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(Counter)Productive Practices

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The Plan

- Our Perspective
- Three Go-To Practices
  - Collecting information from the class
  - Asking a student to clarify their contribution
  - Asking students to revoice their peer's contribution
- Productive and Counterproductive Examples of These Practices
- Discretionary Spaces that Provide Opportunities to Increase Access and Equity
- Criteria for Decision Making
- Challenge

Student-Thinking-Centered Classrooms

NOT teacher transmitting information to their students

Teacher working with students as the students co-construct their ideas about mathematics

A **M**athematical **O**pportunity in **S**tudent **T**hinking (**MOST**) is a high-leverage student contribution made during whole-class interaction ("teachable moment")

Building on a MOST takes full advantage of that opportunity by engaging the class in making sense of the MOST to better understand the mathematics of the MOST

BuildingOnMosts.org

Teacher works with students as they co-construct their' ideas about the mathematics of the MOST

Collecting Information from the Class

- At the core of a student-centered classroom
  - “What do you think?” (Ideas)
  - “Does anyone have a different solution?” (Strategies)
- When we collect matters

Productive Collecting

finding out what students are thinking in a general way

- Launching a task
  - "What do you know about [th
  - "What might someon get started?"
  - "What are you
- Compari nes
- ussion
  - have a different way of thinking about this?"
  - hear from someone who hasn't contributed yet."

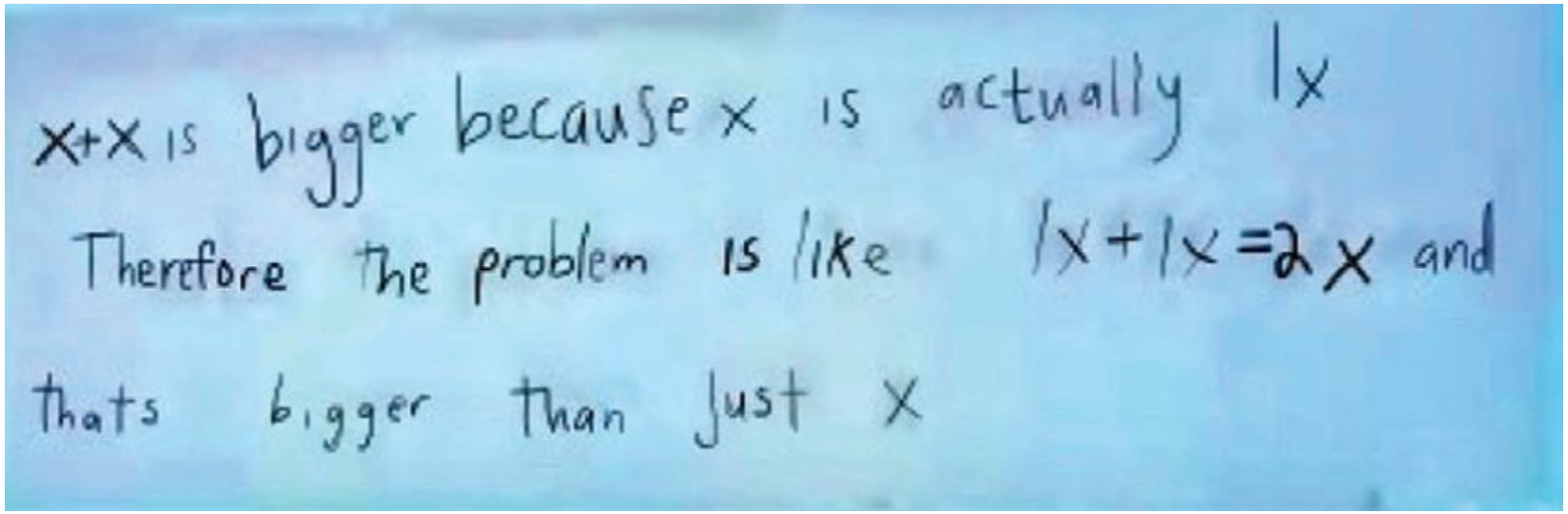
Counterproductive Collecting

- Have a MOST (Mathematical Opportunity in Student Thinking)
- Additional collecting often diminishes the sense-making opportunity

A "Collecting" Example

Variables Problem: *Which is larger, x or x + x? Explain your reasoning.*

Tony's (incorrect) claim about the Variables Problem:



The teacher: "I want to know what you all think about this proposal. Does Tony's claim hold up mathematically, or does it not?

Andrew: Um, I'm saying no 'cause if x was a negative number then the negative and the negative would be smaller than just a normal x, so…

Teacher: So you're saying you have to think about x being a negative number. Ming?

