Establishing Student Mathematical Thinking as an Object of Discussion

PME-NA 43

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Keith R. Leatham
Blake E. Peterson
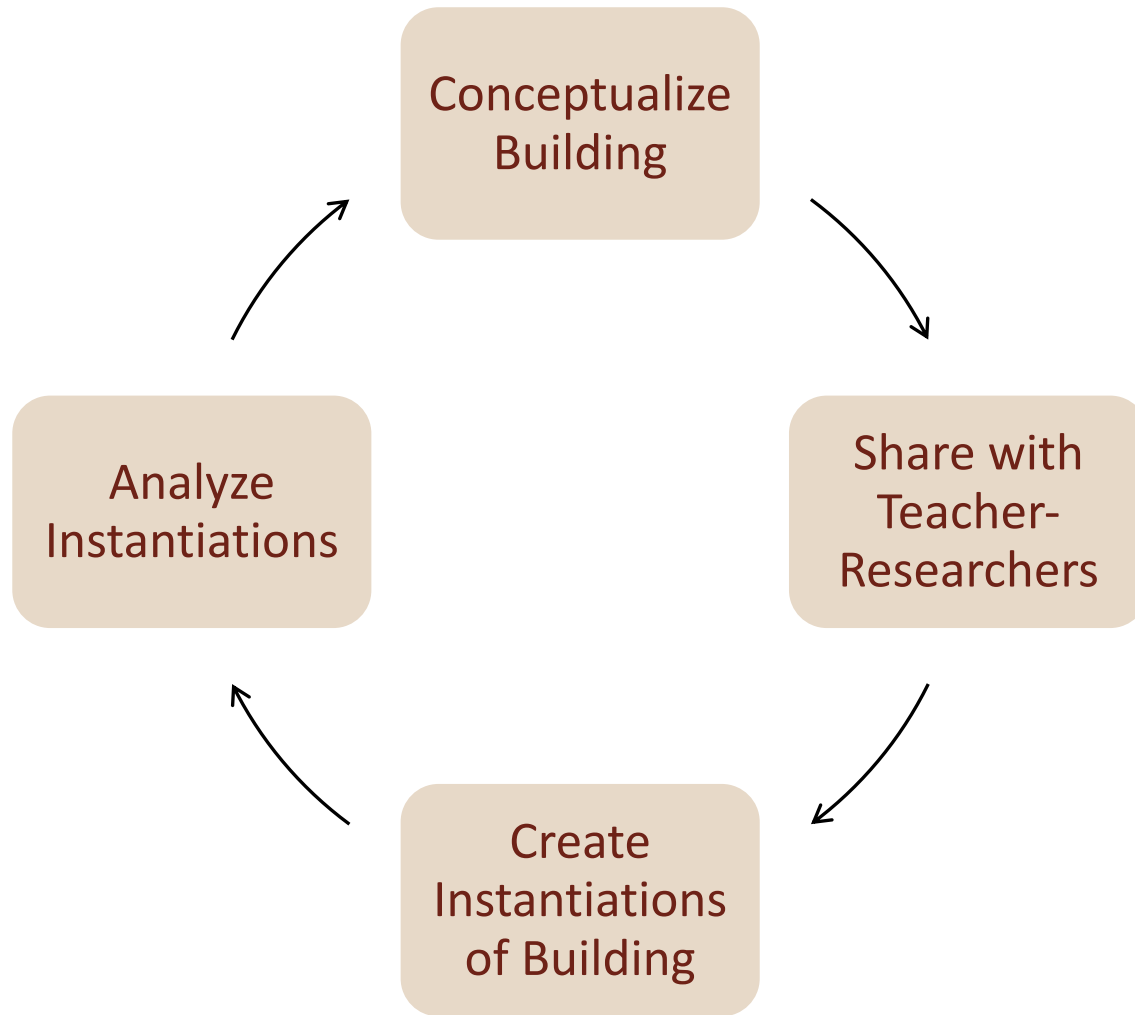
Laura R. Van Zoest
Ben Freeburn

Shari L. Stockero

Building on MOSTs: Investigating Productive Use of High-Leverage Student Mathematical Thinking

