Making the MOST of Student Mathematical Thinking

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Leveraging MOSTs: Developing a Theory of Productive Use of Student Mathematical Thinking

- 4-year collaborative research project funded by the National Science Foundation (DRL-1220141, DRL-1220357, DRL-1220148)
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- Graduate Assistants
  - Lindsay Merrill, Brigham Young University
  - Mary A. Ochieng, Western Michigan University
  - Could it be you?

The Motivation for Our Work

- Graduates of our teacher education programs were successful in eliciting student thinking
- That student thinking was not being used to further students' mathematical understanding
- Saw "teachable moments" not get acted on
- Need to better understand these moments to prepare teachers to take advantage of them

Teachable Moment

- Describe an instance, either from your own mathematics classroom or from one you have observed, when a mathematics-related "teachable moment" occurred.
- Share your instance with the person next to you.
- What is it about these instances that make them mathematics-related "teachable moments"?

From the literature…

- "critical moments in the classroom when students created a moment of choice or opportunity" (Jaworski, 1994, p. 527)
- "novel student idea[s] that prompt teachers to reflect on and rethink their instruction" (Schifter, 1996, p. 130)
- "potentially powerful learning opportunities" (Davis, 1997, p. 360)
- "significant mathematical instances" (Davies and Walker, 2005, p. 275)
- "[student's] comment provides the fodder for a content-related conversation" (Schoenfeld, 2008, p. 57)
- "crucial mathematic hinge moment[s]" (Thames and Ball, 2013, p. 31)

