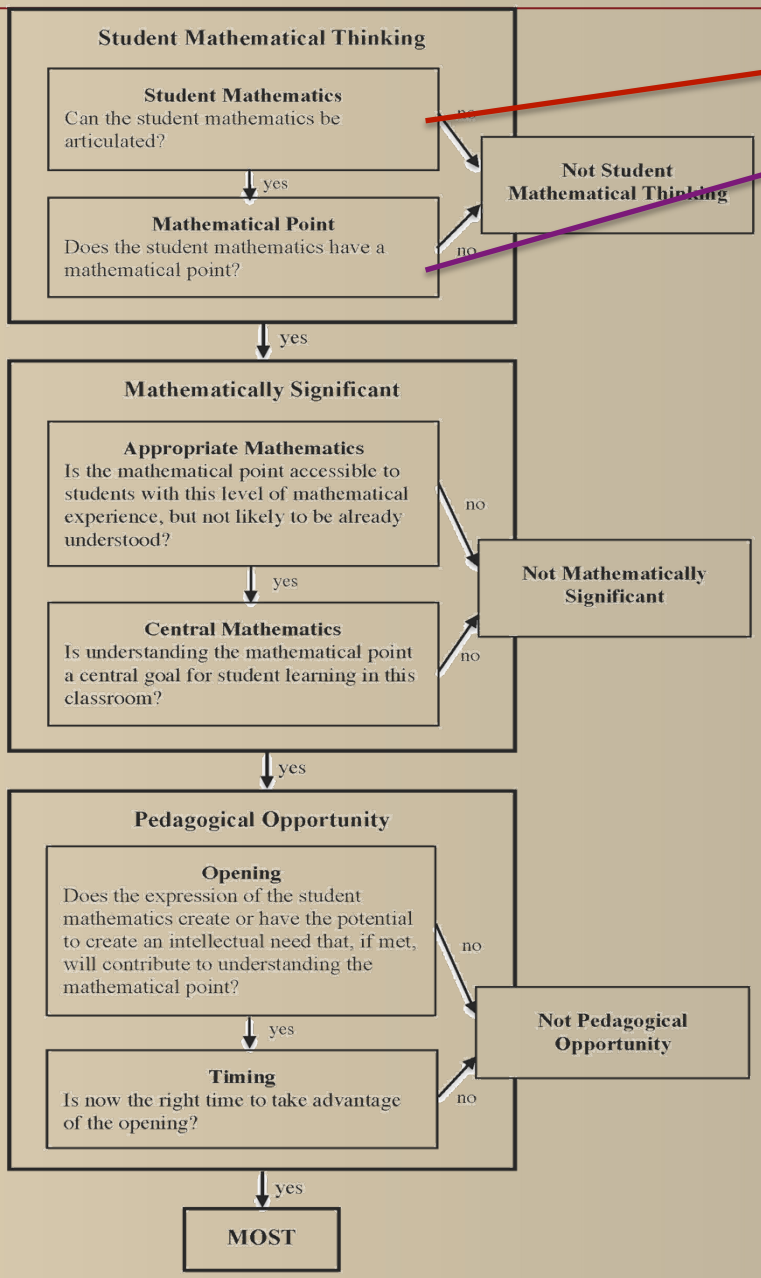

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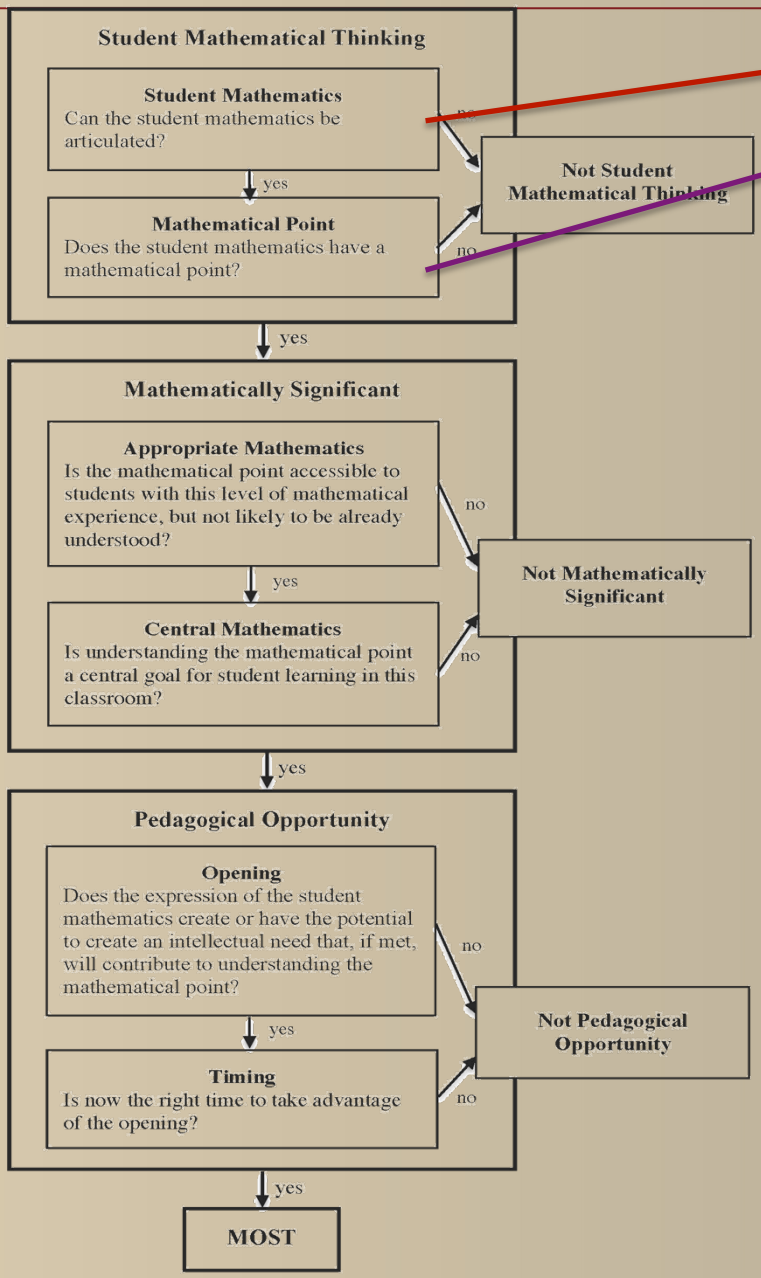

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MP: To evaluate a linear expression for a given value of x, the value is substituted for the x variable and appropriate operations are performed.