A MOST is a **M**athematical **O**pportunity in **S**tudent **T**hinking
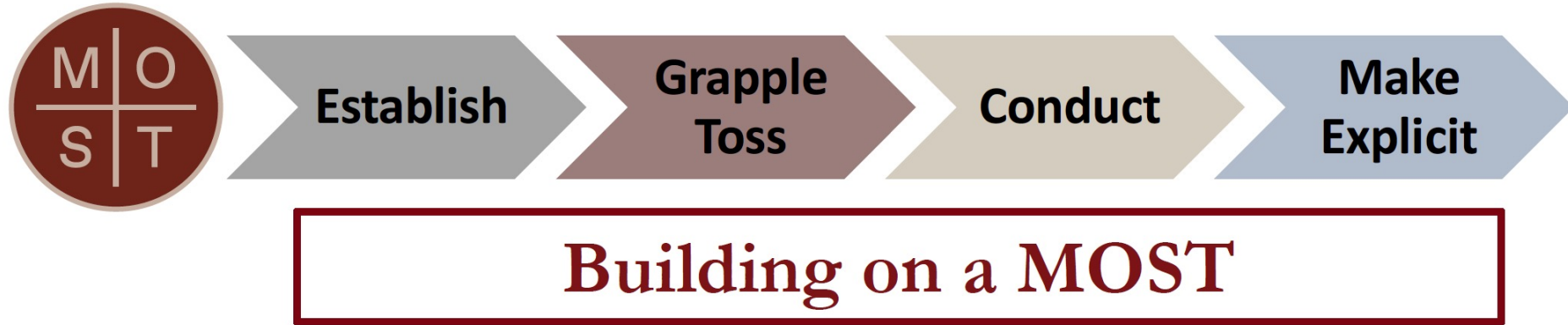
Building on a MOST is engaging the class in making sense of the MOST to better understand the mathematics of the MOST.

CONVERSATIONAL BUBBLE
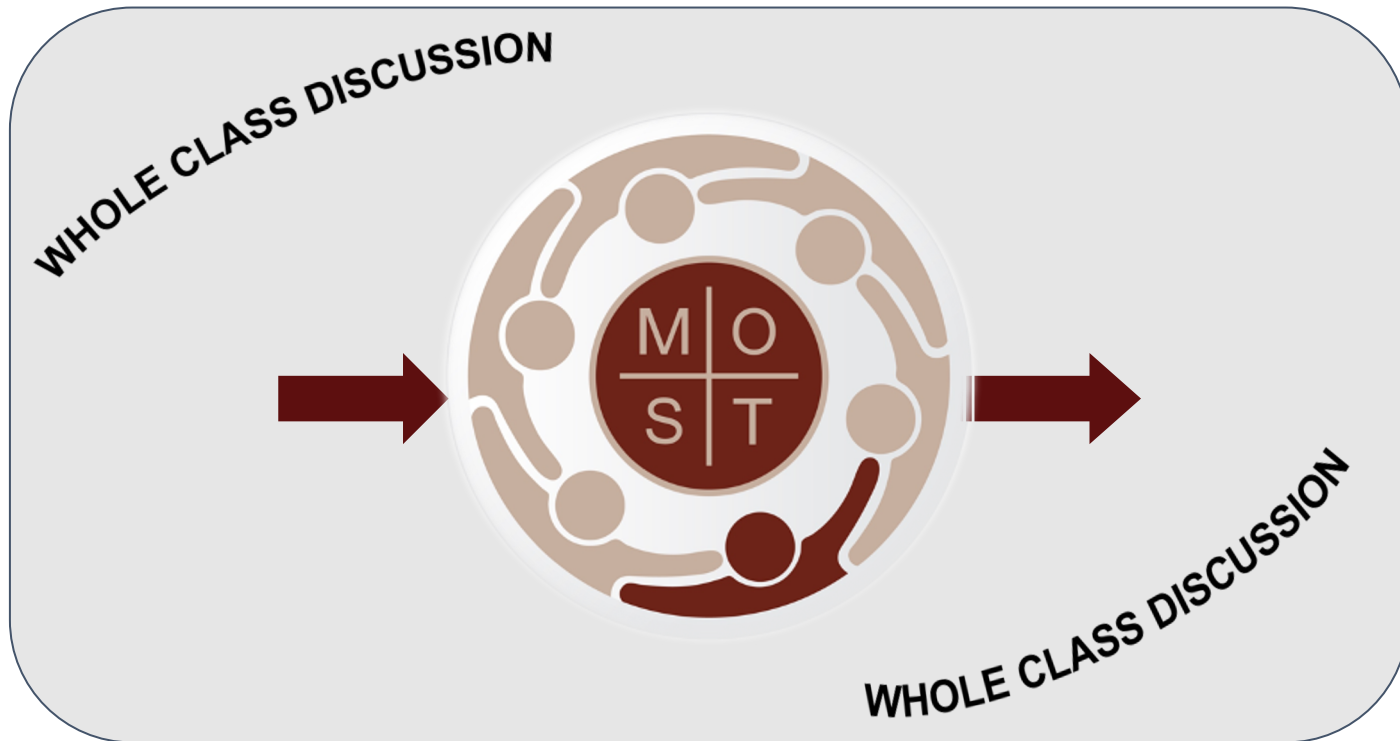
Why focus on just Establish?
- foundational to any productive use of student thinking
- illustrative unpacking
- only have 20 minutes

