Collaborative Research: Leveraging MIPOs: Developing a Theory of Productive Use of Student Mathematical Thinking

Project Summary

Leveraging MIPOs (Mathematically Important Pedagogical Opportunities) is a Full Research and Development proposal addressing the Teaching Strand. The collaborative project focuses on secondary school mathematics. *Leveraging MIPOs* helps teachers provide high quality STEM education for all students by improving teachers' abilities to use student thinking during instruction to develop mathematical concepts. A *Theory of Productive Use of Student Mathematical Thinking (PUMT Theory)* will articulate what the practice of productively using student mathematical thinking looks like, how one develops this practice, and how that development can be facilitated.

Intellectual Merit: Research in mathematics teacher education suggests the benefits of instruction that builds on student thinking (e.g., Fennema, et al., 1996; Stein & Lane, 1996), but such instruction is complex and difficult both to learn and to enact (e.g., Ball & Cohen, 1999; Sherin, 2002). Many teachers, especially novices, fail to notice or to act on opportunities to use student thinking to further mathematical understanding (Peterson & Leatham, 2009; Stockero, Van Zoest, & Taylor, 2010). A growing number of teachers believe that using student thinking to develop mathematical concepts is important and needs to be encouraged; however, neither teachers nor those who educate them have a clear understanding of how to do this (e.g., Van Zoest, Stockero, & Kratky, 2010). Not all student thinking is about important mathematics, nor does it always provide leverage for accomplishing important instructional goals. Thus, it is critical that teachers and teacher educators have access to a *PUMT Theory* that articulates the nuances of using student thinking productively.

The *Leveraging MIPOs* project combines three universities' expertise and access to diverse populations to develop a *PUMT Theory.* It does so through four interrelated phases: (1) *Student thinking*—testing and refining a preliminary MIPO framework by expanding the data set to include more diverse populations; (2) *Teachers' interactions with student thinking*—assessing teachers' perceptions of using student thinking and how they make decisions about which thinking to pursue; (3) *Teachers' learning about student thinking*—using a series of teacher development experiments to improve teachers' abilities to productively use student mathematical thinking during instruction; and (4) *Shareable products*—creating useful products that are in forms that encourage feedback for further refinement.

This project advances knowledge by enhancing the field's understanding of: (1) the student thinking that teachers have available to them in their classrooms; (2) teachers' perceptions and use of student thinking during instruction; and (3) teachers' learning about student thinking and how best to support it. Using student thinking productively is at the center of ambitious teaching (e.g., Cohen, 2011), thus the *PUMT Theory* and associated supports produced by the project will be valuable resources for teachers, teacher educators and researchers in mathematics education, as well as in other fields. Studying student thinking and teachers' interactions with it in diverse classrooms increases the ability of the resulting theory to benefit all students.

Broader Impact: This project provides critical tools for teachers, teacher educators, and researchers that make more tangible the often abstract but fundamental goal of productively using students' mathematical thinking. These tools will enhance teachers' practice of productively using student thinking during instruction, thus improving students' opportunities to learn mathematics. Graduate students, teachers and students from underrepresented groups will be intentionally recruited to participate in project activities. Focusing on cooperating teacher-student teacher pairings in the teacher development experiments will broaden the impact to the current and future student teachers who work with this cooperating teacher. A three-prong dissemination plan—focused on teachers, teacher educators, and researchers—will contribute to broad availability of the results of the work. To increase the applicability of this work, significant time has been built into the project for refining products and results so that they can be used and built upon by other teacher educators and researchers. There is also some indication (e.g., Stockero, Van Zoest, Leatham, & Peterson, 2011) that the ideas developed here have generalizability to other subject areas and grade levels.

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Importance

Research in mathematics teacher education suggests the benefits of instruction that builds on student thinking (e.g., Fennema, et al., 1996; Stein & Lane, 1996), but such instruction is complex and difficult both to learn and to enact (e.g., Ball & Cohen, 1999; Feiman-Nemser, 2001; Sherin, 2002). Many teachers, especially novices, fail to notice or to act on opportunities to use student thinking to further mathematical understanding (Peterson & Leatham, 2009; Stockero, Van Zoest, & Taylor, 2010). Despite a growing number of teachers who are convinced of the value of encouraging the sharing of student mathematical thinking and research showing that focusing teachers on student mathematical thinking can influence their practice (e.g., Fennema, et al., 1996; Jacobs, Lamb, & Philipp, 2010), neither teachers nor those who educate them have a clear understanding of what thinking can best be used to develop mathematical concepts (Peterson & Leatham, 2009; Van Zoest, Stockero, & Kratky, 2010). Not all student thinking is about important mathematics, nor does it always provide leverage for accomplishing important instructional goals. Thus, it is critical that teachers and teacher educators have access to a *Theory of Productive Use of Student Mathematical Thinking (PUMT Theory)*—a theory that articulates what the practice of productively using student mathematical thinking looks like, how one develops this practice, and how that development can be facilitated.

The *Leveraging MIPOs* project combines the expertise and access to diverse populations of three universities to develop such a theory. It expands and refines the existing Mathematically Important Pedagogical Opportunity (MIPO) framework (e.g., Leatham, Peterson, Stockero & Van Zoest, 2011), and uses it as a tool for developing a *PUMT Theory.* The core research questions of the project are: (1) What is the nature of high-leverage student thinking that teachers have available to them in their classrooms? (2) How do teachers use student thinking during instruction and what goals, orientations and resources (Schoenfeld, 2011) underlie that use? (3) What is the learning trajectory for the teaching practice of productively using student thinking? and (4) What supports can be provided to move teachers along that learning trajectory?

The fact that using student thinking productively is central to ambitious teaching (e.g., Cohen, 2011) ensures that the *PUMT Theory* and associated supports produced by the *Leveraging MIPOs* project will be valuable resources to teachers, teacher educators and researchers in mathematics education, as well as in other fields. Studying student thinking and teachers' interactions with it in diverse classrooms enhances the ability of the resulting theory to benefit all students. A well-crafted dissemination plan and strategically-chosen, diverse project participants will contribute to use of project products on a large scale.

Foundations for the Project

Mathematically Important Pedagogical Opportunities (MIPOs)

We conceptualize high-leverage instances of student mathematical thinking that occur during instruction—instances that provide rich opportunities for developing important mathematical ideas—as Mathematically Important Pedagogical Opportunities (MIPOs). As such, MIPOs form the core of our developing *PUMT Theory.* In our prior work (e.g., Leatham, Peterson, Stockero & Van Zoest, 2011a, 2011b, Leatham, Stockero, Van Zoest, & Peterson, 2010,), we have located MIPOs in the intersection of three important criteria: *mathematically important*, *student thinking*, and *pedagogical opportunity*. Here we define these criteria in the context of our work.