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Three Characteristics

- Students
- Mathematics
- Pedagogy

Three Characteristics

- Students—Student Thinking
- Mathematics
- Pedagogy

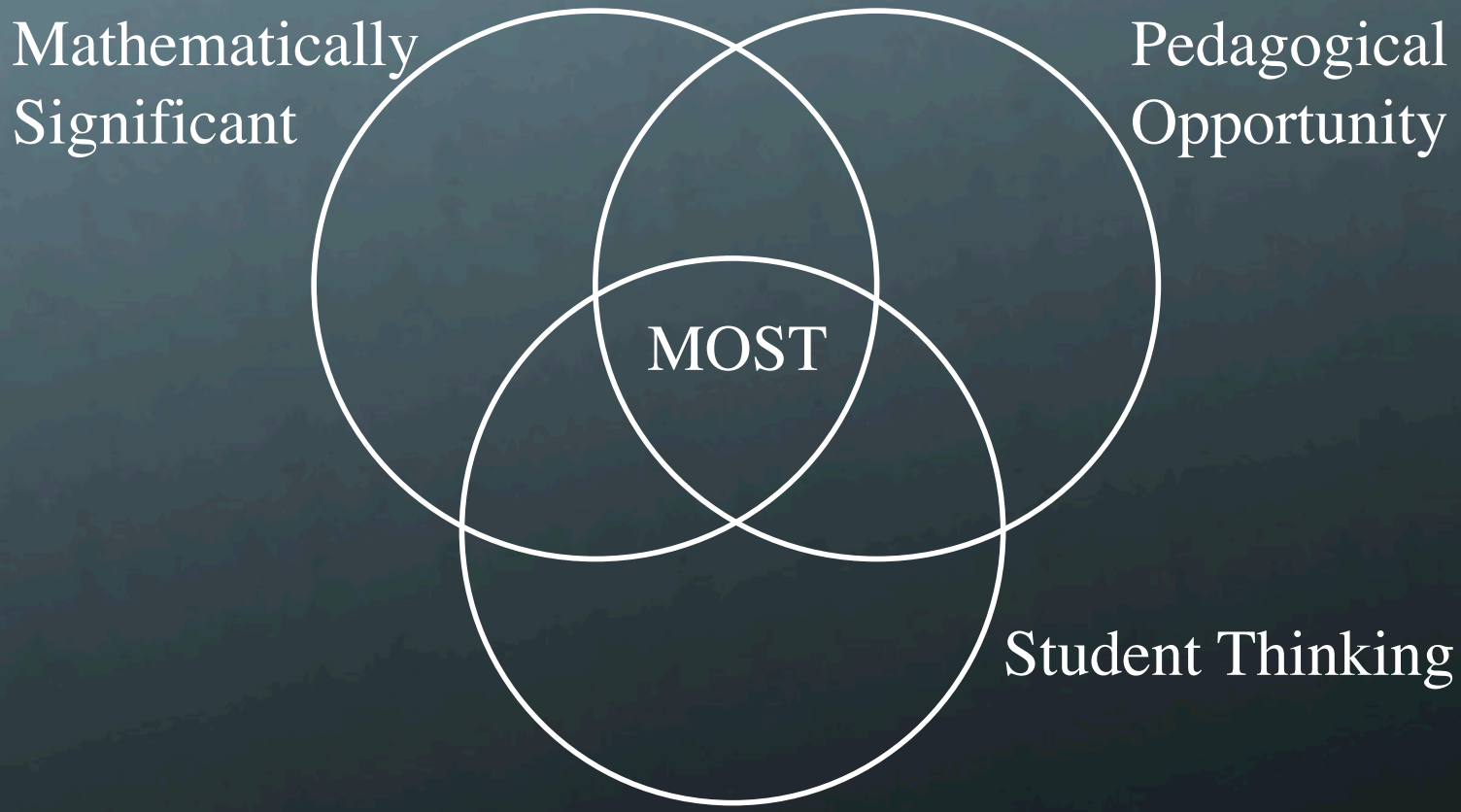
Three Characteristics

- Students—Student Thinking
- Mathematics—Mathematically Significant
- Pedagogy

Three Characteristics

- Students—Student Thinking
- Mathematics—Mathematically Significant
- Pedagogy—Pedagogical Opportunity

Relationship of MOST Characteristics



MOST

Mathematically significant
pedagogical Opportunity
to build on Student
Thinking

Mathematical
Opportunities
in Student
Thinking

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MOSTs Are "In-the-Moment Opportunities"

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But opportunities for what?

MOSTs are opportunities for…

…the teacher to make student thinking the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

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Teacher: *How did you get that?*
Jordan: *I added 95.*
Teacher: *Are you supposed to add here?*

MOSTs are opportunities for…

Jordan: *I added 95.*
Teacher: *Does anyone have an idea for why Jordan added 95?*

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Jordan: *I added 95.*
Teacher: *Does anyone have an idea for why Jordan added 95?*

Jordan: *I added 95.*
Teacher: Why did you add?