ESTABLISH
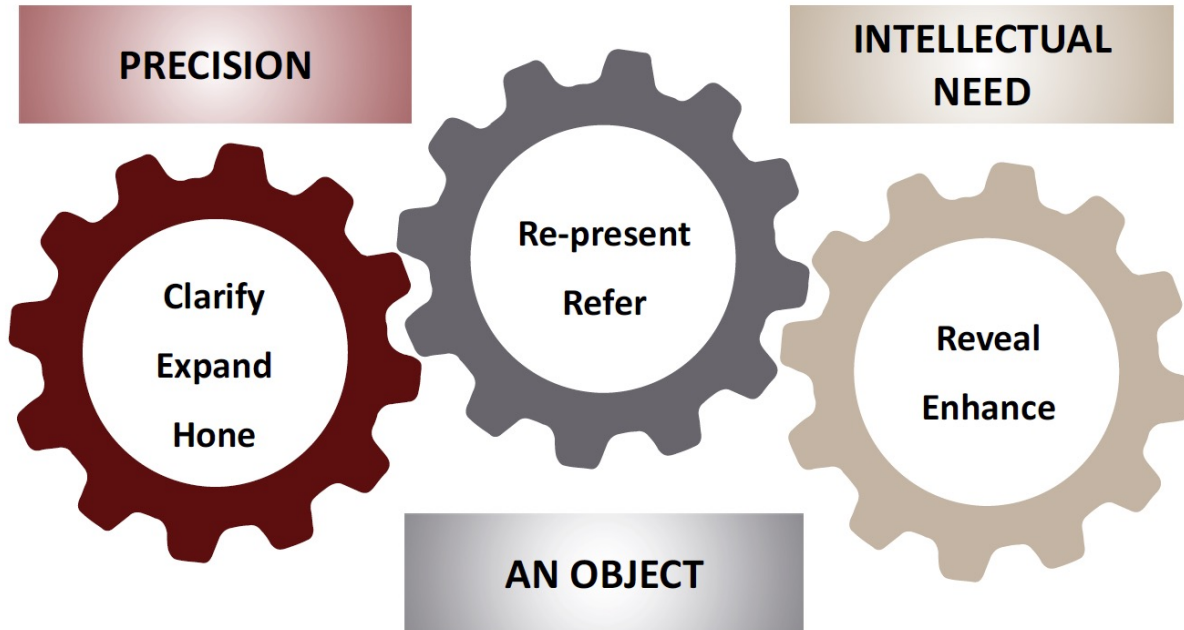
Precision – *ensure that the MOST is precise enough for students to engage with it*

- Clarify

- Expand

- Hone

ESTABLISH

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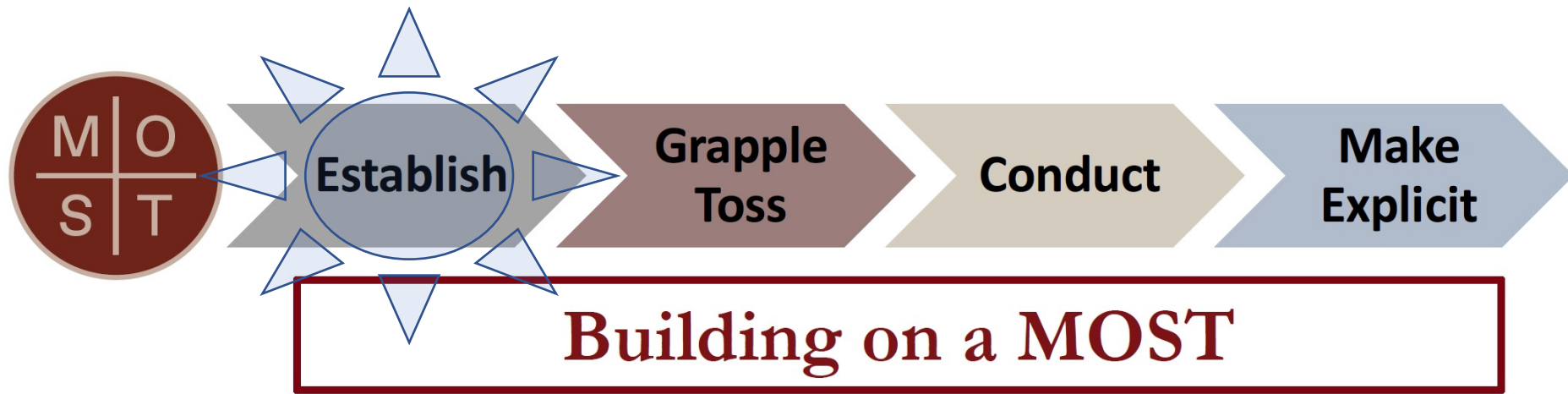- Clarify – make clear WHAT the student has said
  - Articulate inferences such as informal language and pronouns
  - Clarify logical structure