Mathematically important. We use *mathematically important* in the context of teaching and learning mathematics. Thus, we define an instance to be mathematically important in a given classroom if it is centered on an idea related to mathematical goals appropriate for student learning in that classroom. These goals could be determined by the teacher, by an external source, such as the NCTM *Standards* (NCTM, 2000), or they could be inferred by an observer who is knowledgeable in the field of mathematics education. In the narrowest sense, the instance would be related to a mathematical goal for the lesson in which the instance occurs, but more broadly, it could also be related to the goals for a unit of instruction, an entire course, or for understanding mathematics as a whole. In the first case, the instance may focus on a particular mathematical idea or connections among ideas within the lesson, while in the latter cases, the instance might involve making connections to other areas of mathematics, including ideas from prior or future courses, or developing mathematical ways of thinking. A key criterion in all cases, however, is that the ideas be accessible to the students in the class.

Student thinking. We recognize our inability to access directly the thoughts of others. Instead we make inferences based on our observations of what they say and do. Thus, when we use the phrase student thinking we refer to observable evidence of student thinking, which we define as any instances where a student's actions provide sufficient evidence to make reasonable inferences about their thinking.

Note that we make a distinction between observable and observed. There are many cases where student thinking is observable, but not observed by the teacher. Thus, for the purposes of our work, observable refers to thinking that could be observed by someone (the teacher, other students, a researcher) who witnessed the instance, either by being present or engaging with a record of the interactions.

We also realize that the more complex an understanding one has of mathematics and what it takes for students to learn it, the more likely one is to recognize and make sense of subtle instances of student mathematical thinking. The purpose of the MIPO construct is to provide a lens and a common language for recognizing and agreeing on a critical core of high-leverage mathematical thinking that all teachers can aspire to notice when it occurs in their classrooms.

Pedagogical opportunity. In addition to involving important mathematics and student thinking, a MIPO requires a *pedagogical opportunity*. We define pedagogical opportunities as observable student actions that provide evidence that students are engaged with or thinking about the content of an instructional goal and, thus, provide an opening for working toward that goal. To be clear, we are defining *pedagogical opportunities* in a very specific way, in that we focus only on opportunities that are grounded in observable student actions. This does not imply that opportunities to learn cannot occur in less public ways (e.g., a student individually struggling with ideas in a mathematical task), but only that observable evidence is necessary for a pedagogical opportunity to present itself to the teacher.

Pedagogical *opportunities* can be cultivated by the teacher, but cannot be created independently of the students. Teachers routinely make pedagogical *moves* that are designed to create opportunities for students to engage with the content of an instructional goal, such as posing quality tasks, asking probing questions, assessing students' progress and modifying their instruction in response to additional information. Well-executed pedagogical moves can, in fact, increase student engagement and thus the likelihood that pedagogical opportunities will occur in a teacher's class, but the opportunities themselves come from observed students actions, not the teacher. For example, a teacher introducing a theoretical student error into the class discussion to help clarify an issue with which she felt her students were struggling would be a pedagogical move. This move would not become a pedagogical opportunity until the observable actions of a student or students in the class provided evidence of how they might be interacting with the error. It is student actions that provide insight into student engagement with an instructional goal that create an opening for the teacher to work toward achieving that goal.

The intersection of mathematically important, student thinking, and pedagogical opportunity. MIPOs occur at the intersection of *important mathematics*, *student thinking*, and *pedagogical opportunities*. In this intersection, observable evidence of student thinking related to mathematical goals for a given classroom provides pedagogical openings for working towards those goals. A teacher may use these openings in a variety of different ways, from inserting a teacher explanation to asking follow-up questions to orchestrating a class discussion. When a teacher sees a MIPO as an opportunity to step in and explain, it could be classified as a naïve use (Peterson & Leatham, 2009) in that the teacher has used the MIPO only as a trigger to lecture about the mathematical topic. A more productive use of a MIPO is to orchestrate a discussion around the mathematics at hand. This could be done by posing questions to the student(s) who generated the ideas or by asking other members of the class questions about those ideas. These questions might also focus on connections between the mathematics of the observed student thinking and other concepts that are related to the mathematical goals of the classroom.

The MIPO conceptual framework. Figure 1 is a depiction of our conceptual framework for the relationship among important mathematics, student thinking, and pedagogical opportunities. Details about the MIPO framework can be found in Leatham, Peterson, Stockero and Van Zoest (2011a). Here we provide a glimpse into the framework by elaborating on region A, region E and the center MIPO region.

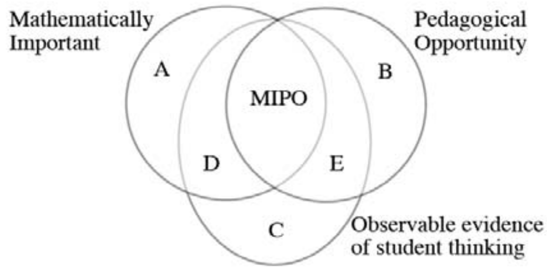

Figure 1. *Conceptual framework for the relationship among important mathematics, student thinking and pedagogical opportunities.*

Region A represents situations that are mathematically important, but neither provide evidence of student thinking nor a pedagogical opportunity. A teacher presenting important mathematical information would fall into this region, as would situations in which a teacher makes a pedagogical move to engage students with important mathematics, but students fail to provide observable evidence of having done so. In general, these are situations where important mathematics is present, but observable student actions related to the mathematics are not. For example, if a teacher were to make a mistake on the board related to important mathematics in the lesson, it would be a mathematically important moment. If the teacher corrects the error and moves on without evidence that students had engaged with the error, this moment would not provide an opportunity to work toward an instructional goal related to that important mathematics. In contrast, a student action in response to the error that provides evidence of his or her mathematical thinking, such as asking questions that illuminate the key mathematics behind the error, would provide an opening for using that thinking to work towards a mathematical goal for student learning. This would put the moment in the intersection of the three areas in Figure 1, making it a MIPO.

Region E represents pedagogical opportunities that provide insight into student thinking, but are not mathematically important. For example, a student might say, "I don't see why I need to think by myself for one minute before I talk with my group." This comment is not directly related to mathematics, but it does provide observable evidence of student thinking and would provide an opening for discussing the general instructional goal of allowing individuals time to formulate their own thoughts before being influenced by others. A quite different example is a middle school student who, during a discussion about division by zero, asks, "Is this like that L'Hopital's Rule my sister keeps talking about?" Although L'Hopital's Rule is an important mathematical idea, and actually is related to division by zero, it is so far removed from the mathematical goals for this group of students and the students' background knowledge that it would not meet our criterion for mathematically important in the context of this classroom. It does, however, provide observable evidence of student thinking and an opening for the instructional goal of developing students' interest in further study of mathematics.

The center region of the diagram represents situations in which observable evidence of student thinking about a mathematical goal for the students provides an opening for working towards that goal. The confluence of these three criteria is what creates a MIPO. Student actions that frequently fall into this region include those in which students question or comment on a mathematical idea, verbalize their incomplete thoughts as they try to make sense of a mathematical idea, express incorrect mathematical thinking, make an error of substance, or notice a mathematical contradiction. Of course, for these actions to be MIPOs, they must be mathematically important and provide an opening for working toward a mathematical goal for the class.