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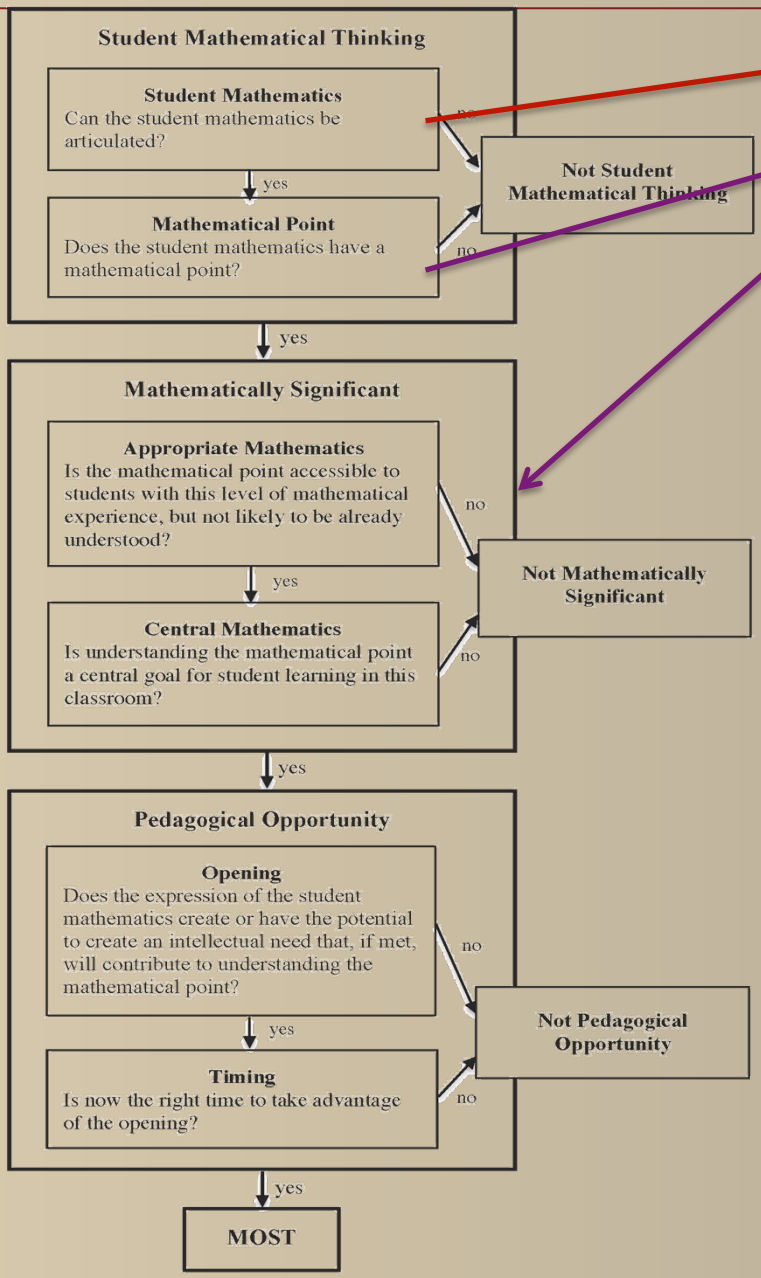

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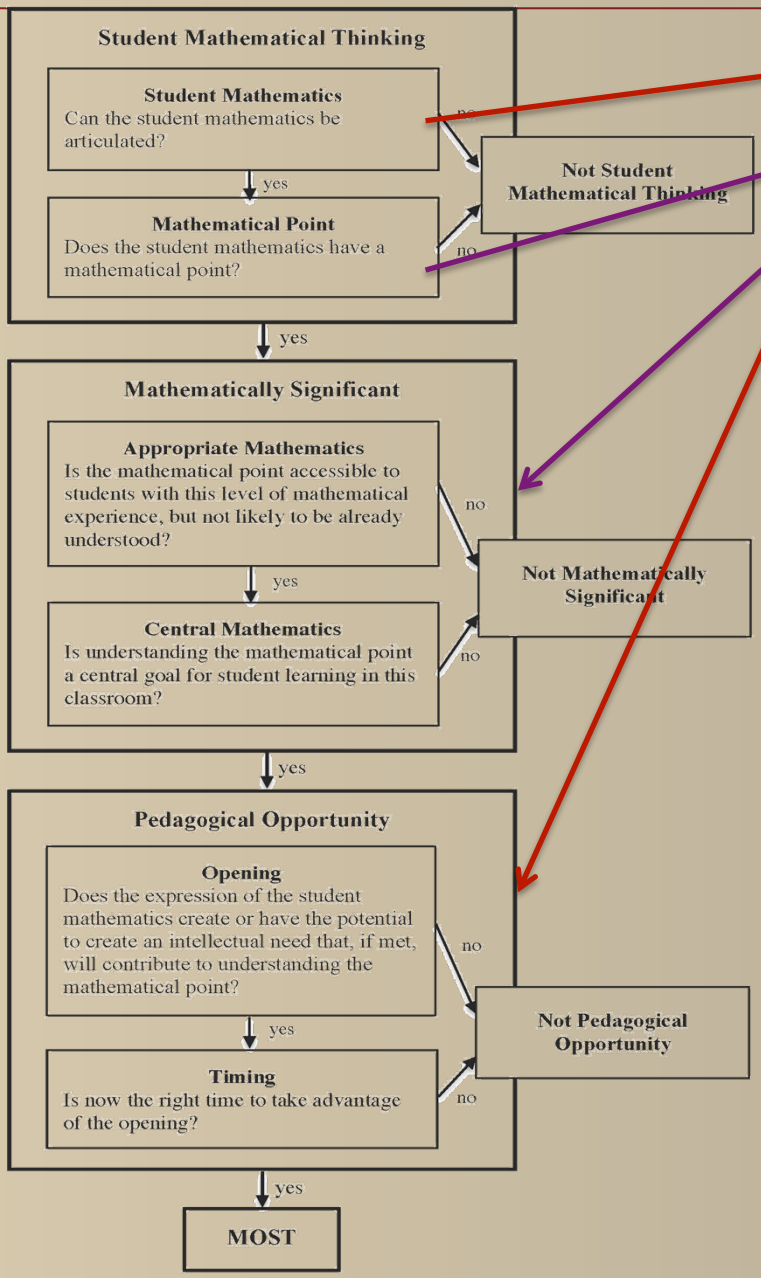

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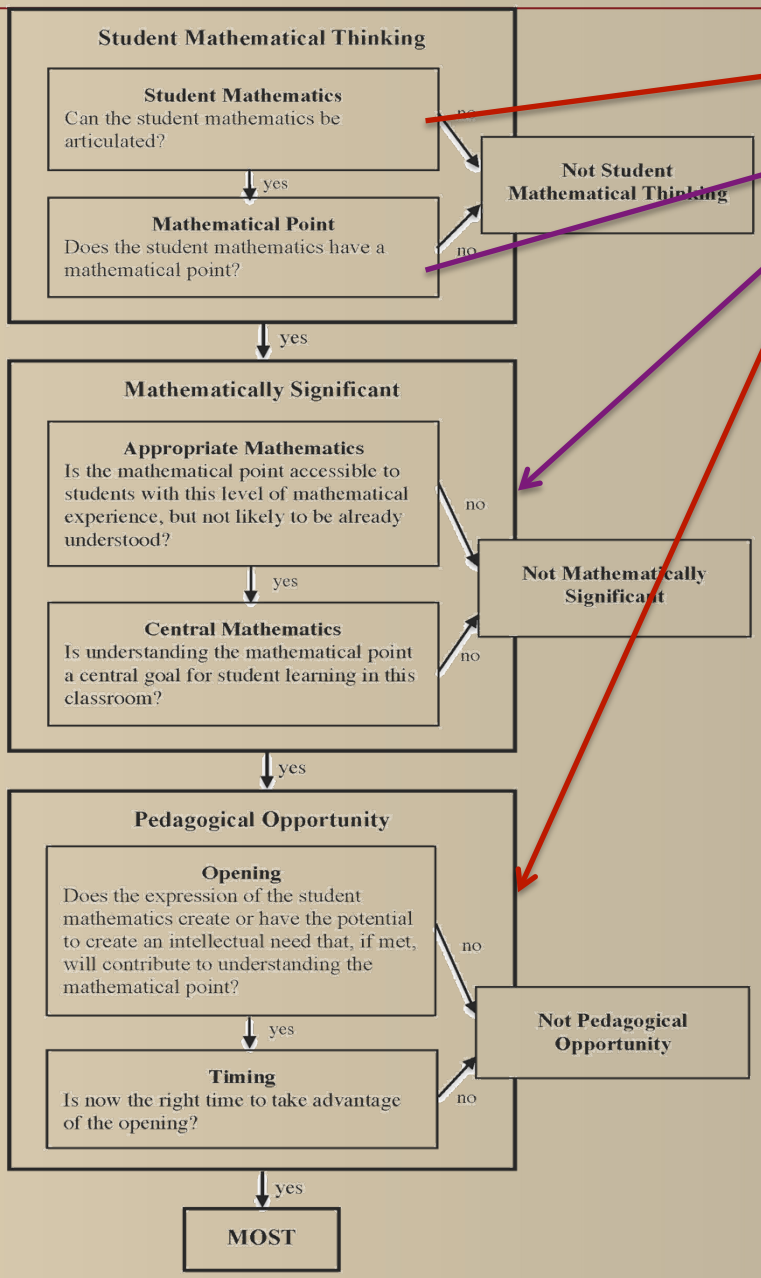

