CLASSROOM MATHEMATICS DISCOURSE: BROADENING PERSPECTIVES BY INTEGRATING TOOLS FOR ANALYSIS

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This working group explores tools for analyzing mathematics classroom discourse across two projects with different, but complementary perspectives. The goals of the working group include generating interaction about the theoretical lenses that we use to analyze and discuss classroom mathematics discourse and the relationships between these different theoretical frameworks. Participants will engage with the individual frameworks in the first two sessions and discuss interactions of the two frameworks in the third session.

Keywords: Classroom Discourse, Instructional Activities and Practices, Teacher Knowledge

History of the Working Group

This working group focuses on the overarching question, *What are the relationships between various tools and frameworks for analyzing classroom mathematics discourse?* The question emerges from two perspectives: the general proliferation of theoretical frameworks for analyzing classroom mathematics discourse, and specifically the interactions between two research groups interested in determining interesting intersection points between their two analytical frames. The group does not have an existing history at PME-NA, but stems from three prior working groups: the *Mathematics Classroom Discourse* (Herbel-Eisenmann, Staples, Choppin, & Wagner, 2005-2007), *Investigating Mathematically Important Pedagogical Opportunities* (Leatham, Stockero, Van Zoest, & Peterson, 2010), and *Measuring Instruction in Relation to Curriculum Use* (Kim, Remillard, Steele, Blunk, Piecham, & Lewis, 2012). We model the organization of this proposed working group on that of the 2012 group, which brought together researchers from three projects to engage participants in conversations around their frameworks for measuring curriculum use, analyzing data using each framework, and having a cross-cutting conversation about the affordances and constraints of each approach.

Our working group brings together researchers from two projects interested in capturing important interactions in mathematics classrooms and describing the ways in which these interactions might represent generalizable, visible patterns that move a teacher's mathematical and social goals forward in interesting ways. Such interactional moments would have great potential as common sites of inquiry and as tools for planning and reflection by teachers, teacher educators, and educational researchers. The Leveraging MOSTs: Developing a Theory of Productive Use of Student Mathematical Thinking Project (MOST) is an NSF-funded collaboration between researchers at Western Michigan University, Brigham Young University, and Michigan Technological University that aims to support teachers in identifying and leveraging important instances of students' mathematical thinking. The project focuses on the identification of MOSTs: **M**athematically Significant Pedagogical **O**penings to Build on **S**tudent **T**hinking and progressively investigates MOSTs across four contexts: student thinking, teacher interaction with student thinking, teachers' learning about student thinking, and shareable products for teacher learning.

The Mathematics Discourse in Secondary Classrooms (MDISC) Project is an NSF-funded collaboration between Michigan State University and the University of Delaware to develop a set of case-based professional development materials focused on secondary mathematics classroom discourse. Specifically, the materials to support teachers in becoming more *purposeful* about fostering *productive* and *powerful* discourse in the mathematics classroom. *Productive* discourse focuses on the ways in which teacher-student and student-student discourse moves the mathematics forward in the classroom. *Powerful* attends to the ways in which discourse can help further and attend to social goals and positioning. At the heart of the materials are six Teacher Discourse Moves (TDMs), which are identifiable discourse tools that can serve to structure productive and powerful discourse in secondary classrooms. The theoretical concepts of mathematics register and positioning are used as lenses through which to interpret what happens when one uses the TDMs.

Both the MOST and MDISC projects have developed conceptual frameworks for identifying specific classroom discourse moments that appear to be powerful and have the potential to influence student learning. In this working group, each project will briefly introduce their theoretical framework to participants and invite engagement with the framework through the collective analysis of classroom episodes. Following this deep engagement with each project's framework, the working group will facilitate a conversation comparing the affordances and constraints of the frameworks, identifying interesting intersection points between them, and more broadly discussing the importance and challenges of identifying and analyzing classroom mathematics discourse, with a particular focus on secondary classrooms.

MOST Project Overview

The MOST project focuses on the work of facilitating and researching teachers' mathematically-productive use of student thinking. We developed the MOST construct based on characteristics of "teachable moments" that emerged from the literature (Davies & Walker, 2005; Davis, 1997; Jaworski, 1994; Schoenfeld, 2008)—student thinking, significant mathematics and pedagogical openings. We define these characteristics in the following sections.