Pivotal Teaching Moments—A Subset of MIPOs

Stockero and Van Zoest's earlier work (e.g., Stockero, Van Zoest & Taylor, 2010; Stockero & Van Zoest, 2011) defined *pivotal teaching moments* (PTMs) as instances in classroom lessons in which an interruption in the flow of the lesson provides the teacher an opportunity to modify instruction in order to extend or change the nature of students' mathematical understanding. This initial work informs the proposed work, as we have come to understand these high-leverage instances of student mathematical thinking as unplanned MIPOs.

In the earlier work, video of beginning secondary mathematics teachers' instruction was analyzed to identify and characterize PTMs in mathematics lessons and to examine how such moments played out. As a result of this work, the research team was able to identify a preliminary framework (Figure 2) for characterizing PTMs, the potential they had for advancing students' mathematical understanding, the teacher decision, the way in which the decision was implemented, and the likely impact on student learning. This framework provides a starting point for our analysis of MIPOs in the first phase of the project and informs our work in subsequent phases as we develop a model for improving teachers' abilities to productively use student mathematical thinking.

<table>
<tr><th colspan="2">Pivotal Teaching Moment</th><th colspan="2">Teacher Decision</th><th rowspan="2">Likely Impact on Student Learning</th></tr>
<tr><th>Type</th><th>Potential</th><th>Action</th><th>Implementation</th></tr>
<tr><td>Extending</td><td rowspan="5">Significant
Moderate</td><td>Extends math and/or makes connections</td><td rowspan="3">Skillfully
Moderately
Poorly</td><td>High positive</td></tr>
<tr><td>Incorrect math</td><td>Pursues student thinking</td><td>Medium positive</td></tr>
<tr><td>Sense-making</td><td>Emphasizes meaning of the mathematics</td><td>Low positive</td></tr>
<tr><td>Contradiction</td><td>Acknowledges, but continues as planned</td><td rowspan="2"></td><td>Neutral</td></tr>
<tr><td>Confusion</td><td>Ignores or dismisses</td><td>Negative</td></tr>
</table>

Figure 2. Framework for understanding pivotal teaching moments, corresponding teacher decisions and the likely impact on student learning.

The PTM data was drawn from beginning teachers in the Midwest; Phase 1 of the *Leveraging MIPOs* project focuses on expanding the existing data set to include secondary school mathematics classrooms intentionally chosen for their diversity, including diversity of students, teachers, and mathematics. This is critical as factors such as cultural and language differences may affect the frequency and nature of the student thinking that is made public in a classroom. Studying diverse contexts will contribute to a *Theory of Productive Use of Student Thinking (PUMT Theory)* that applies to all students.

Teacher Decision Making

An important first step in capitalizing on MIPOs is recognizing that such moments exist (Peterson & Leatham, 2009). Without this awareness, teachers may experience *inattentional blindness* (Simons, 2000)—a phenomenon described in the psychology literature as a failure to focus attention on unexpected events. Recognition is also related to the idea of *framing* (Levin, Hammer & Coffey, 2009), the way in which a teacher makes sense of a classroom situation. From this perspective, whether a teacher notices the value in an instance depends on how he or she frames what is taking place during instruction. If, for example, a teacher views a student error as something that needs to be corrected, he or she is unlikely to consider the mathematical thinking behind the error or whether the error could be used to highlight a specific mathematical idea. On the other hand, a teacher who views an error as a site for learning is more likely to consider both the mathematics underlying the error and how it could be used to develop mathematical understanding. Based on this information, we hypothesize that helping teachers to recognize MIPOs will be an important component of the *Leveraging MIPOs* project.

In the context of teacher professional development, Remillard and Geist (2002) described *openings in the curriculum* as moments in which teachers' questions, observations, or challenges require the facilitator to make a decision about how to incorporate into the discussion the mathematical or pedagogical issues that are raised. The nature of the facilitator's decision determines the extent to which the teachers' ideas advance the learning of the group. Similarly, when a MIPO occurs in a classroom lesson and the teacher recognizes it as such, he or she must make a decision about how to respond. Depending on the teacher's decision, the opportunity to build on student thinking may or may not be taken advantage of to support the development of students' mathematical understanding.

Acting on MIPOs in productive ways requires teachers to have an orientation towards *noticing* (e.g., Sherin, Jacobs & Philipp, 2010) and using student thinking (e.g., Van Zoest, Stockero & Kratky, 2010). Phase 2 of the *Leveraging MIPOs* project—understanding teachers' interactions with student thinking—draws on Schoenfeld's theory of goal-oriented decision making, which describes teachers' decisions as being predictable based on their orientations, resources, and goals (Schoenfeld, 2008, 2011). Addressing teachers' beliefs about student thinking, their mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008), and what it is they want to accomplish in their classrooms will be critical to the success of the teacher development experiments in Phase 3 and the production of products in Phase 4 that will have an impact on what teachers actually do in the classroom.

Other Related Work of the PIs

Two of the PIs are engaged currently in additional research that will inform our initial model for teacher learning about productive use of student mathematical thinking. Stockero is conducting a study to understand whether engaging prospective teachers in research-like analysis of unedited video recordings of secondary school mathematics instruction helps them learn to better notice important mathematical moments that occur during a lesson. The results of a pilot study indicate that student teachers became better able to recognize mathematically important moments during their instruction and began to adapt their instruction in response to important student comments. A preliminary analysis of work with prospective teachers early in their teacher education program shows similar results in terms of noticing. Subsequent analysis will focus on understanding what specific facilitator moves might prompt changes in noticing. This work will help inform the TDE in Phase 3 by providing additional data for understanding MIPOs and teachers' decisions in response to them.

Leatham has been working with two graduate students who have built on previous work related to eliciting, recognizing and using students' mathematical thinking (Peterson & Leatham, 2009). Their studies—one with an expert teacher (Toponce, 2011), the other with two student teachers—focus on secondary school teachers' decisions to use or not use their students' mathematical thinking in whole-class discussions. These studies will inform our work on teachers' decisions, including our methodology, as we will be collecting data in similar ways and asking similar questions of both expert and novice teachers.

Research & Development Design

The *Leveraging MIPOs* project combines the expertise and access to diverse populations of three universities to develop a *Theory of Productive Use of Student Mathematical Thinking (PUMT Theory).* We accomplish this objective through four interrelated phases: (1) *Student thinking*—testing and refining the existing MIPO framework by expanding the data set to include more diverse populations; (2) *Teachers' interactions with student thinking*—assessing teachers' perceptions of using student thinking and how they make decisions about which thinking to pursue; (3) *Teachers' learning about student thinking*—using a series of teacher development experiments to improve teachers' abilities to productively use student thinking during instruction to develop mathematical concepts; and (4) *Shareable products*—creating products that are useful to others in forms that encourage feedback for further refinement. Table 1 summarizes the work of these phases, which are further described in the following sections.