Ming: But if it was a negative then you'd have a negative plus negative, not um, just x.

Teacher: So you're thinking that if x were negative, it would say something different than it says right now? Other thoughts? Joya?

Joya: Well there's one x, it's just x so if you add another x then it's—there's two x's now. So, which, basically showing that there's one of something. 'Cause if there's nothing it's gonna be zero x.

Teacher: Interesting, alright a couple more. Tammy.

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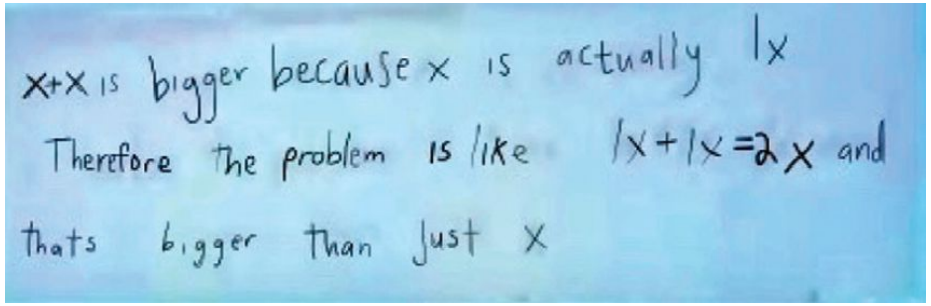

Imagine instead…

Andrew: Um, I'm saying no 'cause if x was a negative number then the negative and the negative would be smaller than just a normal x, so…

Teacher: So you're saying you have to think about x being a negative number. ~~Ming?~~ [writes down Andrew's counterclaim below Tony's] What do the rest of you think that [pointing to the counterclaim] has to do with Tony's claim?

[Ming and Joya share their contributions as part of a focused mathematical discussion]

Discretionary Space re Collecting

- Attend to whose ideas are collected (or not)
- Avoid excluding ideas expressed in less familiar ways
  - different vocabulary or syntax
  - informal notation
  - unfamiliar contexts

Asking a Student to Clarify their Contribution

- Productive if something in the contribution is actually unclear
  - Vague language, such as a pronoun without a clear referent
  - Part of their reasoning is implicit and needs to be made explicit
- Unproductive if students are asked to clarify something that is already clear

Percent Discount Problem:
The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?

A "Productive Clarifying" Example

- Dean shared his solution: *Uh, so I think that the original, it will be the original price. And the reason for this is because if we do it, as it says, it increases by 50%. So, 50% of 10 is $5. We would add $5. So, $15 would be the price; and then if it decreases by 50%, we would subtract 5, and then that would be 10.*

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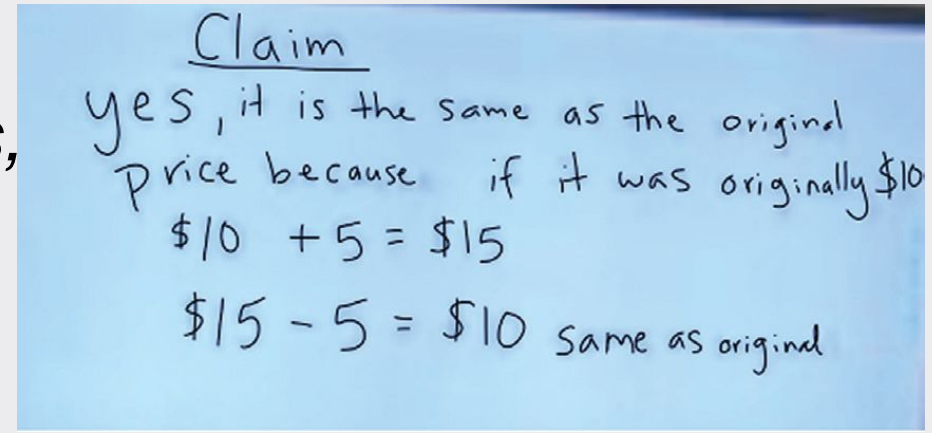
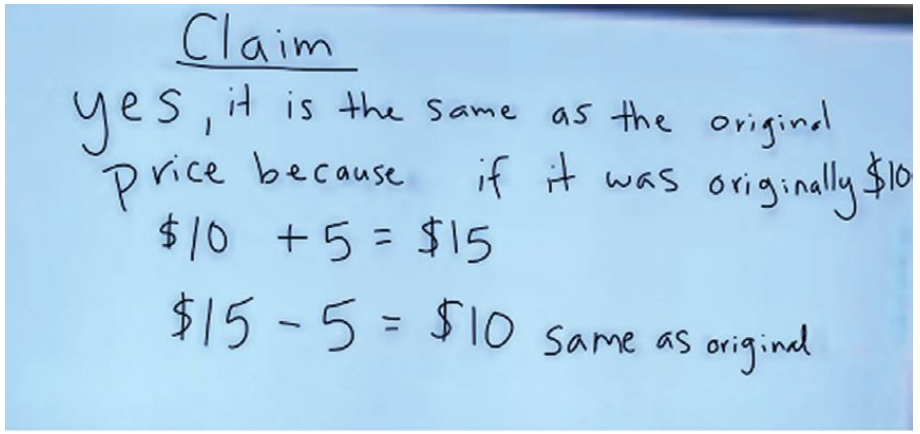
- Teacher: *So you're saying yes, the final price is the same as the original price?*
- *Dean: Yes*