Student Mathematical Thinking

Because the MOST construct is designed to help articulate productive use of student mathematical thinking, we begin by defining what we mean by *student mathematical thinking*. Foremost, the thinking underlying a MOST must come from a student. In addition, the thinking must be mathematical. Although we recognize our inability to access directly the thoughts of students, we make inferences based on our observations of what they say and do. Thus, when we use the phrase *student mathematical thinking* we refer to observable evidence of student mathematical thinking, which we define as any instance where a student's actions provide sufficient evidence to make reasonable inferences about what they are thinking mathematically. In the classroom setting, this evidence most commonly is visible in actions such as verbal utterances, gestures, or written work (including on the board).

Mathematically Significant

In order to be a MOST, the mathematics in an instance must warrant use of limited instructional time; that is, it must be what we call *mathematically significant*. We use the term *mathematically significant* in the context of teachers engaging a particular group of students in the learning of mathematics. Specifically, mathematically significant instances contain mathematical ideas that, when they become the object of discussion, can be used to further the students' understanding of mathematics. To begin the mathematically significant analysis, we formalize the student mathematical thinking to articulate the mathematics of the instance (MI)—the important mathematical idea to which the student thinking is related. We then consider the instance in relationship to two key criteria: (1) the appropriateness of the MI for the mathematical development level of the students, and (2) the centrality of the MI to the mathematical goals for the students.

Meeting the *appropriate mathematics* criterion requires two things. First, the MI must be accessible to the students given their prior mathematical experiences; they must have adequate background knowledge to engage with the mathematical idea. Second, students at that mathematical level would not be expected to have mastered the MI. If they had, pursuing that idea would not likely further their understanding of mathematics. The *central mathematics* criterion requires that the MI be related to a central mathematical goal for student learning in that class. The mathematical goals for the classroom encompass both mathematical content and mathematical practices. A goal can meet the centrality criterion by being a lesson goal or by being a broader mathematical goal that is central to the discipline of mathematics. The further a goal is from the lesson, the more central it has to be to the discipline of mathematics to meet this criterion.

Pedagogical Opening

Conscientious teachers continuously seek evidence of their students' engagement with a wide variety of instructional goals. They take cues from actions big and small, making adjustments and pushing students to elaborate, explain and justify their thinking. Not all student actions, however, are "critical moments" (Walshaw & Anthony, 2008, p. 527) that create "potentially powerful learning opportunities" (Davis, 1997, p. 360). In the interest of differentiating student actions that meet this higher threshold, we define *pedagogical openings* as observable student actions that provide compelling opportunities to work toward an instructional goal. Determining whether an opening has been presented requires considering both the *positioning* and the *timing* of an observable student action. Building on the notion from the discourse analysis literature (e.g., Davies & Harré, 1990), we define *positioning* as the way in which an observable student action positions that student with respect to the content of an instructional goal. Students are positioned well with respect to an instructional goal if their action has a "deep" connection with the content of that goal as opposed to remaining "at a surface level." Whereas good positioning is determined by a particular student's engagement with the content of an instructional goal, good *timing* is determined with respect to the preparation of other students in the class to engage with the idea being raised in ways that support, rather than supplant, overall instructional goals.

Putting the Theory into Action

When determining whether a MOST has occurred, the focus of our analysis is an "instance"—an observable student action or small collection of connected actions (such as a verbal expression combined with a gesture). Typically an instance is one conversational turn or physical expression (such as writing a solution on the board), but it can involve multiple turns.

Determining whether an instance qualifies as a MOST involves a systematic analysis of whether the instance embodies the three MOST characteristics (see Figure 1); if any criterion is not met, the analysis ends and the instance is not a MOST. The analysis begins with questioning whether the students' mathematics can be articulated. Focusing first on this characteristic stems from the perspective that what students say or do during a lesson is critical and should inform the teacher's actions. If the students' mathematics can be articulated, it is formalized into a statement of the MI. The MI is then analyzed to determine whether the instance is mathematically significant; that is, whether it satisfies the appropriate and central mathematics criteria.