- Expand – make the student contribution complete
  - Elicit reasoning
  - Elicit essential information

- Hone – make the student contribution concise
  - Remove extra verbiage or extraneous information
  - Capture ideas succinctly using symbols or other shorthand

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ESTABLISH

An Object – *ensure that the MOST takes on a measure of permanence and identity so that it can be referred to during the remainder of building*

- Re-Present

- Refer

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- Re-Present – create a representation of the object
  - Repeat or revoice
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  - Name it
  - Gesture toward it

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ESTABLISH

Intellectual Need – *ensure that the intellectual need is leveraged to maximize student engagement*

- Reveal

- Enhance

ESTABLISH

Intellectual Need – *ensure that the intellectual need is leveraged to maximize student engagement*

- Reveal – make hidden intellectual need visible

- Enhance – increase student buy-in

    - Establish commonality
    - Establish difference
    - Show enthusiasm
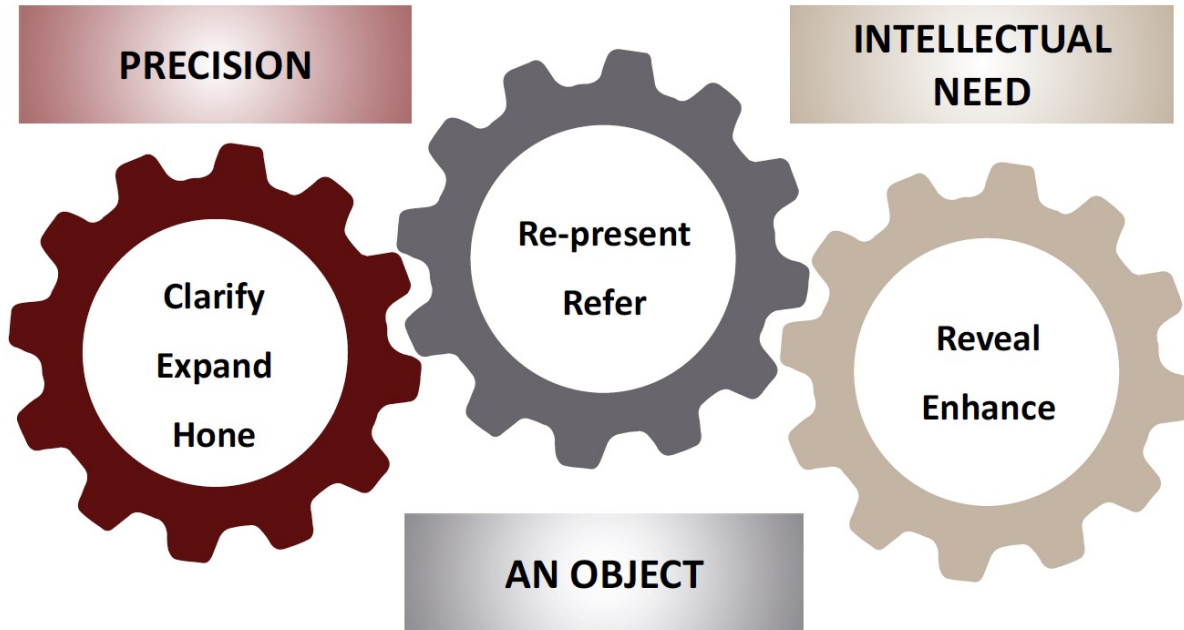
Three Observations

Establishing precision operates on the pieces whereas establishing an object operates on the object as an entity.