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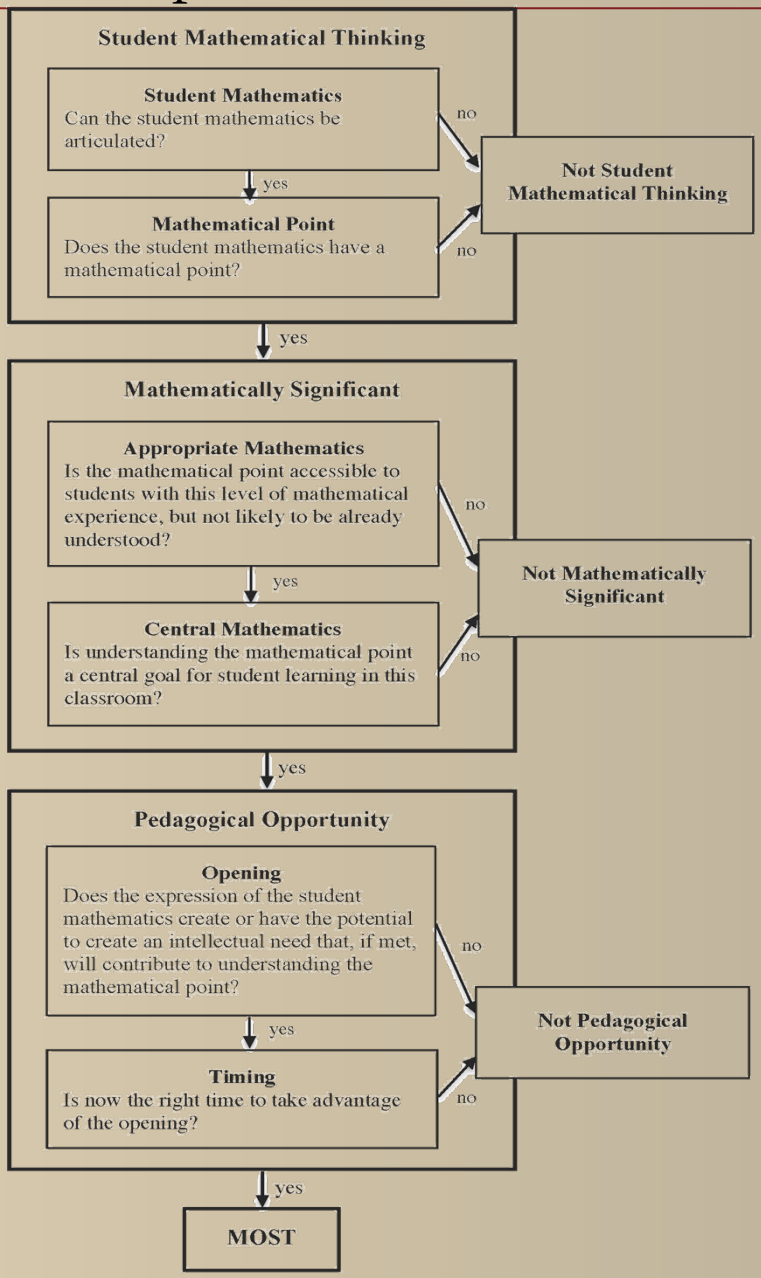
Example 2



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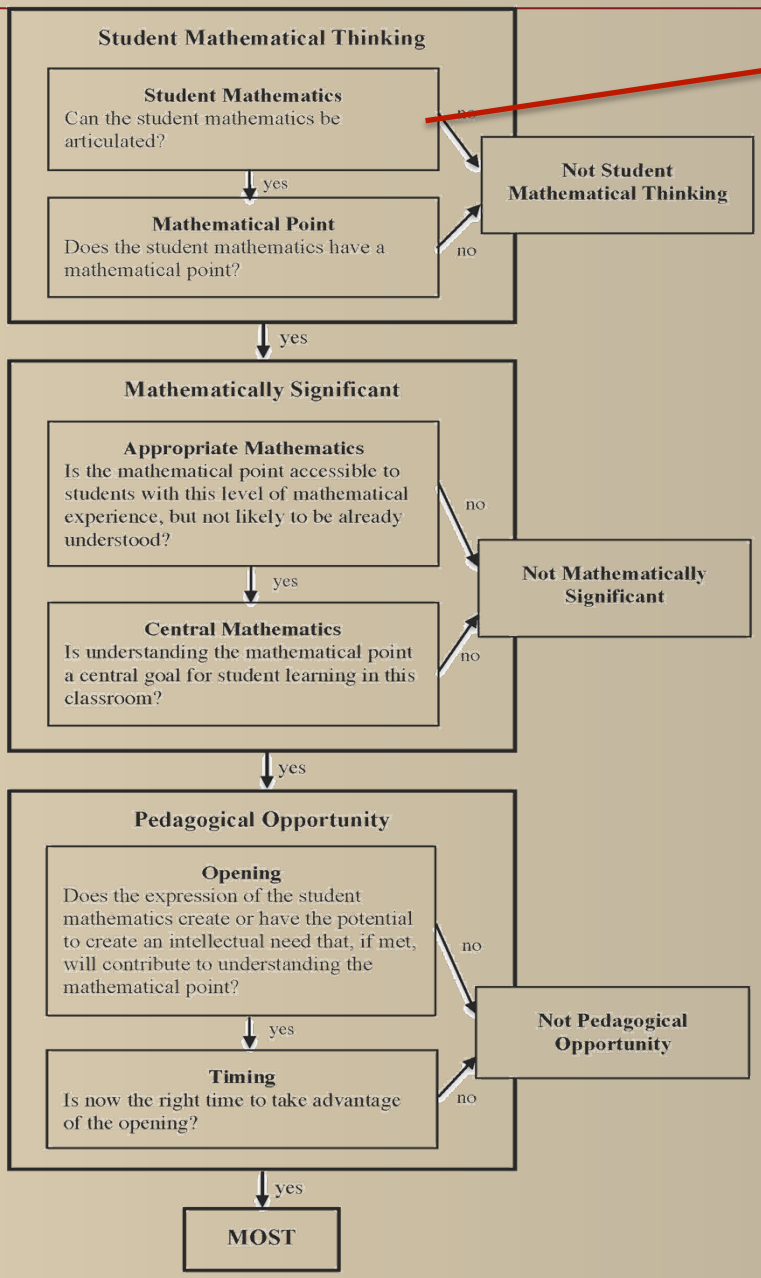
Is this instance a MOST?