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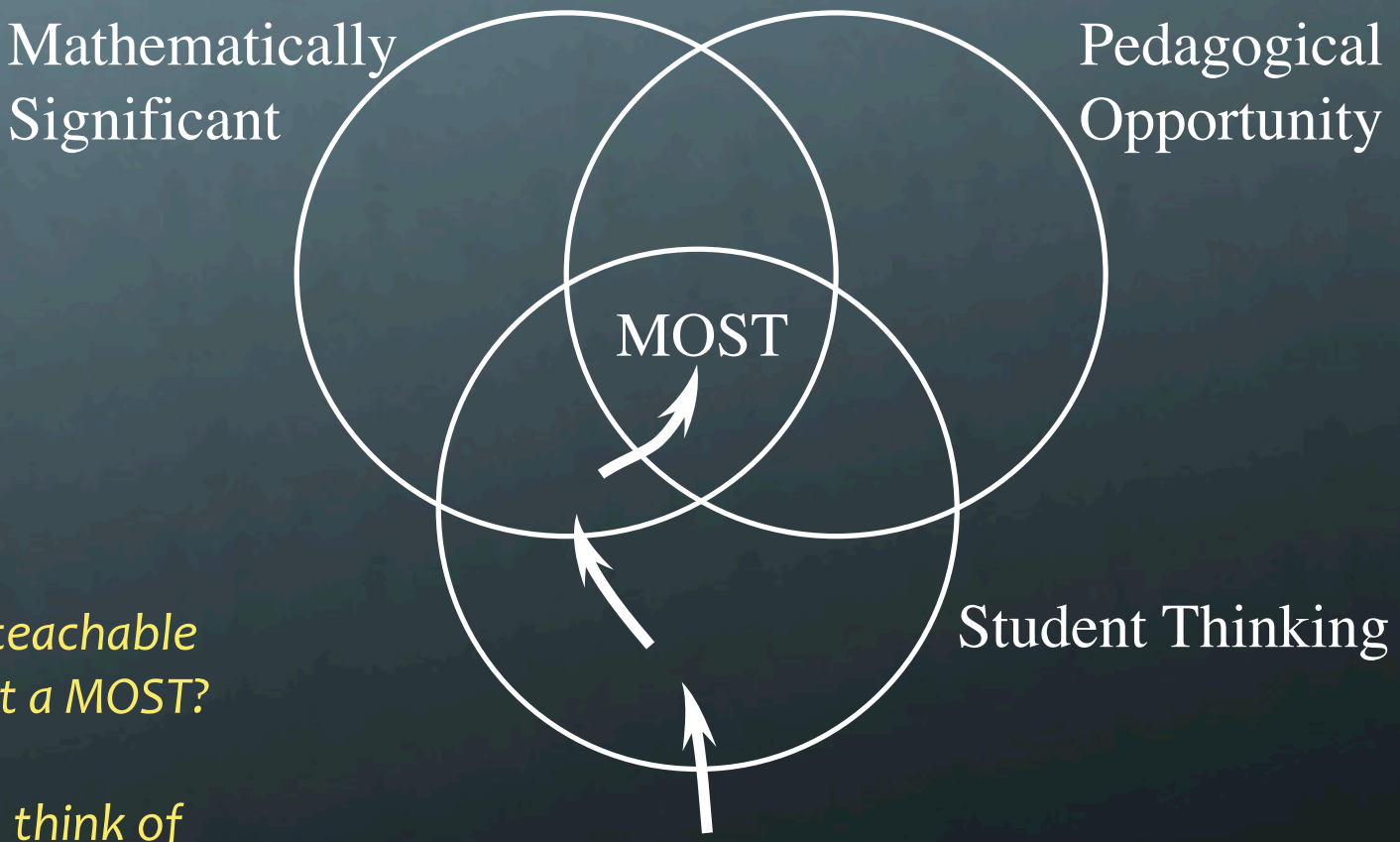
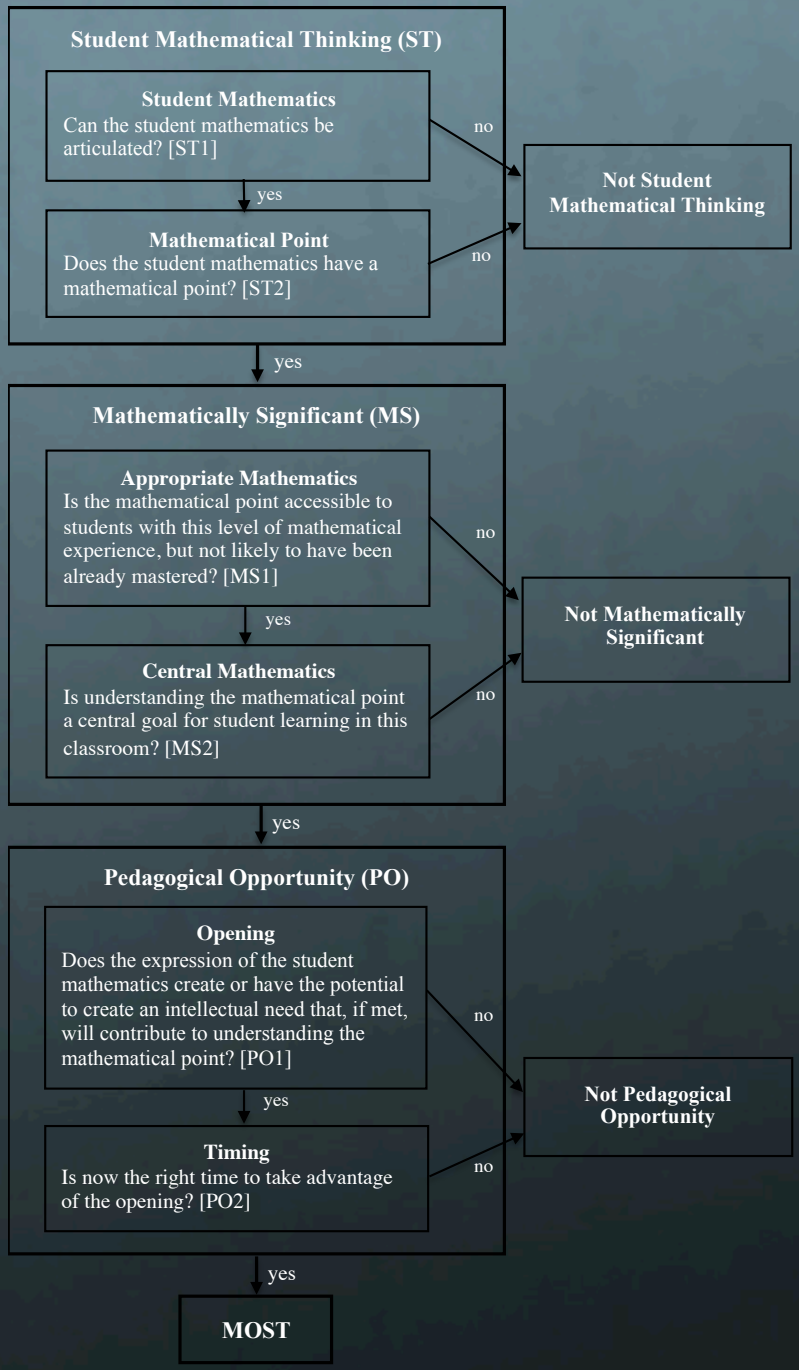
Mathematical Opportunities in Student Thinking

Is it a MOST?