In practice, the aspects are intermingled.
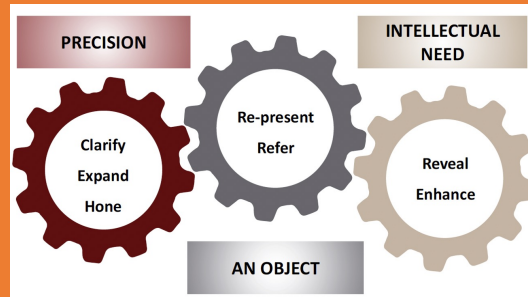
The actor matters.

Discussion

What are your reactions to this unpacking of the Establish element of building?

How might "establishing" play a role in other teacher practices?

How might this work contribute to other research endeavors?

What additional feedback do you have for us?

Thank you!

BuildingOnMosts.org

The MOST Analytic Framework
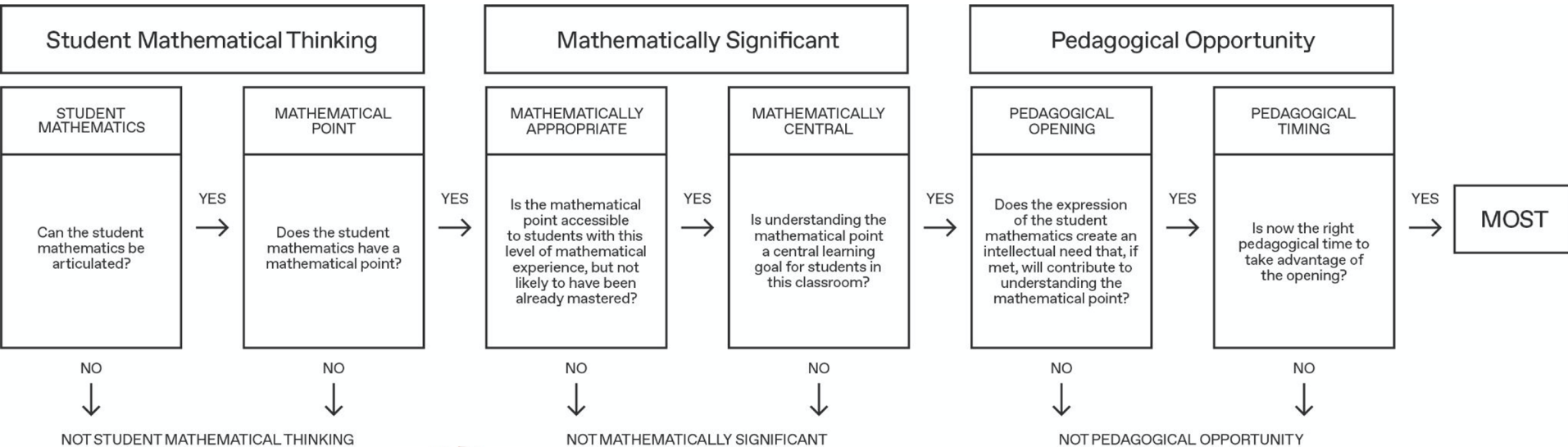
GRAPPLE TOSS

Toss the **established object** with a **specific sense-making action** determined by the nature of the MOST.

Question

What do you think, [established object]?
e.g., What do you think, can a linear equation have two y-intercepts?

Claim / Solution

What do you find mathematically compelling or conflicting about [established object]?

Revealed

What is going on here, [action on established object]?
e.g., What is going on here? Why is 9 wrong?

Multiple Claims / Solutions

How do you decide which of [established object] are correct?