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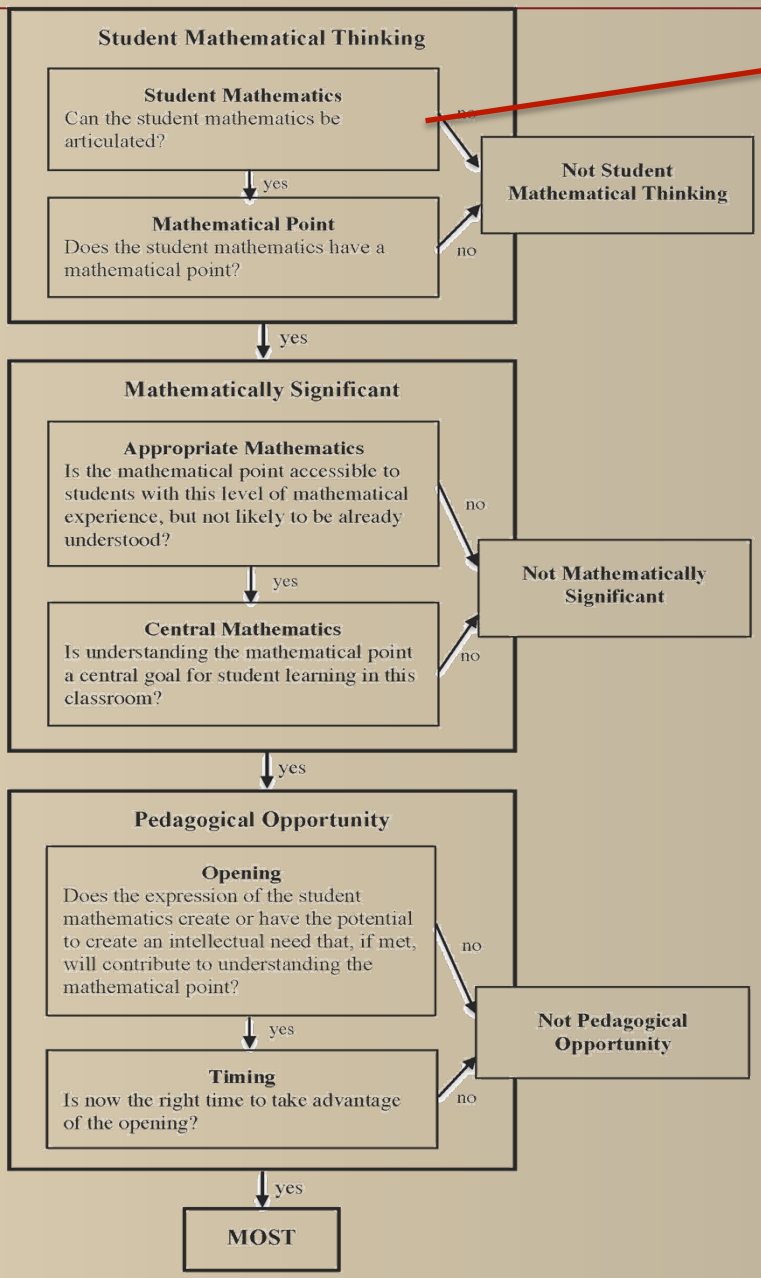

SM: The equation for a line if you are given any table is the amount between each number in the y-column times the x-variable plus the down payment equals the y variable. [using the points from the table (2, 19) and (3, 21) to calculate slope and (0, 15) to find the y-intercept to get 2x+15=y]

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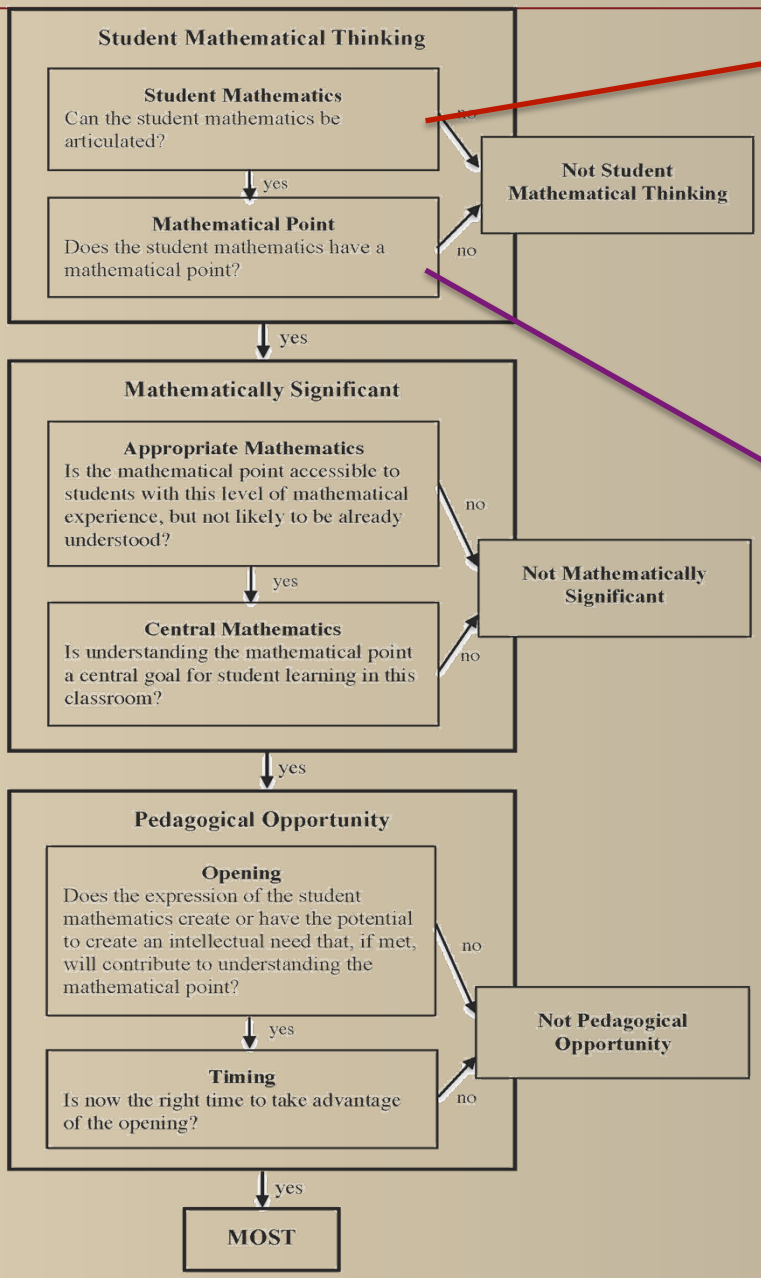