Table 1. *Project work plan.*

Phase	Timeline	Primary Research Questions	Data
Phase 1: Student Thinking	Fall 2012-Summer 2013 (Year 1)	RQ1: What is the nature of high-leverage student thinking that teachers have available to them in their classrooms?	Teacher questionnaire Videotaped classroom observations Existing classroom video and/or transcripts
Phase 2: Teachers' Interactions with Student Thinking	Fall 2013-Fall 2014 (Years 2 and 3)	RQ2: How do teachers use student thinking during instruction and what goals, orientations, and resources underlie that use?	Teacher questionnaire Videotaped classroom observations Teacher interviews Researcher field notes
Phase 3: Teacher Learning about Using Student Thinking	Spring 2015-Fall 2015 (Years 3 and 4)	RQ3: What is the learning trajectory for the teaching practice of productively using student thinking? RQ4: What supports can be provided to move teachers along that learning trajectory?	Video and audio recordings of classroom instruction Video clips of real-time noticed MIPOs Teacher interviews Records of TDE sessions Researcher field notes Records of researchers' discussions about TDE activities
Phase 4: Shareable Products	Spring 2016-Summer 2016 (Year 4)	Final products related to all research questions	No additional data

Phase 1: Student Thinking (Fall 2012-Summer 2013)

The purpose of the initial phase of the project is to test and refine the previously described MIPO framework through the purposeful creation of a data set of videotapes and transcripts of secondary classroom mathematics discourse that reflects the student mathematical thinking that can occur in diverse classrooms and through continuing analysis of that data set. The existing MIPO framework has emerged from analysis of a data set of convenience. We analyzed classroom discourse from published research papers and from extant videotaped lessons of preservice and inservice teachers from the researchers' previous and ongoing research projects. For the MIPO framework to be robust, however, it must describe mathematics discourse across a broad range of classroom settings. Thus, we will draw on existing and new data to investigate the nature of student mathematical thinking that emerges in diverse classroom settings. (RQ1: What is the nature of high-leverage student thinking that teachers have available to them in their classrooms?)

During this phase we will test and refine the MIPO framework through the analysis of a data set of videotaped mathematics lessons that was constructed to reflect variation of students, teachers and mathematics in U.S. classrooms. The students will vary by geographic location, community type, socioeconomic status and race and ethnicity. Teachers will vary in terms of gender, race and ethnicity, experience and teaching approach. Levels of mathematics classes from grades 7-12, pre-algebra through Calculus, and classes using a range of curricula will be represented. One strength of the geographic diversity (and connections) of the PIs on this project is our ability in aggregate to access such a diverse collection of classrooms. For example, Van Zoest has access to urban and suburban multi-ethnic classrooms in the Midwest; Peterson to predominantly American Indian classrooms in the West and predominantly Hispanic classrooms in the Southwest; Stockero to Rural low-income classrooms in the North; and Leatham to multi-ethnic classrooms in the Southeast. We anticipate adding approximately 50 videotaped classroom observations to the existing data set.

During Phase 1 of the project, the primary form of data collection will consist of videotaped classroom observations with accompanying field notes. In addition, each teacher will be asked to fill out a brief questionnaire about the demographics of their school and classroom and the nature of their teaching. The questionnaire data will allow us to compare the student thinking that emerges in diverse classrooms.

Each classroom episode will be coded and analyzed according to the student thinking made evident in the classroom and the ways in which this thinking is made public and utilized during the lesson. The analysis will begin with multiple researchers, including PIs and graduate students who have secondary school teaching experience, viewing the episodes to identify instances of observable evidence of student thinking. The research team will use the evolving MIPO framework to individually, and then collectively, characterize each instance in terms of why it is or is not a MIPO, considering both the importance of the mathematics and the extent to which it provides a pedagogical opportunity. Instances that are identified as MIPOs will be further coded for attributes such as type, teacher decision, and potential for supporting student learning (see PTM framework in Figure 2). The questionnaire data will be used to investigate patterns in the data, including the types of classrooms in which MIPOs are likely to occur, and the similarities in student thinking that might exist among classrooms that share demographic characteristics.

The research team will meet weekly through videoconference to collaborate in the coding and analysis. All coding and analysis will be facilitated by using the Studiocode video analysis software (SportsTec, 2011), which allows multiple researchers to easily compare coding by merging multiple video coding timelines. The researchers will discuss any discrepancies in the coding until agreement is reached. Here, as in subsequent phases, the external evaluators will serve an important role in providing formative feedback on our coding and analysis. The work of Phase 1 will support us in modifying and refining the MIPO framework as we expand our data set and identify examples and non-examples of MIPOs that represent the diversity of classrooms settings where student thinking is revealed and could be used. This information about the nature and form of student thinking in instructional situations will inform our developing *PUMT Theory*.

One final aspect of Phase 1 involves developing and pilot-testing the interview protocols and assessment instrument we will be using later in the project. Because we will have access to numerous classrooms settings during Phase 1, it will be an ideal time to refine the interview protocols. We will select 4-8 teachers over the course of Phase 1 to pilot our protocols for Phase 2. In addition, the external evaluators will begin to develop an assessment tool to measure teachers' knowledge of productive use of student mathematical thinking.

Phase 2: Teachers' Interactions with Student Thinking (Fall 2013-Fall 2014)

At the conclusion of Phase 1, we will have inferences about the types of student thinking that emerge in mathematics classrooms. Phase 2 uses the refined MIPO framework from Phase 1 as a tool for investigating how teachers think about *using* that thinking during instruction and how they make decisions about which student thinking to pursue. (RQ2: How do teachers use student thinking during instruction and what goals, orientations and resources (Schoenfeld, 2011) underlie that use?) The result of this work will be an understanding of how teachers conceptualize using their students' thinking during instruction and a hypothetical learning trajectory for the mathematics teaching practice of productively using student thinking during instruction to develop mathematical concepts—an important component of a *PUMT Theory.*

The data for this phase of the project will be clinical interviews with teachers, which will be grounded in video clips of classroom practice. As a means of continuing to explore the productive use of student mathematical thinking in diverse classrooms, these teachers will be chosen to reflect the diversity of students, teachers and mathematics discussed in Phase 1. The collaboration between the three universities will play a significant role in giving us the ability to gain access to the diversity of teachers and classrooms desired. We anticipate identifying 24-32 teachers (some of whom may be Phase 1 teachers) to participate in two to three clinical interviews each.

In preparation for each clinical interview, teachers will participate in a brief pre-lesson discussion about their plans for the day's lesson before being observed teaching. The discussion and observation will inform the clinical interview, which will focus on the practice of using student thinking and be grounded in video excerpts from the observed lesson. Interview questions regarding specific classroom episodes will focus on teachers' decisions to use or not use observable evidence of student thinking (as described in the MIPO framework). The overall purpose of these interviews is to understand teachers' orientations, resources, and goals (Schoenfeld, 2011) related to using student mathematical thinking during instruction—in essence, what "using student mathematical thinking during instruction" means for each teacher.