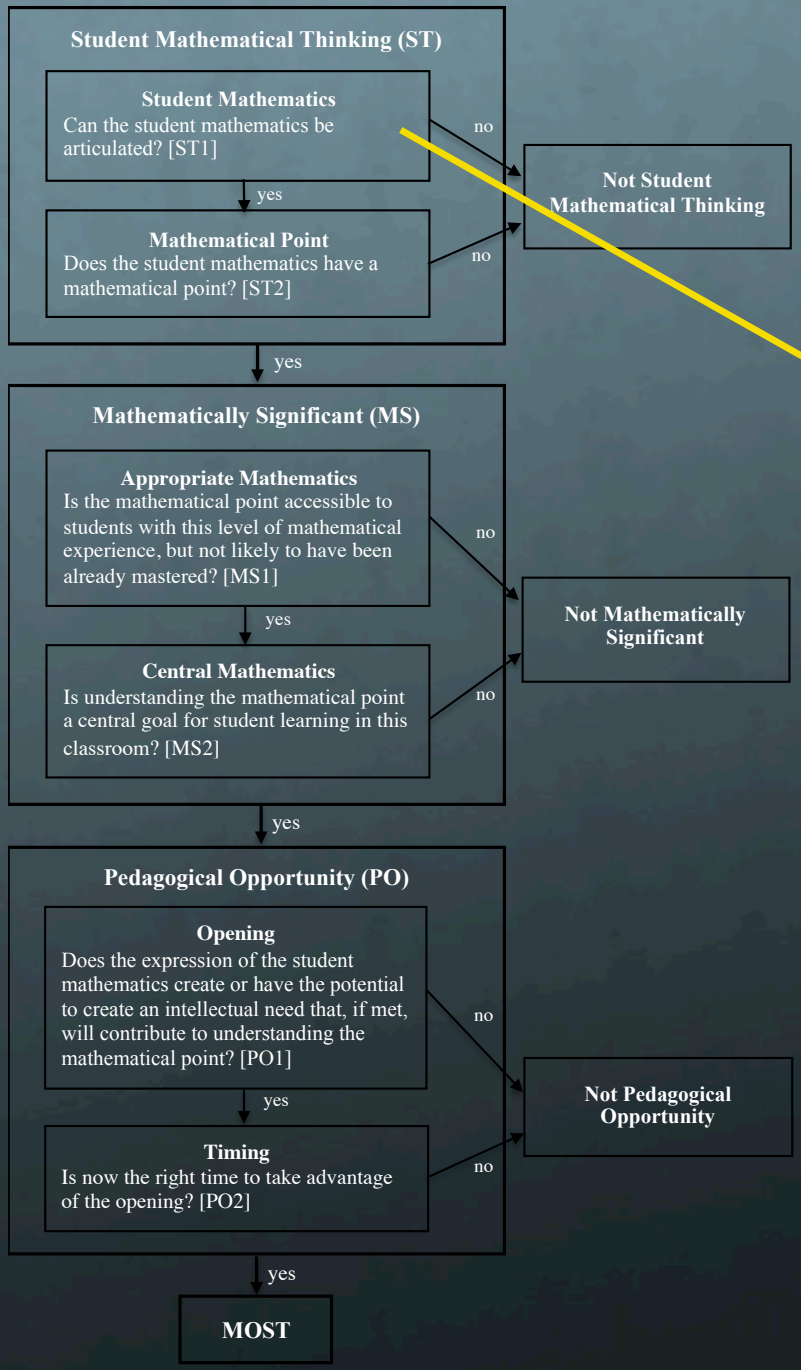


Is your teachable moment a MOST?

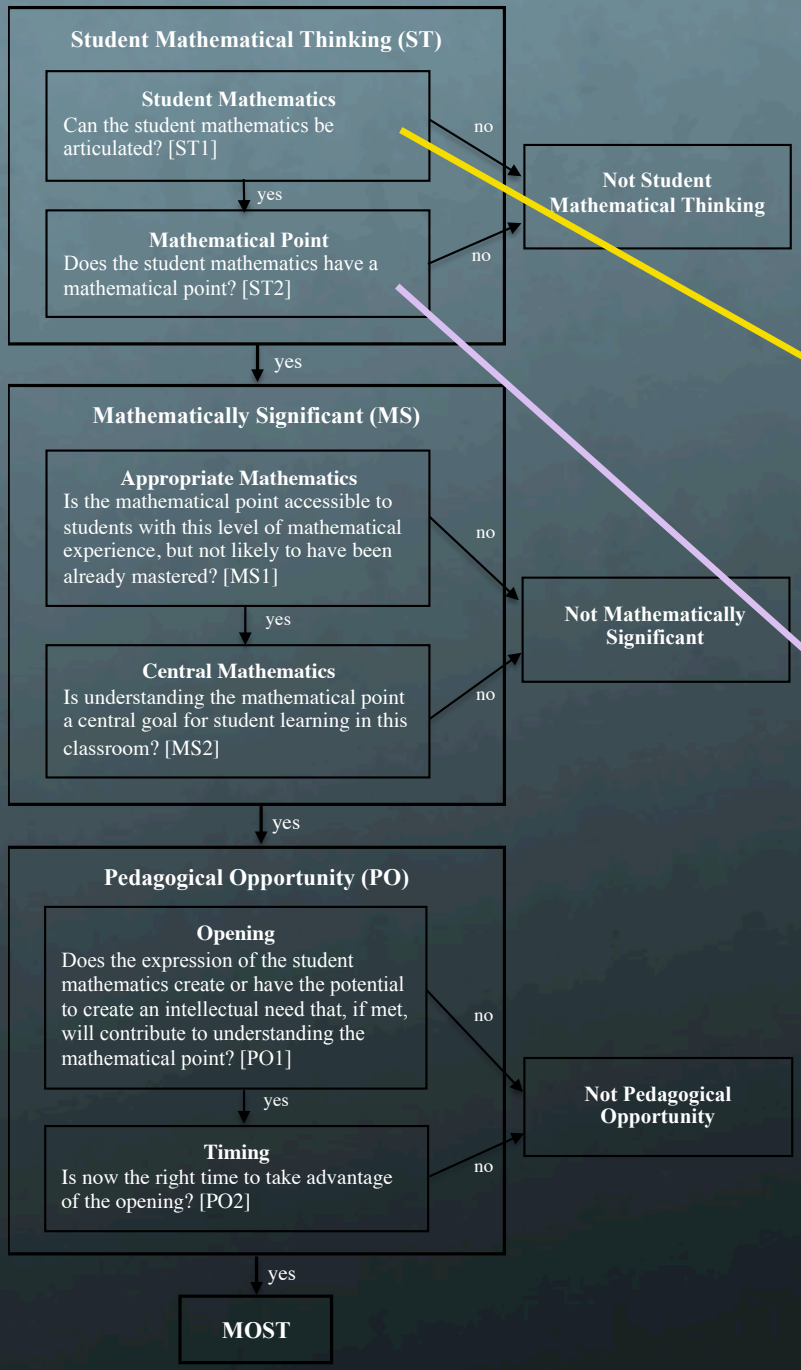
Can you think of another MOST?