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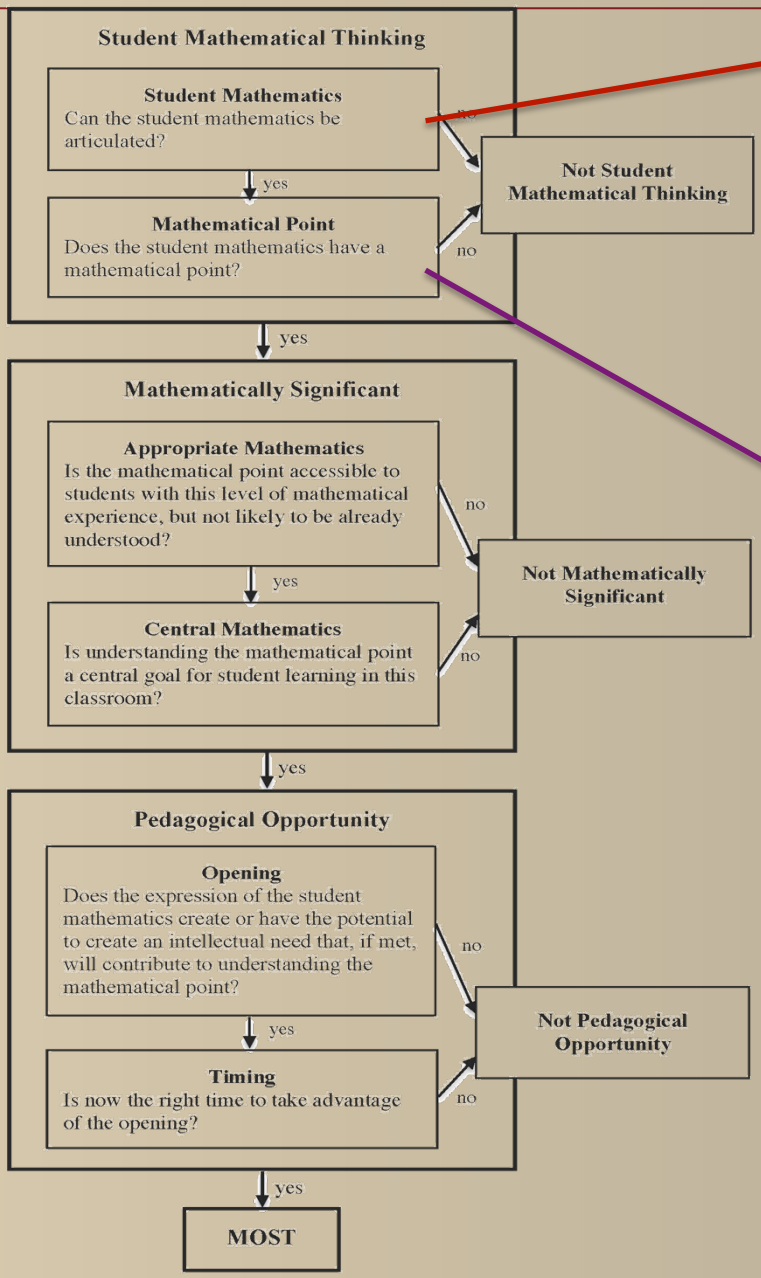

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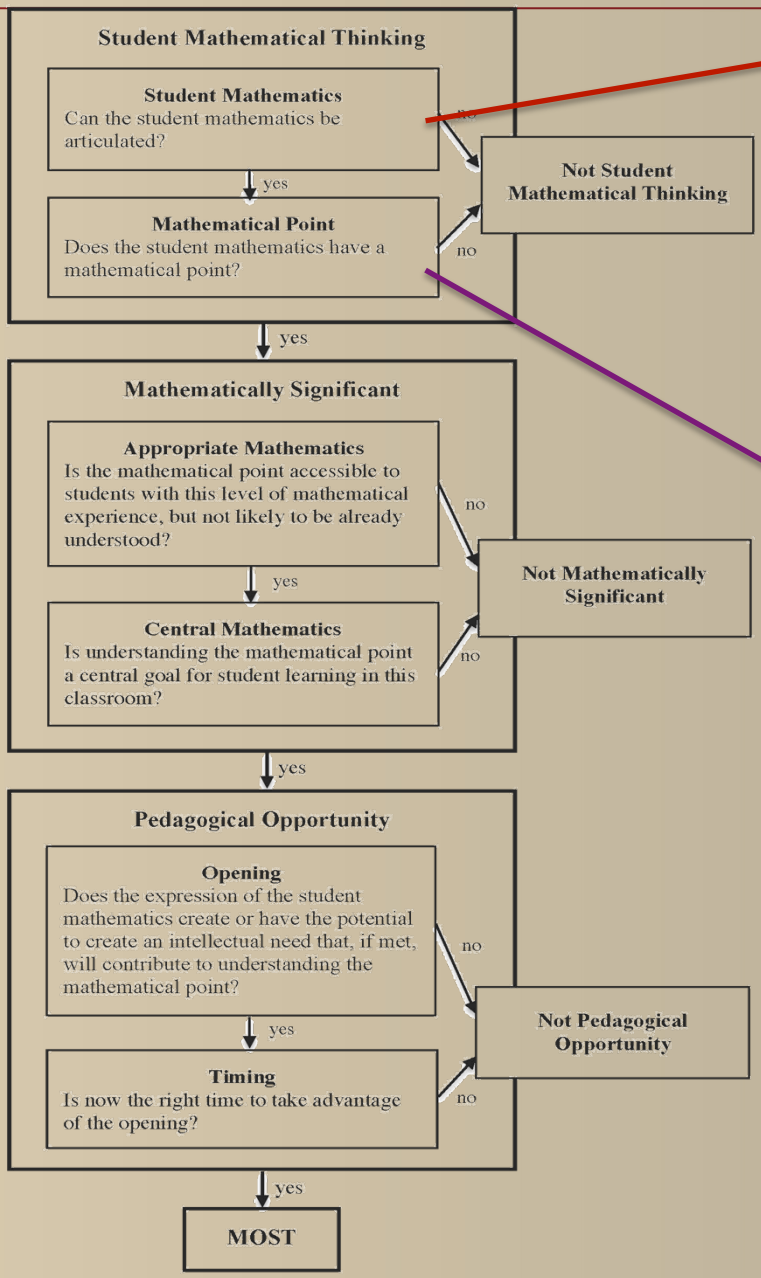

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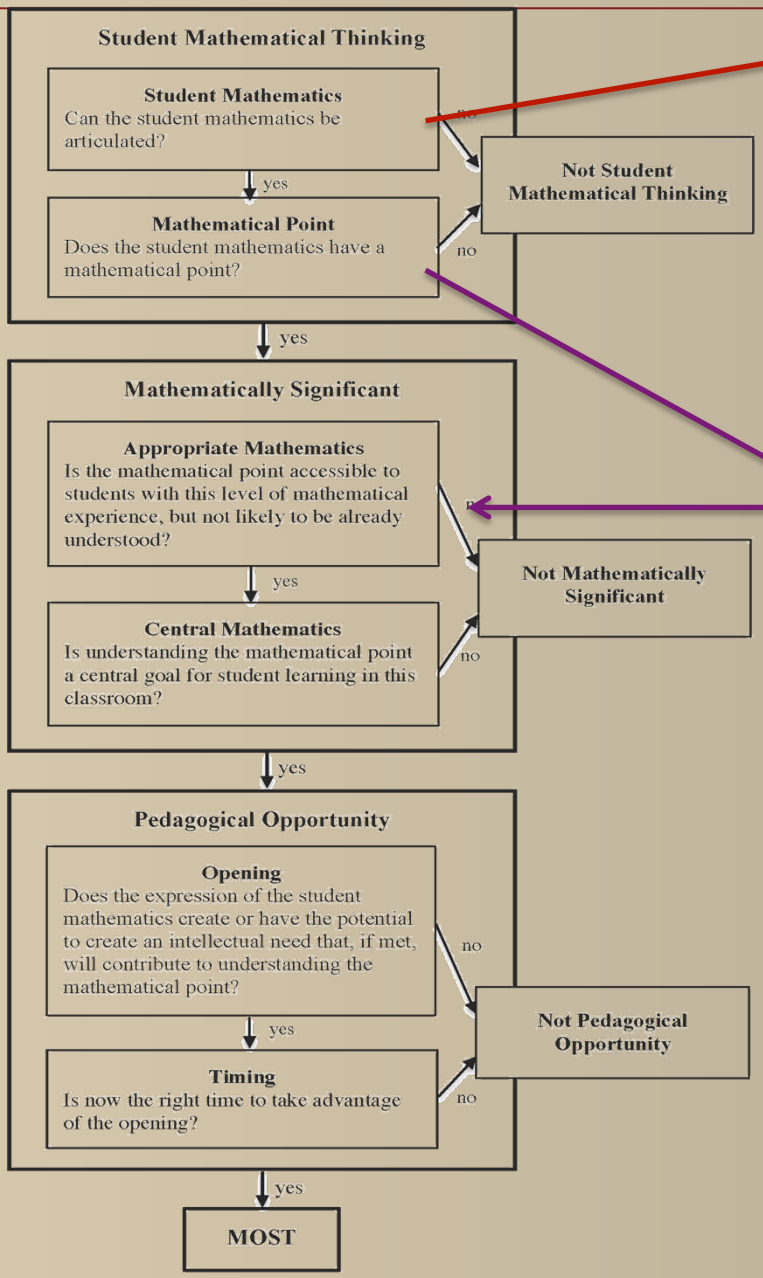