The researchers will interview 6-8 teachers during the first five weeks of fall semester 2013. During the second five weeks of the semester, the researchers will use an iterative, collaborative process (facilitated by weekly videoconferencing) to conduct an initial analysis of these interviews, using the MIPO framework (see Figure 1 and Leatham, Peterson, Stockero, & Van Zoest, 2011a) as a lens to view productive use of students' thinking. Based on the results of this initial analysis, the interview protocol will be refined and an initial theory of teachers' conceptions of productive use of student thinking will be articulated. During the last five weeks of fall semester, the next 6-8 teachers will be interviewed. The researchers will continue their collaborative analysis of the now expanded data set during the beginning of spring semester 2014. During the final 10 weeks of spring semester, additional teachers (8-20) will be interviewed until the developing theory appears to be saturated (consistent with Strauss and Corbin's (1998) concept of theoretical saturation). The external evaluators will provide important feedback to help us determine when the theory has been saturated. In addition, the external evaluators and PIs will continue to develop the assessment tool to measure teachers' knowledge of productive use of student mathematical thinking and will pilot test the instrument with a subset of Phase 2 teachers.

The research team will spend the summer and fall of 2014 continuing to collaboratively analyze the data to develop an understanding of how teachers think about using their students' mathematical thinking during instruction. As we characterize the various ways that teachers conceptualize using student thinking we will hypothesize a learning trajectory for the mathematics teaching practice of productively using student thinking during instruction to develop mathematical concepts. We will ask the following types of questions of the data in order to develop this trajectory: What does it look like when an individual does not seem to have this practice at all? A little? What aspects of the practice seem to be pervasive among all teachers? What seem to be the primary differences between those who are highly skilled in this practice and those who are not? Are certain categories of MIPOs (e.g., obvious, planned for, convenient) most often viewed as important to use? Might "naïve use" (Peterson & Leatham, 2009) be a good starting place for teachers to refine their practice? What does it look like when a teacher seems to want to develop the practice but does not have the skill or knowledge to do so? What does it look like when a teacher actively looks for MIPOs? What about when someone actively tries to create and act on them? Answering questions such as these will inform the development of the hypothetical learning trajectory that will be the foundation for Phase 3 of the project.

Phase 3: Teachers' Learning about Using Student Thinking (Spring 2015-Fall 2015)

Phase 3 centers on a teacher development experiment (TDE) and has two distinct goals: (a) improve the participating secondary school mathematics teachers' abilities to use student thinking during instruction to develop mathematical concepts and, thus, improve their ability to provide high quality STEM education for all students; and (b) use evidence gathered during the TDE to test out and refine our developing *PUMT Theory* (RQ3: What is the learning trajectory for the teaching practice of productively using student thinking? RQ4: What supports can be provided to move teachers along that learning trajectory?).

Phase 3 draws on the design research methodology outlined by Cobb and colleagues (e.g., Cobb, Zhao & Dean, 2009). Key features of this approach include: (a) interdependence of research and instructional design; (b) ongoing and retrospective analyses; and (c) the development of testable conjectures about both a hypothetical learning trajectory (HLT) for teachers and means to support teachers' learning. The HLT for the mathematics teaching practice of productively using student mathematical thinking developed in Phase 2 will be the HLT for our participants in Phase 3. We will leverage the MIPO framework as a tool for professional development based on testable conjectures about the HLT. We anticipate that some of these conjectures will be directly related to the theory (e.g., “Teachers on the cusp of developing skillful practice can both recognize the important mathematics in students’ thinking and the pedagogical opportunity that thinking affords, but lack a skill set that allows them to coordinate this knowledge into meaningful pedagogical moves”), while others will be in support of getting to a place where the theory can be applied (e.g., “Supporting teachers’ use of wait time will create space for students’ mathematical comments to be processed both by the teacher and the other students in the class so that the comments can then be used productively during instruction to develop mathematical concepts”).

It is important to note that in this TDE, rather than the learning being connected to teaching a specific mathematics content topic, the teachers in the teacher development experiment will engage with us in learning what MIPOs are, how to recognize them, and how to take advantage of them in their classroom as a way to improve their practice of using student thinking in support of the development of whatever mathematical ideas are the focus of their teaching. The primary activity of this phase can be thought of as “mutual development of theory” where the teachers are aware of our desire to further understand the practice of productively using student mathematical thinking as well as how one comes to understand the practice. These teachers will take our current theory and “try it on” as they also analyze and strive to increase their own abilities to carry out the practice. Drawing on relevant research, including results of Phases 1 and 2 of the project, we will attend to the teachers’ beliefs about student thinking, their mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008), and what it is they want to accomplish in their classrooms as we seek to design meaningful TDE sessions that are relevant to the teachers’ positions on the HLT. Although the TDE sessions will be grounded in the teachers’ classrooms and the learning of their students, the research lens will be on the teachers and their interactions with the researchers/professional developers who work directly with them.

There will be two cycles of the TDE. Teacher-participants (described below) will complete at least one full cycle. Each cycle will involve a semester-long series of approximately six two-week sessions focused on the teaching practice of productively using of student mathematical thinking. One PI will serve as a researcher/professional development provider for one teacher and any associated student teachers at each site. The bulk of the professional development will take place in the teachers’ classrooms in the context of their teaching. At the beginning of each two-week session, the PIs will meet with the teachers to discuss aspects of their practice related to their current development along the HLT and to provide them with knowledge and resources to begin working on these aspects of their practice. Throughout the two weeks the PIs will interact with the teachers in a variety of ways—classroom observations, face-to-face discussions, videoconferencing, email conversations—regarding questions they have and obstacles they have run into as they implement the ideas in their classroom. The structure, content, and number of our interactions with the teachers will vary because they will be based on individual teachers’ thinking about their students’ thinking (thus building on their thinking in a similar way to how they are attempting to use their students’ thinking). For example, a classroom teacher who already elicits substantial student thinking might benefit from brainstorming about the mathematical importance of that student thinking. On the other hand, a teacher who is having difficulty eliciting student thinking might benefit from reading literature related to teacher questioning or task design, depending on underlying issues. At the end of each two-week session the PIs will observe the teachers in their classrooms to document their progress along the HLT. The research team will then meet via videoconferencing to debrief the session and, as necessary, modify the HLT and refine the plans for subsequent sessions accordingly.

In the first TDE cycle, we will use data from Phase 2 to choose 3-4 cooperating teachers and their student teachers. The cooperating teachers will be chosen to represent a variety of classroom settings, levels of experience, courses, and curriculum types, but will have one thing in common—a demonstrated commitment to using student thinking during instruction to develop mathematical concepts. Although it may be useful to consider what it would take to move a teacher who does not value student thinking to a place where they do, our initial interest is in finding ways to support teachers who are committed to this practice to do so productively. Knowing how to support such teachers is an important precursor to increasing the number of teachers who are ready to be supported.