BUILDING

CONDUCT

```
              ┌─────────────────────────┐
              │   Student Contribution   │
              └─────────────────────────┘
                          │
              Is this a MOST-related Contribution (MRC)?

        Not MRC                          MRC

   ┌──────────────────┐         ┌──────────────────┐
   │ Put contribution │         │ Establish the MRC │
   │      aside       │         └──────────────────┘
   └──────────────────┘                  │
            │              Do students need to make    YES   ┌──────────────────┐
   ┌──────────────────┐    sense of the MRC?*    ───────────▶│ Grapple Toss MRC │
   │  Re-center MOST  │                                      └──────────────────┘
   └──────────────────┘              NO
            │
   ┌──────────────────┐    Are there multiple MRCs    YES   ┌──────────────────┐
   │Re-Grapple Toss   │    or does the MRC present a  ─────▶│ Synthesize MRC(s) │
   │     MOST         │    critical contradiction of         └──────────────────┘
   └──────────────────┘    the MOST?)
                                    NO                Establish Connection

                           ┌──────────────────┐
                           │Connect MRC and MOST│
                           └──────────────────┘
                                    │
                           ┌──────────────────┐
                           │Grapple Toss Connection│
                           └──────────────────┘
```
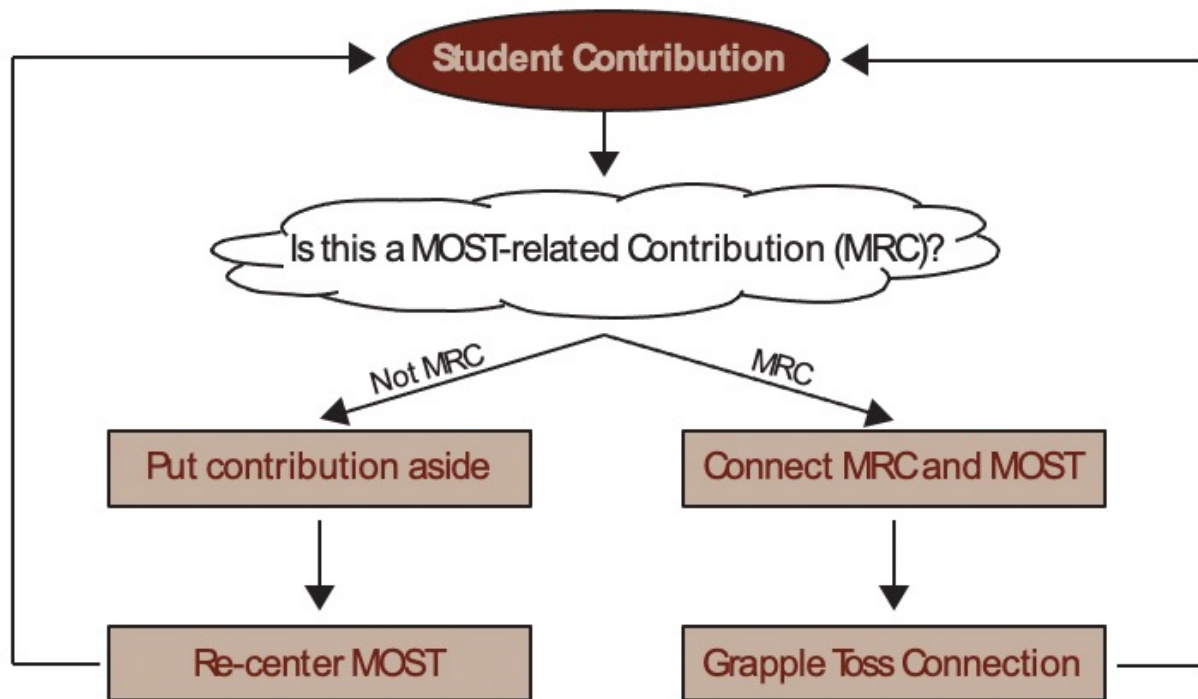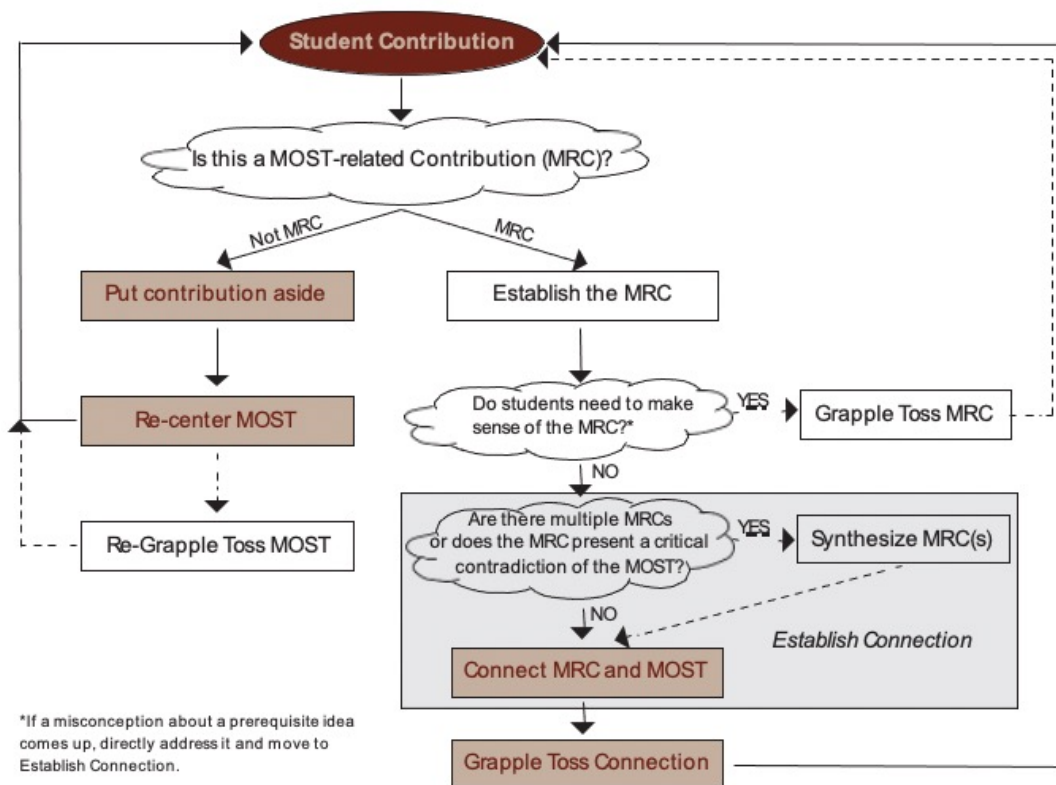
*If a misconception about a prerequisite idea comes up, directly address it and move to Establish Connection.