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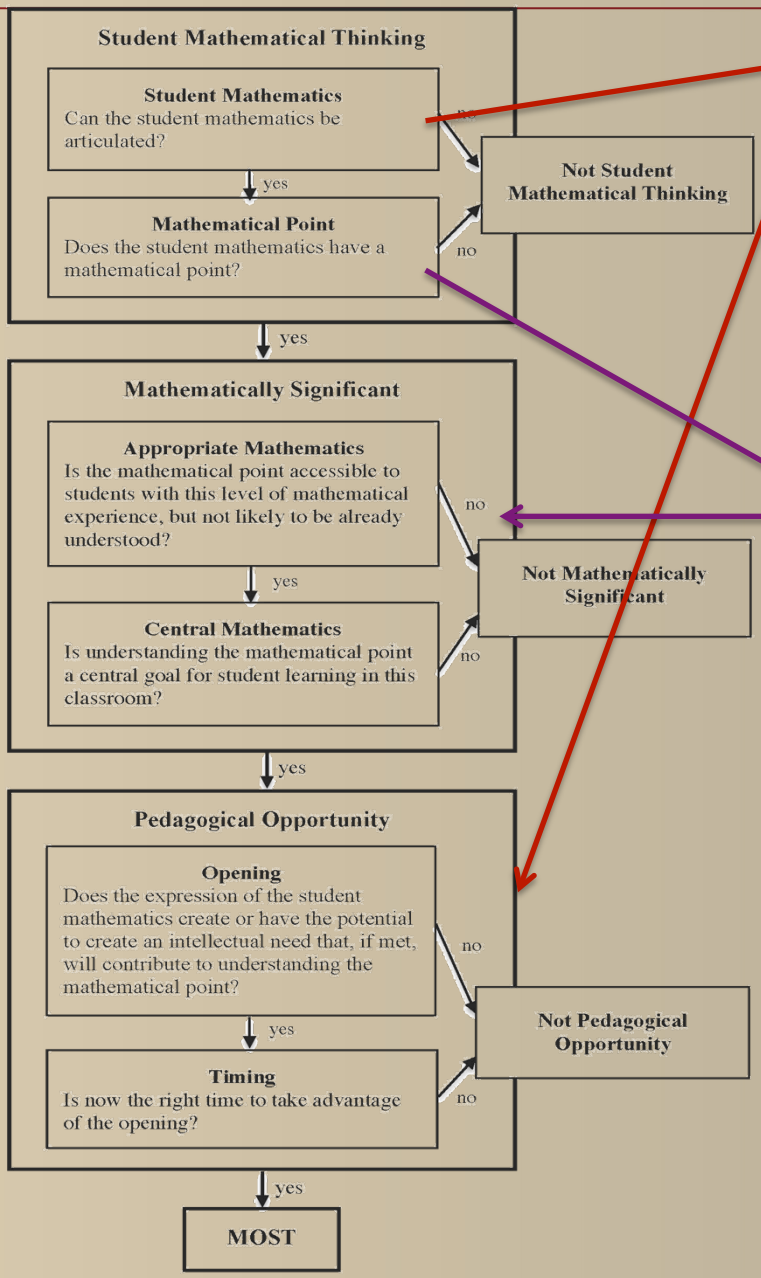

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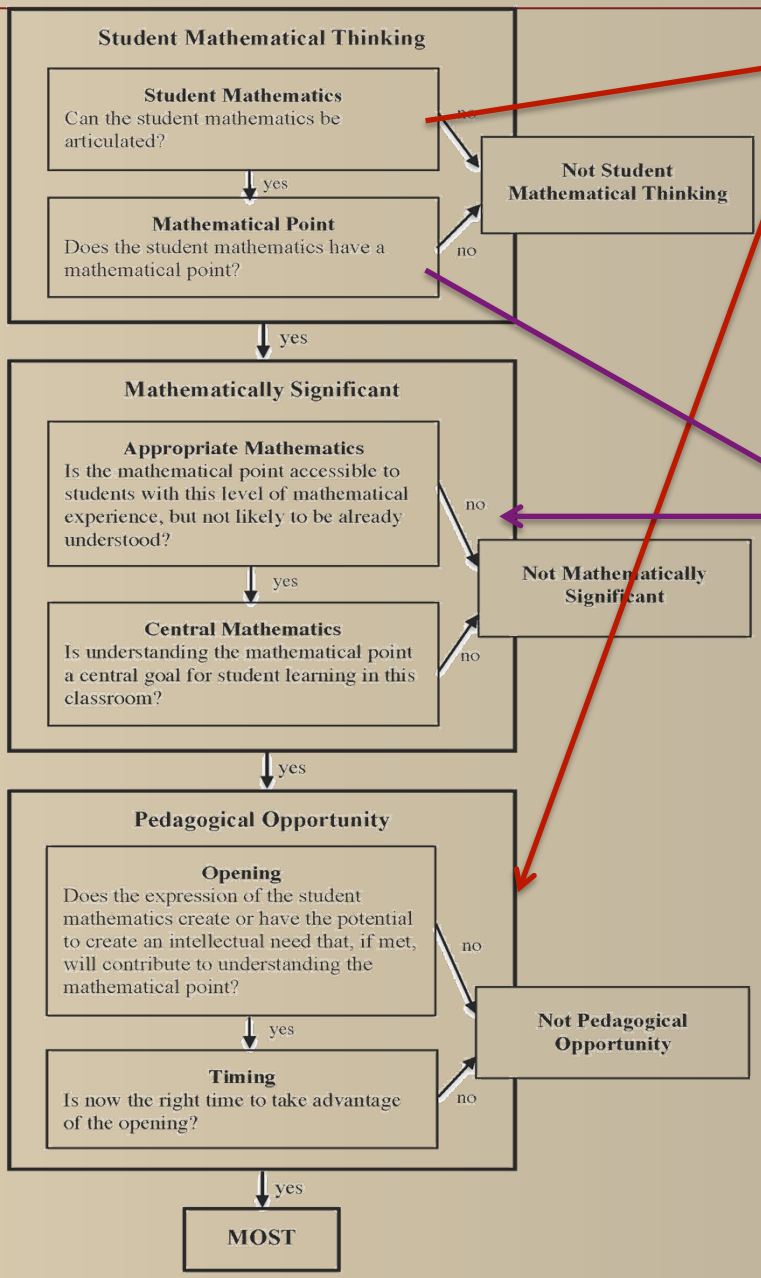