Although choosing cooperating teacher-student teacher pairings during the first cycle introduces some additional complexity, it provides many advantages. First, including prospective teachers allows for multiple teachers in the same classroom to see the same instances of student thinking. This will provide a richer basis for assessing teachers' interactions with student thinking. Second, working with student teachers allows us to investigate the effectiveness of our design with beginning teachers who have their full careers ahead of them. Third, working with student teachers also provides the opportunity to think about what aspects of the learning trajectory could effectively be part of preservice education and which parts are better suited for ongoing professional development. Fourth, the context of working with student teachers provides additional opportunities for us to work with the cooperating teacher as they view their student teachers' learning. Finally, working with cooperating teacher-student teacher pairings also allows for more flexibility in meeting and discussing what occurs in the classrooms immediately after it happens as one teacher can teach while the other(s) meet.

Participants in the second cycle (3-4 teachers) will be chosen intentionally to allow us to test out the refined conjectures from our first cycle of design and analysis. For example, we may find that teachers did not progress along the learning trajectory as quickly as anticipated. In this case, we may want to use the same teachers as the first cycle so we can modify the types of supports we provide and investigate whether these modifications have a more desirable affect on their learning. If, on the other hand, we find that the design was quite successful, we would want to test its robustness by applying that same design to teachers representing different types of classroom diversity (see Phase 1).

The following data will be collected, analyzed, and used to revise the testable conjectures and developing theory: whole-class video and audiotapes of classroom instruction; video clips of participants' real-time identified MIPOs; interviews with participants; records of TDE sessions; fieldnotes; and records of the researchers' discussions about the TDE activities. In addition to being used for research, this data will directly inform and be used in the TDE activities. Of particular interest will be assessments of the teachers' knowledge of productively using student thinking, and ability to do so, prior to, at key points during, and after the TDE. We will use the instrument piloted in Phase 2 to measure their knowledge and analysis of classroom observations to measure their ability. We will incorporate technology that allows us to document teachers' real-time noticing of MIPOs into our Phase 3 work as a means of both data collection and the facilitation of teachers' learning in and from their practice. For example, self-mounted cameras like DejaView allow teachers to capture 30-second video clips of important instances while teaching.

Analysis will use the qualitative data analysis tool Studiocode (SportsTec, 2011) to look for patterns in the teachers' interactions with student thinking and to investigate relationships between the types of support provided to the teachers and changes in those interactions. The multiple data sources, multiple sites, and multiple researchers, as well as the iterative TDE design, implementation and analysis processes, will allow for triangulation of, and greater confidence in, the results. Using design research methodology through two cycles of TDE will allow us to test and revise our conjectures, improving the emerging *PUMT Theory* and augmenting it with information about specific ways to improve teachers' productive use of student thinking during instruction to develop mathematical concepts.

Phase 4: Shareable Products (Spring 2016-Summer 2016)

The goal of Phase 4 is to make the results of the work useful to others by developing and disseminating sharable products. Although dissemination will take place throughout the project and will target a variety of interested stakeholders, this phase will focus on articulating refined products and results that can be directly used and built upon by other teacher educators and researchers. In particular, these shareable products will include: (a) a refined *PUMT Theory* that is grounded in the data from the first three phases of the project and includes a refined MIPO framework and a teacher learning trajectory for the mathematics teaching practice of productively using student thinking during instruction; (b) an *outline of teacher learning activities* found to support teachers in improving their use of students' mathematical thinking during instruction; and (c) an *instrument* that can be used to assess teachers' knowledge of using student mathematical thinking. We intend to use the outline of learning activities to inform our own work in a subsequent project aimed at developing teacher education materials focused on using student thinking to advance mathematical understanding. We will also obtain appropriate permissions so that data collected as part of this project may be used in these materials.

As discussed at the beginning of the proposal, products such as these are needed if STEM teachers are going to develop the critical teaching practice of productively using student thinking during instruction to develop mathematical concepts. These products will be valuable resources for school and university based professional development and research on mathematics education and mathematics teacher education.

Dissemination

The dissemination will focus on making the results of the project available to three main groups: (a) researchers; (b) mathematics teacher educators and district leaders; and (c) classroom teachers. The *Theory of Productive Use of Student Thinking* (*PUMT Theory)* will be the primary product for the researchers, the *outline of teacher learning activities* and the *instrument* to assess teachers' knowledge of using student thinking will be the primary products for the mathematics teacher educators and versions of all three of these shareable products will be made available to classroom teachers. We will intentionally share our results with organizations that focus on improving the mathematics learning of underrepresented populations (e.g., The Benjamin Banneker Association and the Center for the Mathematics Education of Latinos/as). A project website will be developed to serve as a central access point for general information about the project and the sharable products outlined in Phase 4. The venues of dissemination for the three main audiences and the specifics of the products are now outlined.

Research result dissemination will be through papers, conference presentations and working group sessions. Papers related to our *PUMT Theory*, the MIPO framework, and the results of our work with teachers will be published in professional mathematics education journals, such as the *Journal of Mathematics Teacher Education* (JMTE) and the *Journal for Research in Mathematics Education* (JRME). Links to all publications will also be maintained on the project website. Research presentations will be given at conferences such as the NCTM Research Presession, Psychology of Mathematics Education (PME International and North American Chapter) and American Educational Research Association (AERA). We will also plan working group sessions (e.g., PME-NA) to provide opportunities for other researchers to engage with us in analyzing the data and discussing the results.

Dissemination to mathematics teacher educators and district leaders will focus on developing an awareness of our findings and helping these teacher developers incorporate the project results—particularly the outline of teacher learning activities and the accompanying teacher learning trajectory—into their own context. We will plan sessions and working groups at mathematics teacher educator conferences, such as Association of Mathematics Teacher Educators (AMTE) and will submit papers to teacher educator journals, such as NCTM/AMTE's new *Mathematics Teacher Educator* (MTE).

Finally, we think it important that our findings and frameworks are made accessible to those for whom the results can have the largest impact—classroom mathematics teachers. Thus, we will submit teacher-focused articles about productive use of student mathematical thinking during instruction to journals such as NCTM's *Mathematics Teacher* and *Mathematics Teaching in the Middle School*. We will also plan sessions at NCTM's national and regional conferences and affiliate state-level conferences.

Broader Impacts

The *Leveraging MIPOs* project provides critical tools for teachers, teacher educators, and researchers that make more tangible the often abstract but fundamental goal of productively using students' mathematical thinking during instruction—a central tenet of ambitious teaching (e.g., Cohen, 2011). A *PUMT Theory* that articulates what the practice of productively using student mathematical thinking looks like, how one develops this practice, and how that development can be facilitated will enhance teachers' implementation of this practice, thus improving students' opportunities to learn important mathematics.