An "Counterproductive Clarifying" Example

Teacher: Because, and you said you chose, $10?
Dean: Yes, as my original price. $10.
Teacher: Okay [begins to record Dean's contribution on board; see Figure 3] then it would increase by?
Dean: 50%, which would be $5.
Teacher: [continues to record] And that gives you?
Dean: $15.
Teacher: [continues to record] Okay. And then?
Dean: I would subtract it by 5 because it would decrease it by 50%.
Teacher: So then you would do $15 [continues to record] minus $5?
Dean: Yeah.
Teacher: Which would give you?
Dean: 10.
Teacher: [continues to record] 10. Right?
Dean: Yes.
Teacher: [continues to write Dean's contribution on board] And you wrote, same as original on your paper.
Dean: Yes, yes I did.



What made it counterproductive?

- Dean had already clearly said what was needed
- Students were likely to get lost in the details
- Diminished the opportunity to engage the class in making sense of the contribution as a whole
- It was all about Dean ~ other students might lose interest
- May send the message to the contributing student that they did not clearly articulate their idea when, in fact, they did
- Risks taking away the opportunity to grapple with the math ~ “So 50% of 15 is 5?” shifts to fixing an error rather than grappling with a big idea

Discretionary Space re Clarifying a Contribution

- Attend to whose contributions get clarified
- Avoid dwelling on clarifying irrelevant things, such as correcting multi-lingual learners' pronunciation when the meaning is clear

Asking Students to Revoice a Peer's Contribution

- Productive when used to
  - enhance student engagement
  - assess students' understanding of what another student has said
  - ensure that the students are taking away the big ideas from a class discussion
- Counterproductive when trying to establish a high-leverage contribution as the focus of a class discussion
  - risks losing the mathematical opportunity of the original contribution
  - the aspects of the student contribution that are important to revoice may not be obvious to the student revoicing
  - students may contribute their own ideas instead

A “Counterproductive Revoicing” example

Bike Ride Problem: *On Blake’s morning bike ride, he averaged 3 miles per hour (mph) riding a trail up a hill and 15 mph returning back down that same trail. What was his average speed for his whole ride?*

Loret: OK, so why I think it’s 9 is because if you do, I guess that would be 4, 5, 6 [writing out numbers 3 through 15] and you find the number in the middle. So what I did is I just kind of cross them out as I go [crosses out all the numbers except for 9]. 9 is the number that’s in the middle in between 3 and 15. So that is how we find the average, so that’s why I said 9.

Teacher: Could someone revoice how Loret thought about this? Yeah, how’d she think about this?”

Lila: So how I was taught to find the average is you add the two numbers together, so 3 plus 15 is 18. And then you divide it by the amount of numbers given, so 18 divided by 2 is 9.

Discretionary Space re Revoicing

- Attend to
  - whose contributions get revoiced
  - who does the revoicing
- Avoid always asking a high-status student to revoice a traditionally marginalized student's contribution

When to Use and Avoid Using The Three (Counter)Productive Teaching Practices

Teaching Practice	Is Productive When	Is Counterproductive When
Asking clarifying questions	Something in a student contribution needs to be clarified for students to engage in making sense of it	A student contribution is "clear enough" for the class to engage in making sense of it