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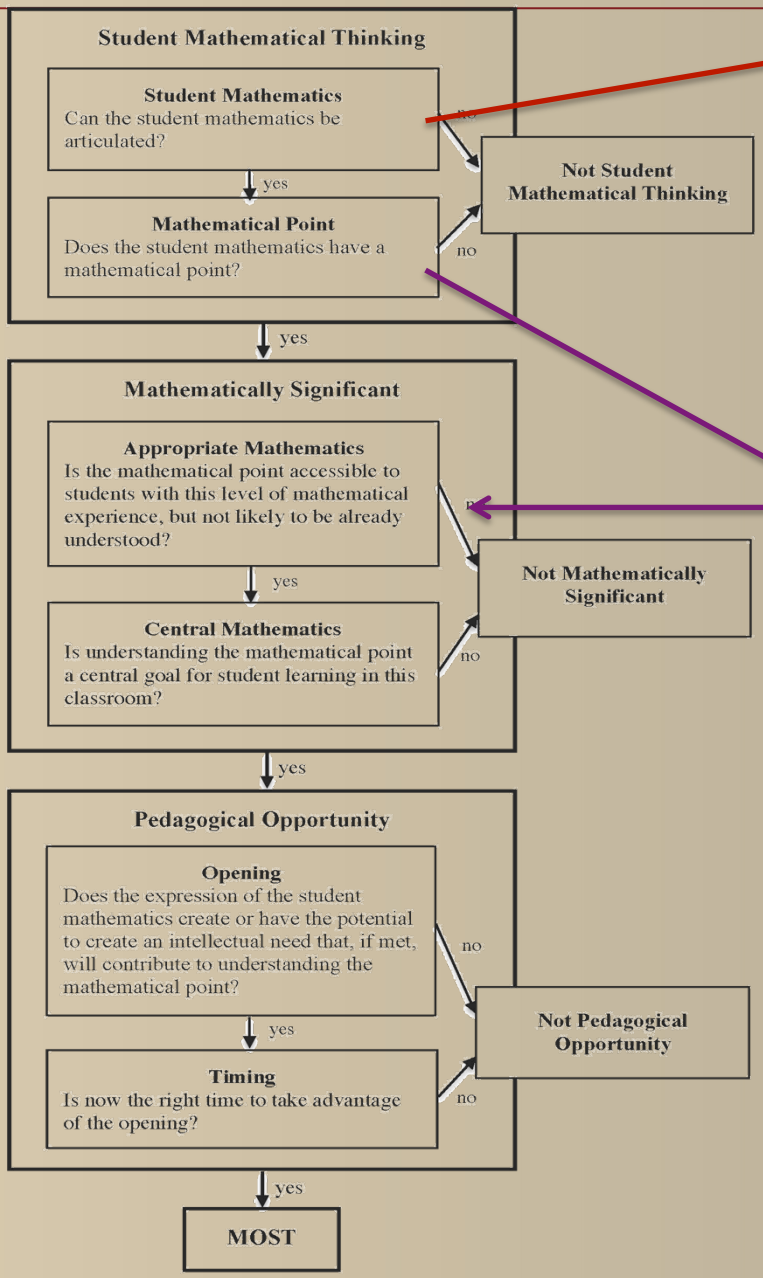

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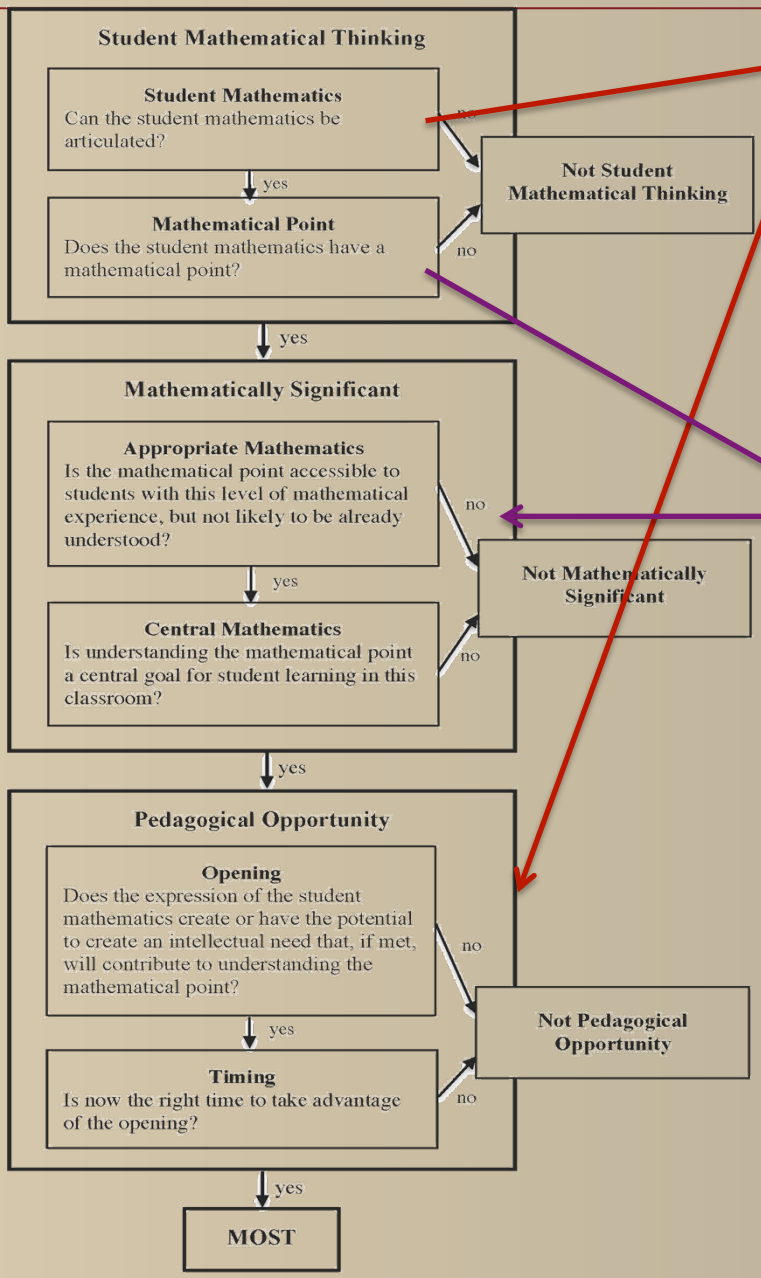