The research design helps to ensure the applicability of the project findings to a wide range of U.S. classrooms. The project will meaningfully attend to issues of diversity through intentional recruitment of participants to reflect the diversity of teachers, students, mathematics, and curricula present in U.S.

schools. This diversity will allow us to explore how various types of classroom diversity influence the types of student mathematical thinking that emerge in a particular classroom. In addition, recruitment of diverse graduate student researchers—including those with mathematics teaching experience—along with the diverse experiences of the collaborating PIs will ensure that divergent perspectives are represented in the data analysis. Studying student thinking and teachers' interactions with it in diverse classrooms enhances the ability of the resulting theory to benefit all students.

Including cooperating teacher-student teacher pairings in the teacher development experiments will broaden the impact of the project as the cooperating teachers work with their current and future student teachers. As the student teachers, cooperating teachers and PIs interact in talking about productively using student mathematical thinking, there will be wide range of experiences and perspectives present among these triads. We will have individuals at various places on the HLT trying to work together to enact the practice. Thus the cooperating teachers will need to think about their own efforts to use student thinking as they teach novice teachers how to do the same. Thus this design broadens the range of applicability of the theory. There is also some indication (e.g., Stockero, Van Zoest, Leatham, & Peterson, 2011) that the ideas related to productively using student thinking developed here have generalizability to other subject areas and grade levels.

To increase the applicability and availability of the results of this work, significant time has been built into the project for refining products and results so that they are in forms that are useful to and can be built upon by other teacher educators and researchers. In particular, we expect the project to result in an outline of professional development activities that have been found to support teachers in learning to productively use student thinking. We plan to use this outline to develop professional development materials in a future project, and anticipate that it will also be used by other teacher educators who do similar work. In the long-term, this could allow the results of the project to impact the practice of a significant number of mathematics teachers. A three-prong dissemination plan—focused on teachers, teacher educators, and researchers—will also contribute to broad availability of the results of the work.

Evaluation

Horizon Research, Inc., will serve as the external evaluator for the Leveraging MIPOs project. HRI is well suited for this role, with experience evaluating a variety of mathematics and science education research, development, and implementation projects, and educational reform initiatives ranging from small, narrowly-focused teacher enhancement efforts to large, national, cross-site evaluations of NSF-funded programs.

The evaluation of Leveraging MIPOs will include both formative and summative components. Formative evaluation will focus on the following questions: (1) To what extent is the project making progress in carrying out project activities as proposed? (2) What is the quality of the project's process for developing a *Theory of Productive Use of Student Mathematical Thinking (PUMT Theory)*, and associated tools and professional development? (3) How well is the project collecting and using evidence to enhance the theory, tools, and professional development? (4) How well is the project operationalizing measurable teacher outcomes of the tools for applying the theory and associated professional development?

Summative evaluation will also address four questions: (1) To what extent have the project's outcomes been achieved? (2) What is the quality of the project's primary products, particularly tools for applying the theory and associated professional development? (3) What is the strength of the evidentiary basis for the project's research findings? (4) What is the potential for broader use of the theory and associated tools and associated professional development?

Multiple methods will be used to collect evaluation data, including observations, interviews, document review, and assessment development and administration. Evaluation staff will observe selected project activities, including meetings of the project's leadership and development team, face-to-face professional development offerings, and a sample of classrooms of teacher participants involved in that professional development. Interviews will be conducted with project leaders, focusing on the development process, including collection and use of evidence, and with participating teachers, focusing on the quality and utility of the theory and professional development to support its use in practice.

As part of assessing the quality of the research methods, HRI will conduct an external validity check of the project's coding observation and interview data. HRI will also work closely with the project's leadership and development team to identify key outcomes for teacher participants in order to develop embedded professional learning tasks for collecting evidence to measure and document these outcomes. This process will provide opportunities for the project to clarify intended knowledge and practice outcomes for teachers who work with the theory, and to consider what evidence best indicates the nature and extent of teacher participants' progress on these outcomes.

HRI will also coordinate an independent review of key products the project develops for content accuracy, coherence, consistency with best practice, and alignment with the project's goals. Finally, HRI will conduct a formal review of the project's research reports, using standards of evidence developed for a prior Math Science Partnerships project (NSF Grant #0445398).

HRI will prepare annual reports addressing all evaluation activities and findings, including a final summative report. Formative feedback will be provided on a more frequent basis through a series of memos delivered to inform key project decisions and mid-course corrections, with regular contact via videoconference, phone, and email as appropriate.

Expertise

The Leveraging MIPOs collaborative project draws on three universities' expertise and access to diverse populations. The institutions themselves have varied strengths that support the project: Brigham Young University's focus on undergraduate research and its Masters in mathematics education program that attracts experienced classroom teachers, Michigan Technological University's strong focus on technology and improving STEM education, and Western Michigan University's Ph.D. program in mathematics education. BYU will be the managing partner, maintaining all financial records and filing reports. WMU will recruit and host the PhD students. MTU will handle aspects of the project related to technology, including hosting the project website, providing videoconferencing access, and storing all the project data on a secure server.

The core research team will be the four PIs and three full-time PhD students. Together, the PIs provide a range of experience and expertise that is briefly summarized below. The PhD students will be specifically chosen for their ability to provide the perspective of underrepresented populations and their recent experience as classroom mathematics teachers and professional development providers. In addition, other graduate students and faculty who have an interest in the project will be involved in various aspects of the research. The project PIs have been meeting via videoconference approximately weekly to work on research related to MIPOs for almost two years and have established effective ways to collaborate across long distances. Continuing this practice, the research team will meet at least bi-weekly, often weekly, to discuss project activities and to collectively discuss the data analysis and conclusions. The Advisory Board and External Evaluators will join us at critical junctures to provide their input.

Project PIs

Leatham brings expertise in research on learning to teach and on teacher beliefs. Much of his most recent work has focused on how teachers learn to use students' mathematical thinking. He brings to the project a strong working knowledge of literature and methodology related to teachers' beliefs that will be particularly useful as we make sense of the data collected from teachers in Phase 2. Leatham has used qualitative research methodologies to conduct research with teachers in classroom and school settings extensively throughout his career

Peterson brings to the project a rich mathematical background from his Ph.D. in mathematics. More recently he has focused on improving preservice teacher education. He has studied the way Japanese cooperating teachers interact with student teachers as they prepare lessons, which led to a restructuring of student teaching at BYU. The study of student teaching in Japan and in the United States has provided insight into identifying MIPOs because of the frequency with which novice teachers miss opportunities to use student thinking to help students understand the mathematics at hand.

Stockero brings to the project extensive experience teaching high school mathematics, as well as experience designing and teaching university methods courses, supervising student teachers, and providing professional development to practicing K-12 teachers. She has engaged in substantial analysis of classroom video as part of her research on understanding teacher learning and has used the Studiocode software to collect and analyze classroom video and as a tool to conduct grounded teacher interviews. She has also used self-mounted cameras as a means of collecting data on in-the-moment teacher noticing.