The teacher makes student mathematical thinking the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.



The teacher makes *student mathematical thinking* the object of consideration by the class in order to engage the class in making sense of that thinking to better understand *an important mathematical idea*.



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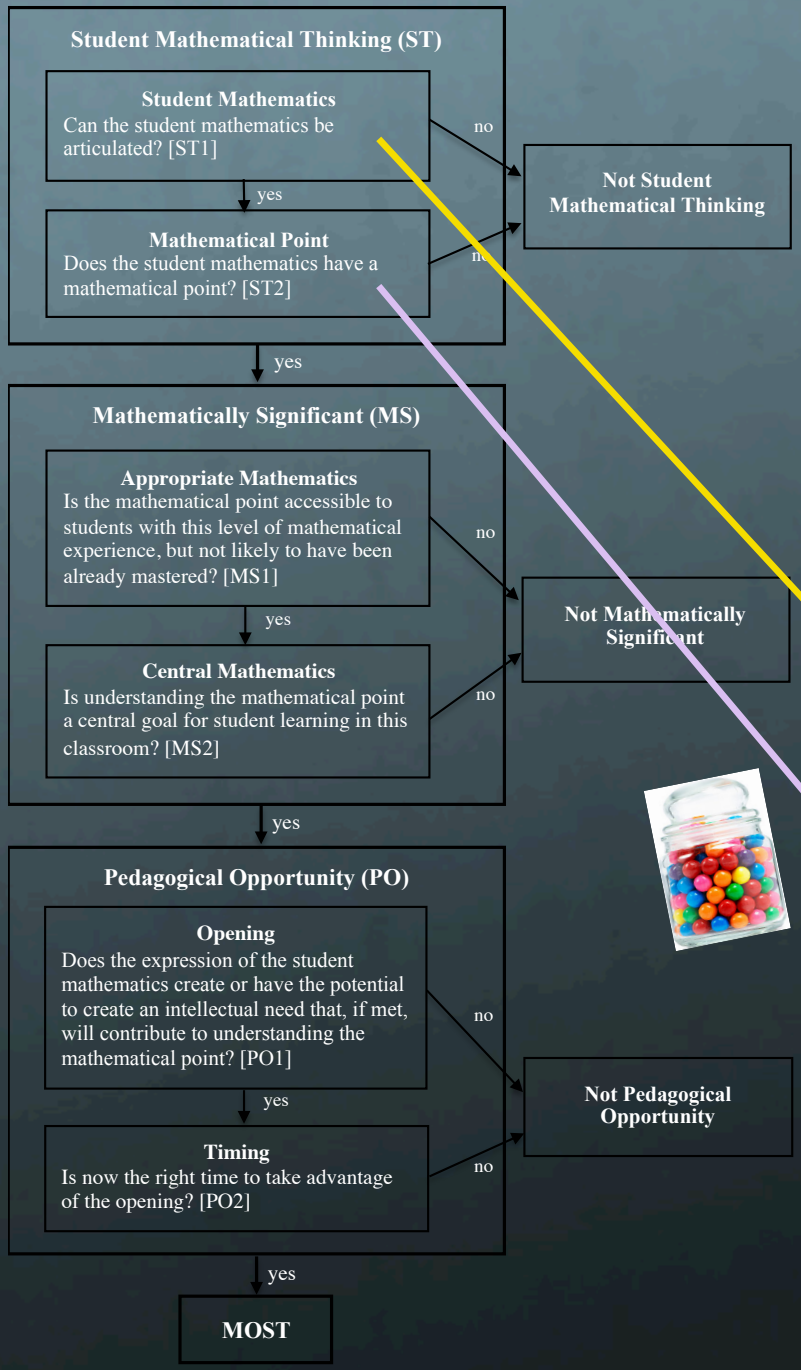
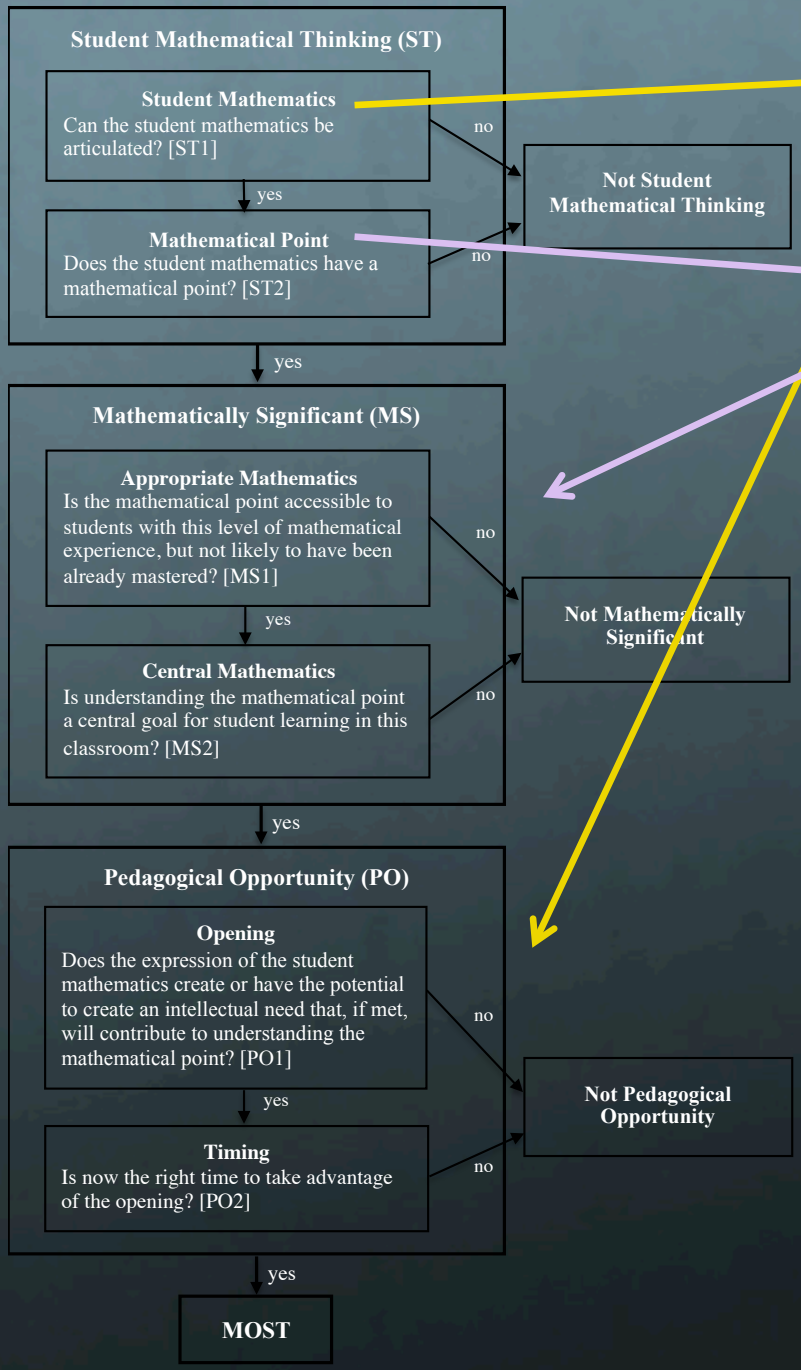


The teacher makes *student mathematical thinking* the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important *mathematical idea*.

Jordan: I added 95.

SM: Adding 95 to both 5 and 13 will give the same ratio as 5 to 13.

MP: Ratios grow at a constant rate that is multiplicative rather than additive.



SM: Adding 95 to 13 will give the same ratio as adding 95 to 5.

MP: Ratios grow at a constant rate that is multiplicative rather than additive.

Jordan: I added 95.