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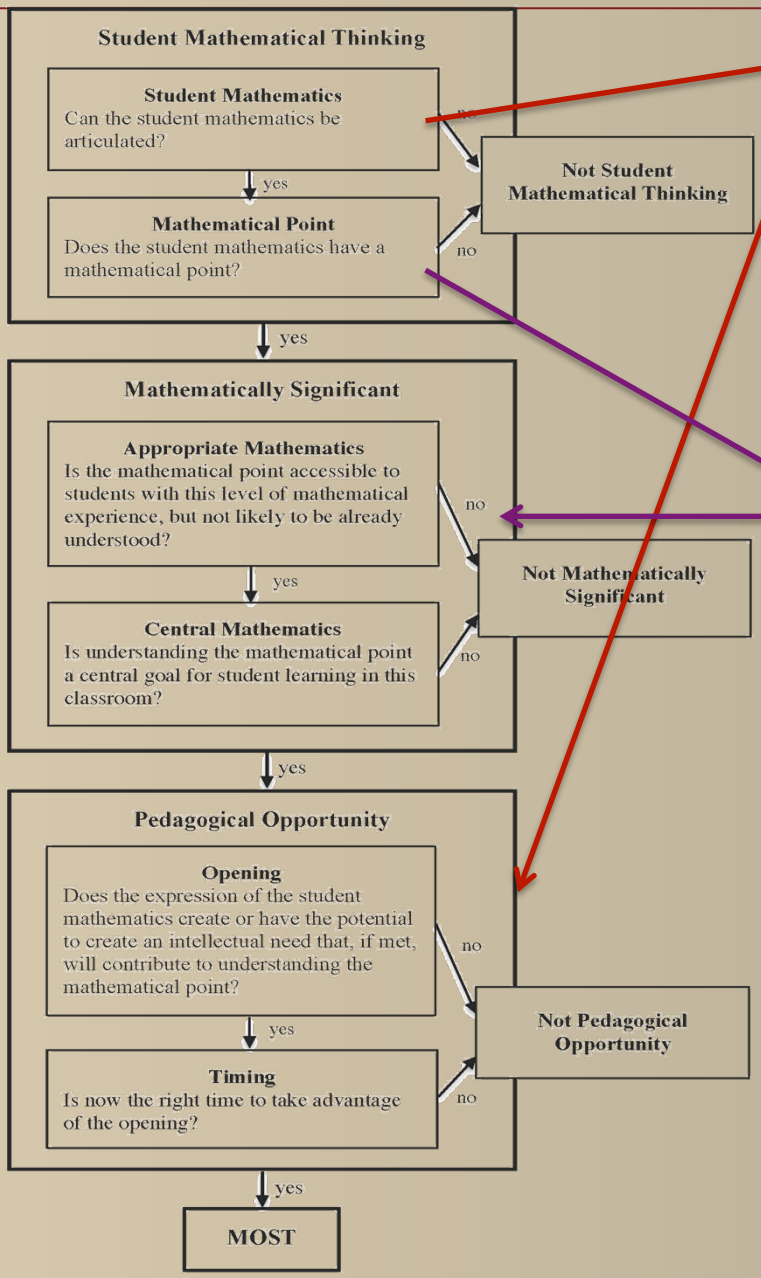

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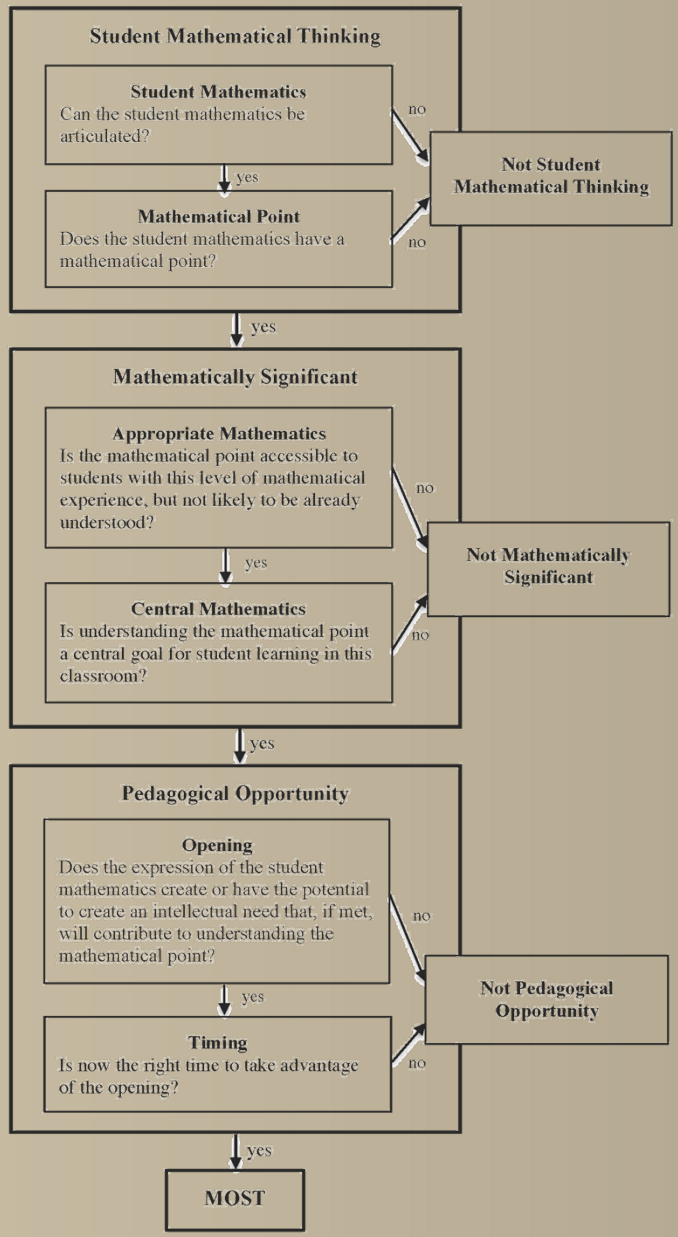
Questions for you

How might this construct help researchers to

a) focus discourse analysis on student mathematical thinking?

b) analyze the extent to which pedagogy is responsive to the instructional dynamic in classrooms?

c) better understand how teachers learn to move beyond eliciting student mathematical thinking to using it productively?


Student Mathematical Thinking
Student Mathematics
Can the student mathematics be articulated?
no
yes
Mathematical Point
Does the student mathematics have a mathematical point?
no
Not Student Mathematical Thinking
yes
Mathematically Significant
Appropriate Mathematics
Is the mathematical point accessible to students with this level of mathematical experience, but not likely to be already understood?
no
yes
Central Mathematics
Is understanding the mathematical point a central goal for student learning in this classroom?
no
Not Mathematically Significant
yes
Pedagogical Opportunity
Opening
Does the expression of the student mathematics create or have the potential to create an intellectual need that, if met, will contribute to understanding the mathematical point?
no
yes
Timing
Is now the right time to take advantage of the opening?
no
Not Pedagogical Opportunity
yes
MOST

Student Mathematical Thinking: An evidence-based inference about student mathematical thinking that can be used to develop a mathematical idea.

Criteria:

Student Mathematics: an inference (that can reasonably be made based on a student's actions) about what a student is thinking mathematically, regardless of the correctness of that thinking.

Mathematical Point: a concise statement of a mathematical idea that mathematics learners could better understand as a result of making the student mathematics of the instance an object for consideration.

Mathematically Significant: When the mathematical point of an instance warrants the use of limited instructional time; used in the context of teachers engaging a particular group of students in the learning of mathematics.

Criteria:

Appropriate Mathematics: When the mathematical point is accessible to students given their prior mathematical experiences but not typically mastered by most of them.

Central Mathematics: When the mathematical point is a central goal for this group of students—central either to the lesson or to the discipline of mathematics.

<u>Pedagogical Opportunity</u> *(to build on student thinking)*: An observable student action that creates an *intellectual need* (Harel, 2013) that can be acted on in that moment to contribute to students' understanding of a mathematical point.

Criteria:

Opening: an instance in which the expression of a student's mathematical thinking creates, or has the potential to create, an intellectual need for students to make sense of the *student mathematics*, thus providing an opportunity to understand the *mathematical point*.

Timing: an opportune time when taking advantage of the opening *at that moment* is likely to further students' understanding of the mathematical point of the instance.

Mathematical Opportunities in Student Thinking

http://LeveragingMOSTs.org