Van Zoest is an experienced teacher educator and researcher who has designed, developed and tested content for university methods courses and professional development. She brings to the project a broad knowledge of current research on Mathematics Knowledge for Teaching (Ball, Thames, & Phelps, 2008) and theoretical frameworks for understanding mathematics teaching (she is currently co-editing a focus issue of *ZDM: The International Journal on Mathematics Education* on this topic). As part of earlier NSF projects and in her recent work with preservice teachers, she has partnered with teachers to improve specific aspects of their practice (e.g., Van Zoest, & Enyart, 1998).

Experienced Teachers

The project will draw on the expertise of three experienced mathematics teachers and/or professional development providers who are interested in pursuing a PhD degree. Two will be directly funded by the project. We will use resources such as the Benjamin Banneker Association to intentionally recruit teachers from underrepresented populations. This will strengthen the research team by diversifying the perspectives through which we look at the data.

External Evaluators—Horizon Research, Inc. (HRI)

Daniel J. Heck, Senior Research Associate, holds a Ph.D. in Education from the University of Illinois at Urbana-Champaign, with a specialization in Educational Psychology. Prior to joining HRI in 1999, he taught high school mathematics and was an Associate Researcher at the University of Wisconsin's Center for Education Research. At HRI, Dr. Heck has directed the Study of the Impact of the Statewide Systemic Initiatives, and Lessons Learned from Research on Systemic Reform projects, and was PI of Developing a Research Agenda for Understanding the Influence of the Common Core State Standards for Mathematics. He leads HRI's evaluations of mathematics education research, development, and implementation projects. He is also the current chair of the NCTM Research Committee.

Kristen A. Malzahn, Research Associate, holds a Masters degree in Education from the University of North Carolina at Greensboro, NC. Prior to coming to HRI in 2001, she taught at the elementary level for five years and was a lead teacher for a mathematics professional development initiative in Durham, NC. At HRI, she has worked on the evaluation of NSF's Local Systemic Change through Teacher Enhancement Initiative, and Deepening Everyone's Mathematics Content Knowledge project, a partnership between the University of Rochester and surrounding rural and suburban school districts.

Joan D. Pasley, Senior Research Associate, holds a Ph.D. in Curriculum and Instruction from the University of North Carolina at Chapel Hill. She has been working with HRI since 1994 on a number of research and evaluation projects, including directing the evaluation of Deepening Everyone's Mathematics Content Knowledge project, a partnership between the University of Rochester and a number of surrounding rural and suburban school districts. She co-authored the recently-released *A Research Agenda for Understanding the Influence of the Common Core State Standards in Mathematics*.

Advisory Board

The project will benefit from an advisory board consisting of experienced mathematicians and mathematics teacher educators with expertise in orchestrating productive mathematics discourse, diversity, teacher noticing, mathematical knowledge for teaching at the secondary school level, and teacher education materials development. Margaret Smith, University of Pittsburgh brings experience working with several large-scale grants that focus on increasing the quality of mathematical tasks and student mathematical discourse and on how teachers orchestrate productive discourse. Kathleen Heid, Pennsylvania State University, brings expertise in mathematical understandings of secondary school teachers and how they use that knowledge in their work. This is important to understanding teachers decisions related to using student thinking during instruction. Randy Phillip, San Diego State University, has worked with mathematics teachers' conceptions of teaching, with noticing, and with the teaching practice of attending to students' mathematics. His input will be particularly useful for our investigation of how teachers notice MIPOs. Susanna Salamanca-Riba, New Mexico State University, is a research mathematician with extensive experience working in primarily Hispanic classrooms. She will provide insights related to the mathematical integrity of the work, along with issues related to diversity. The advisory board will meet with the PIs annually to provide formative feedback on the project activities and products and to review written works that result from the project.

Results of Prior NSF Support

As a subgrant to the *Developing Facilitators of Practice-based Professional Development* project (Mumme & Seago, REC #0243558, 2008-09), Van Zoest and Stockero were co-investigators in a study focused on understanding the durability of documented teacher learning outcomes from using a video case professional development curriculum in an undergraduate mathematics methods course. Four major learning outcomes from the original study (Stockero, 2008a, 2008b) were found to be durable over time: (1) the use of evidence to support analyses of teaching and learning, (2) a focus on individual students' thinking, (3) the adoption of a tentative stance towards practice, and (4) a focus on how teachers' decisions support student thinking (Van Zoest & Stockero, 2008a). In addition, seven sociomathematical and professional norms cultivated in the methods course were found to re-emerge when participants were reconvened to analyze teaching practice (Van Zoest, Stockero, & Taylor, 2009; Van Zoest, Stockero, & Taylor, 2010). These results are significant in that the dispositions and normative behaviors that were embedded in the video case curriculum are focused on supporting student-centered instruction.

The classroom video data from this study were further analyzed to understand PTMs in beginning mathematics teachers' practice, the decisions teachers make when such pivotal moments occur, and the type of learning that results (Stockero, Van Zoest, & Taylor, 2010). This analysis is directly related to the proposed project, as it contributed to the development of the initial MIPO framework and provides many of the initial hypotheses for developing a model of how teachers learn to identify and use MIPOs.

Van Zoest co-directed the Mathematical Sciences Sequential Summer Institute (Hirsch, Alavi, & Van Zoest, ESI-9353513, 1995-2000), a TD program that resulted in 30 teacher leaders from 29 districts in Michigan developing content and pedagogical knowledge consistent with the 1989 NCTM Standards (see Van Zoest, Ziebarth, & Breyfogle, 2002). She co-directed the Local Systemic Change projects Renewing Mathematics Teaching through Curriculum (Van Zoest & Ritsema, ESI-9618896, 1997-2002) and Renewing Mathematics Teaching through Curriculum in the Middle School (Van Zoest & Kline, ESI-9843519, 1998-2002), which provided support to teachers implementing the Core-Plus and Connected Mathematics curricular materials (see Grant, Kline & Van Zoest, 2001). She also received a POWRE grant, Learning to Teach in a Reform Environment (Van Zoest, ESI-9743679, 1997-2002), to study the results of including preservice teachers in professional development around NSF-funded curricular materials which they then used during their student teaching experiences (Van Zoest & Stockero, 2008b). Although these projects are not directly related to the proposed project, they provide a rich background of knowledge and experience relevant to providing effective professional development for mathematics teachers.

Stockero is currently PI on a DRK-12 CAREER grant (DRL-1052958) focused on helping prospective teachers notice mathematically important moments that occur during instruction; this project is in the early data collection stage. It is expected that the results of the early phases of this study will contribute to the *Leveraging MIPOs* project by informing the TDE in Phase 3. She is also co-PI on MTU's Robert Noyce Teacher Scholarship Program (DUE-0934763). This program, aimed at recruiting and preparing STEM teachers to work in high-needs schools, is concluding the second year of activities. As of December 2011, four Noyce scholars graduated, two are scheduled to student teach in Spring 2012, and seven others are enrolled in the program; none have yet accepted full-time teaching positions.

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