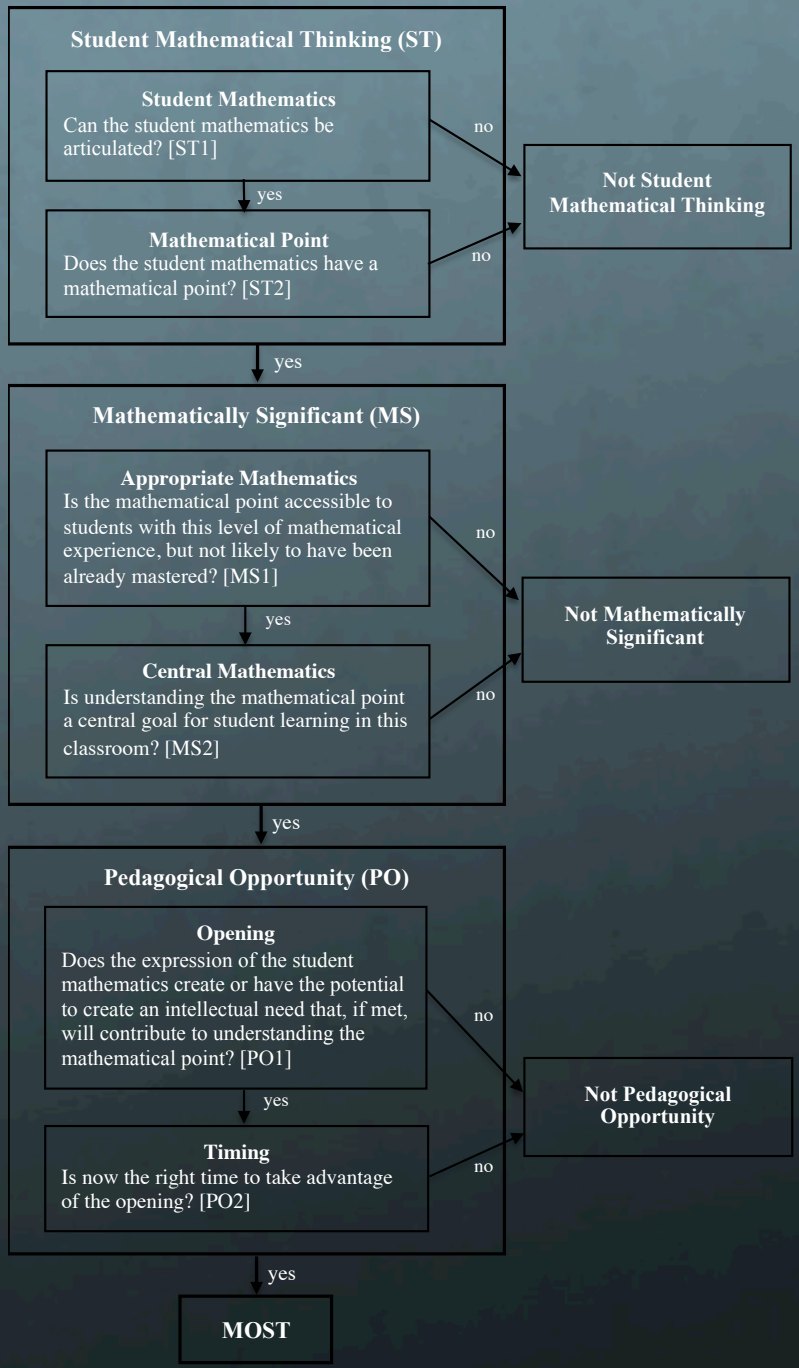
Is it a MOST?

Yes!

Your turn

Context

- 7th grade general mathematics class
- Discussion of how to solve $m - 12 = 5$
- Teacher asked the students how to "make a zero"
- Students replied *"subtract 12 from both sides of the equation"* (following a procedure they had used the prior day to solve similar equations that had a plus sign instead of a minus sign)



SM:

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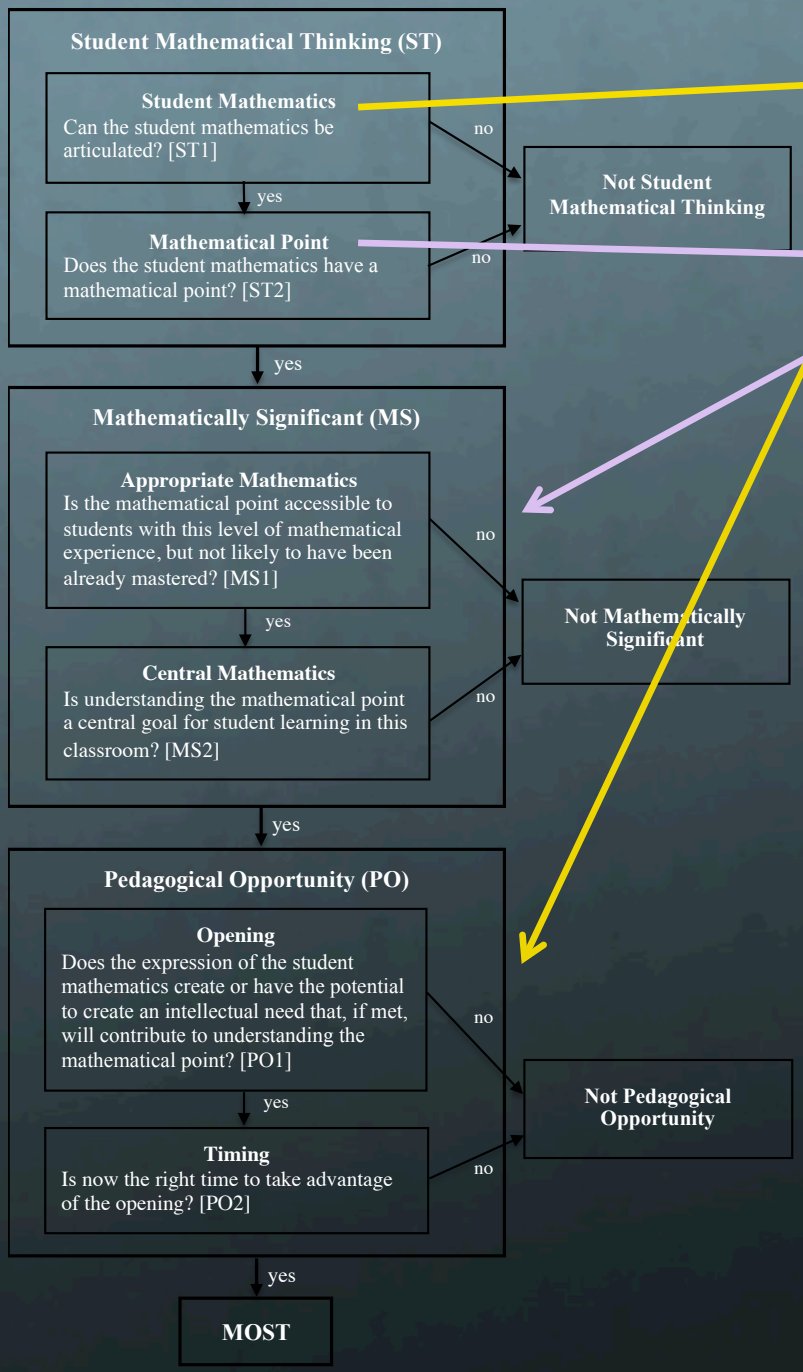
Problem: $m - 12 = 5$