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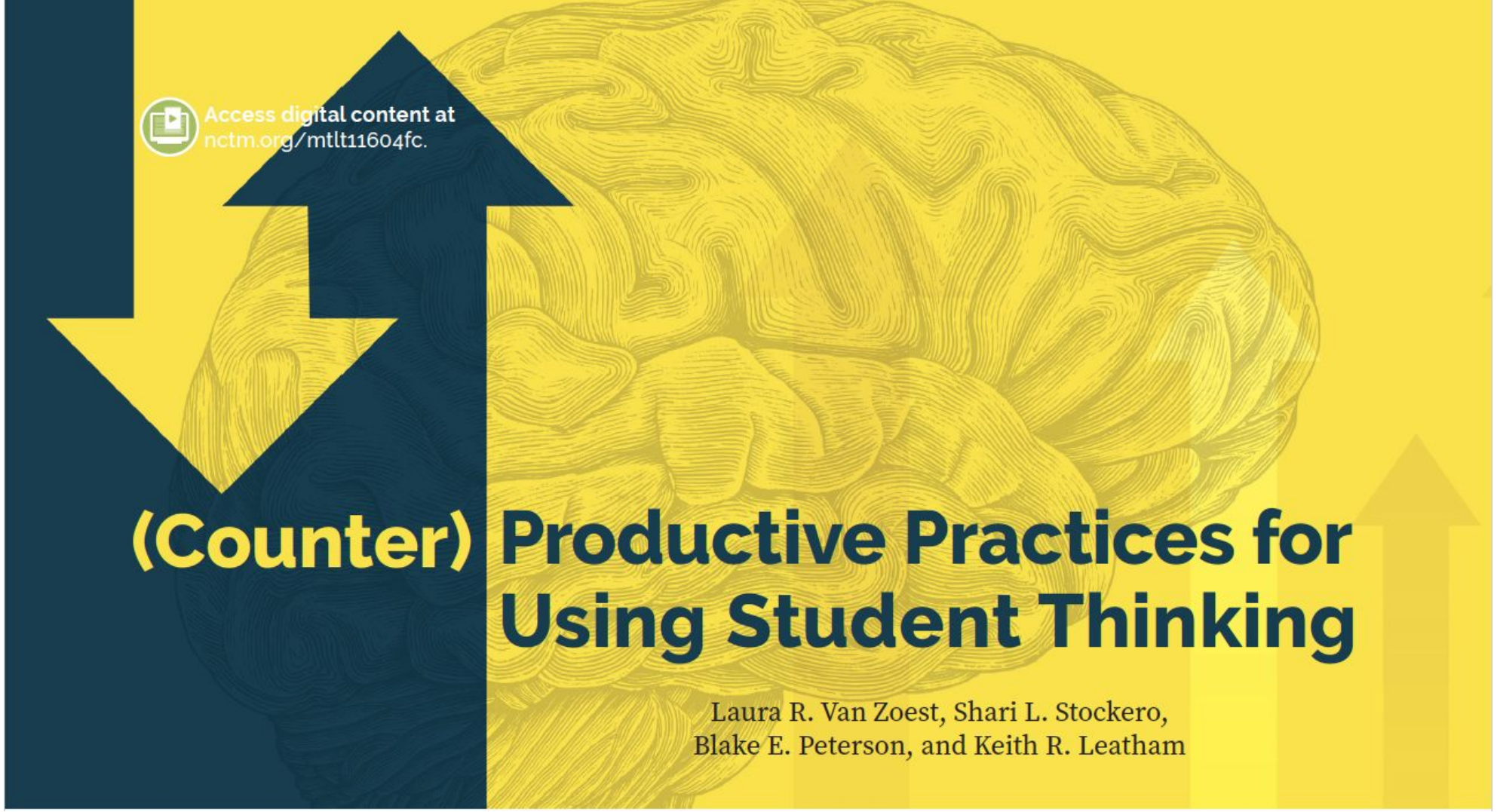
Asking students to revoice	You want to assess whether students understand the focus of the discussion.	You are trying to establish a contribution you want students to discuss.

Challenge

- Videotape your instruction and look for instances where you use these three practices in productive or counterproductive ways.
- Consider other teaching practices that you routinely use: In what situations might they be counterproductive?

Minor adjustments to your practice can have a major effect on students' opportunities for sense making.



Mathematics Teacher: Learning and Teaching PK-12, 116(4), 244-251

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Tomorrow's Building on MOST Presentations

Session: *318 - Using a Public Record to Anchor Joint Sense Making of Mathematics*

Time: 9:30 AM - 10:30 AM

Location: McCormick Place, N230a

Session: *362 - Tackling Tangential Student Contributions*

Time: 11:00 AM - 12:00 PM

Location: McCormick Place, S404 A

Session: *434 - Wait, What Are We Talking About? (Re)focusing Students During Whole-Class Discussion*

Time: 2:30 PM - 3:30 PM

Location: McCormick Place, S406 B