BUILDING

MAKE EXPLICIT

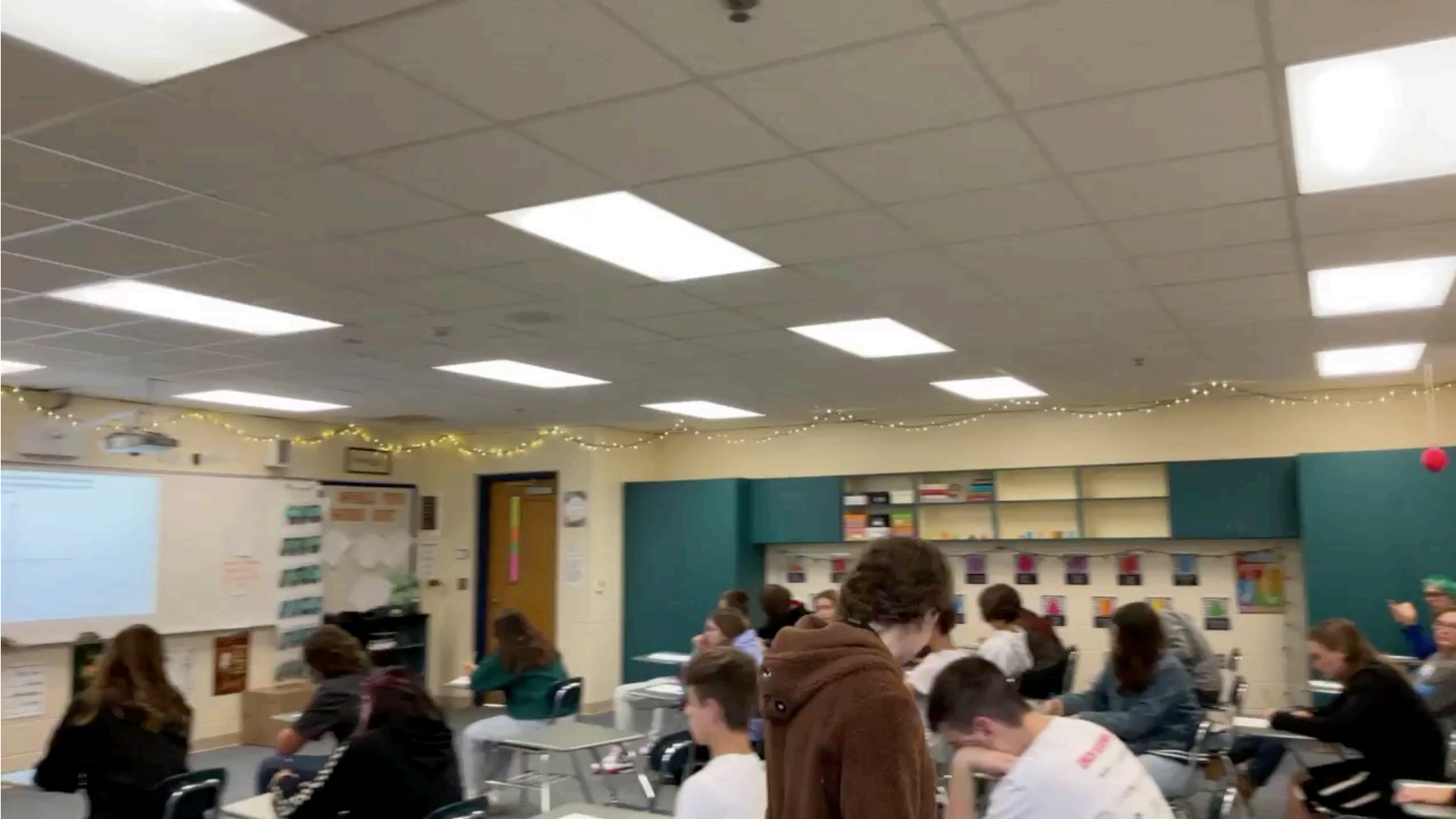**RESOLVE THE MOST**

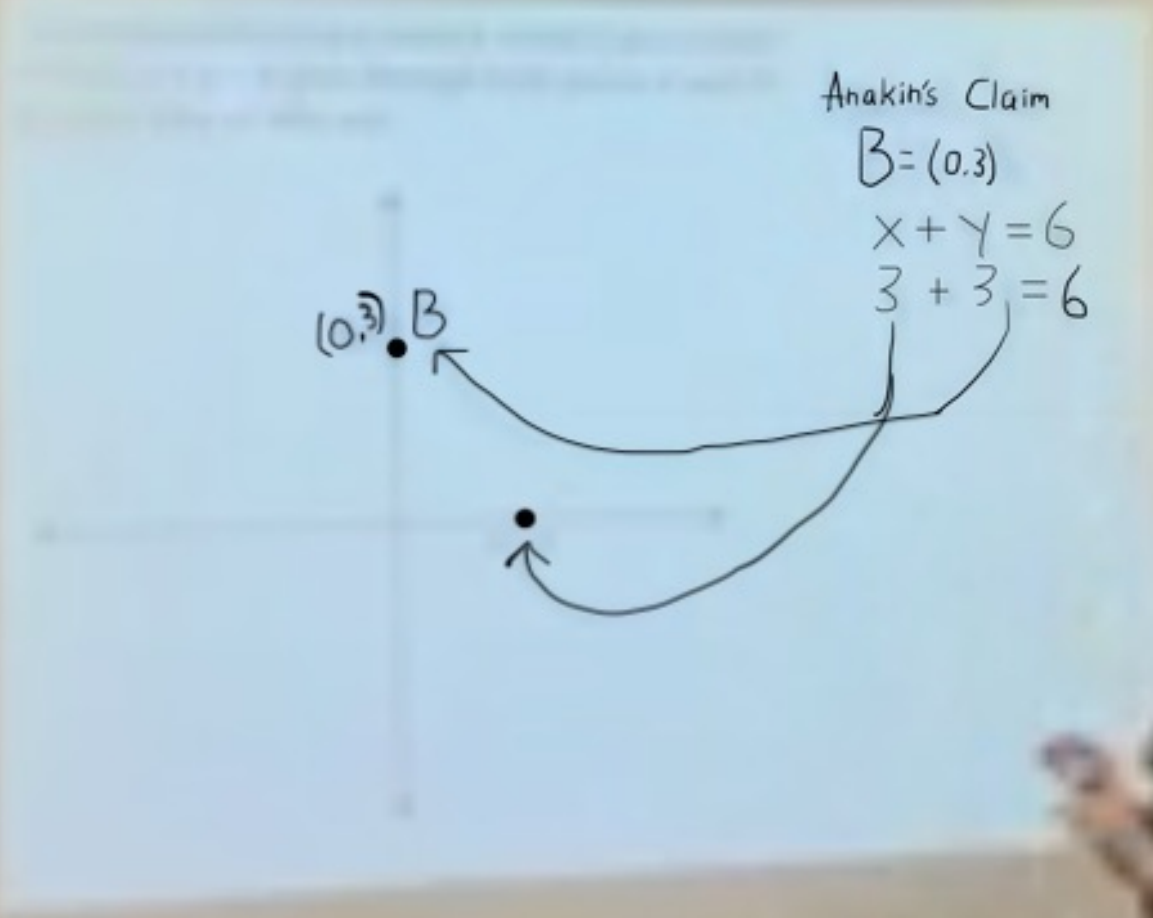Acknowledge OR Check-in

Assess agreement

POSITION AS GLOBAL

Determine actor
Provide scaffolding

TRANSITION

Position MP
Switch activity

Anakin's Claim

$B = (0.3)$

$x + y = 6$

$3 + 3 = 6$

$(0.3) \cdot B$