Student Comment: *subtract 12 from both sides of the equation*

Is it a MOST?



SM: To make zero in the equation $m - 12 = 5$, you need to subtract 12 from both sides.

MP: Inverse operations of addition and subtraction can be used to isolate a term in a linear equation.

Problem: $m - 12 = 5$

Student Comment: *subtract 12 from both sides of the equation*

Is it a MOST?

YES!

		Student Math	Math Point
S	Subtract 12 from both sides of the equation.	To make zero in the equation $m - 12 = 5$, you need to subtract 12 from both sides.	Inverse operations of addition and subtraction can be used to isolate a term in a linear equation.

The Teacher's Response

		Student Math	Math Point
S	Subtract 12 from both sides of the equation.	To make zero in the equation $m - 12 = 5$, you need to subtract 12 from both sides.	Inverse operations of addition and subtraction can be used to isolate a term in a linear equation.
T	OK, here we already have subtraction (indicating the symbol '–' in '$m - 12 = 5$'), so what's the opposite of subtraction?		

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Types of student thinking that tend to generate "intellectual need"

- An incorrect answer that involves a common or mathematically rich misconception
- A correct answer with novel reasoning
- A mathematical contradiction – different answers or different interpretations
- Incomplete or incorrect reasoning
- Why or generalizing questions

Shifting Gears

- Established what MOSTs are (teachable moments with a mathematical focus)
- Explained that MOSTs are an opportunity for the teacher to make student thinking the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea
- Introduced the MOST framework and illustrated its use

So, how do MOSTs relate to other things you have been hearing about?

What do MOSTs have to do with the CCSSM?

Problem	Mathematical Point	CCSS.Math.
Candy Jar	Ratios grow at a constant rate that is multiplicative rather than additive.	Content.6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. Content.6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems…
$m - 12 = 5$	Inverse operations of addition and subtraction can be used to isolate a term in a linear equation.	Content.7.NS.A.1c (a) Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Content.7.EE.B.4a (a) Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers.

How does the MOST Framework relate to the Five Practices?

- **Intertwined and mutually supportive**
  - ***Anticipating*** **and** ***Monitoring*** **improve teachers' abilities to recognize and productively use MOSTs**
  - **The MOST Framework improves teachers' abilities to** ***Select***, ***Sequence*** **and** ***Connect*** **effectively**
- **Different foci**
  - **Five practices focuses primarily on student thinking in response to tasks as it emerges across a lesson**
  - **MOSTs are instances of student thinking that lose their potential if not acted on at the time they occur**

MOSTs Are "Pedagogical Waves"

How does the MOST framework relate to the Eight Practices?

- **Establish mathematics goals to focus learning**—Effective teaching of mathematics establishes clear goals for the mathematics student are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
- **Elicit and use ~~evidence of~~ student thinking:** Effective teaching of mathematics uses ~~evidence of~~ student thinking to ~~assess~~ make progress toward mathematical understanding and to continually adjust instruction in ways that support and extend learning.

Some thoughts

- There is agreement that use of student thinking is an important teaching practice
- Not *all* student thinking is equally valuable
- Not *all* student thinking should be pursued in the same way

Reflection

- What are you confused by?
- What are you wondering?
- What would you like to know more about?

Mathematical Opportunities in Student Thinking

http://LeveragingMOSTs.org