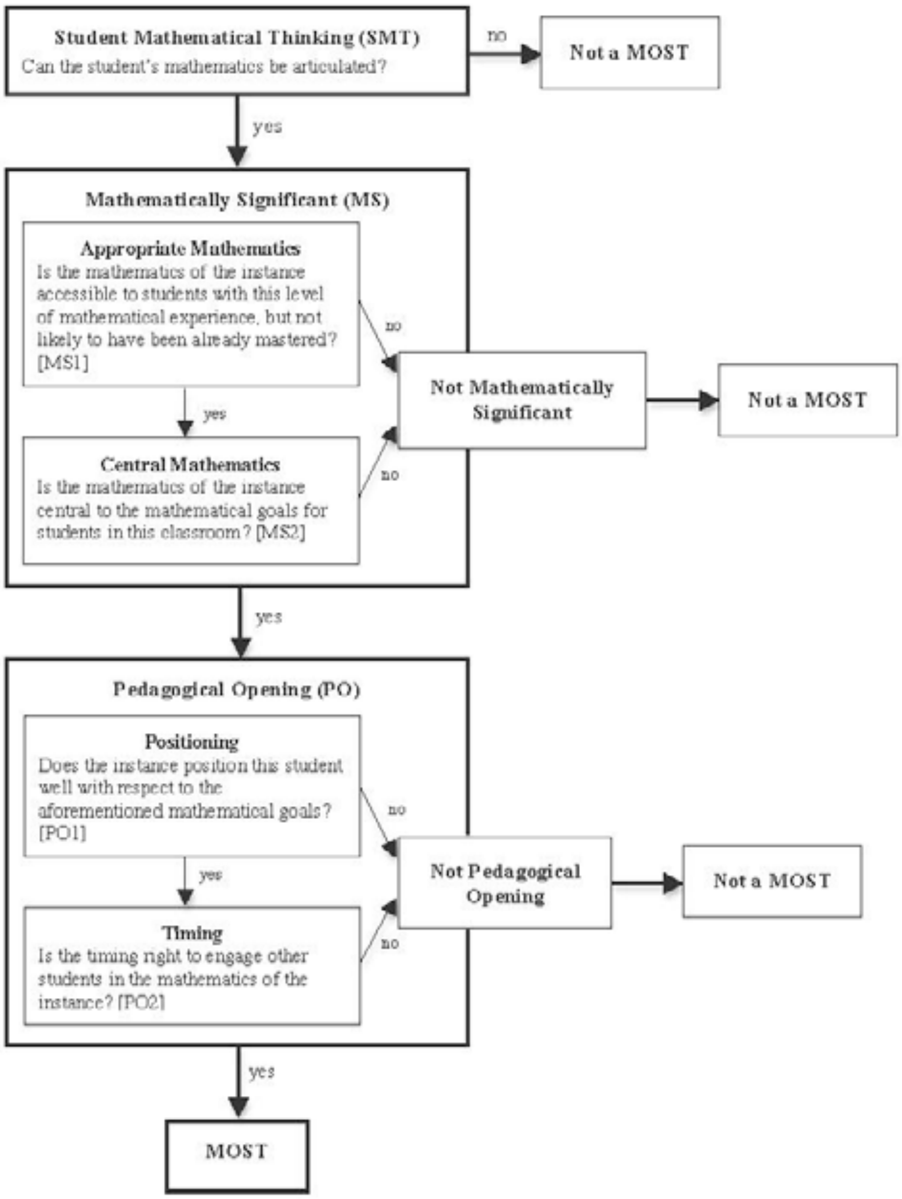

Figure 1: Analysis Process for Determining Whether a Classroom Instance is a MOST.

This mathematical analysis of the instance distinguishes our work from more general work on classroom discourse or even “teachable moments” in that we focus on instances that are likely to advance students’ development of mathematical ideas. If an instance is mathematically significant, it is analyzed in terms of whether the positioning and timing are right to create a pedagogical opening. If so, the instance has met the criteria for all three characteristics and is deemed to be a MOST. We have found that taking this flowchart approach to the analysis of an instance brings structure and simplicity to an often chaotic and complex task.

Conclusion

By clearly defining three critical characteristics that distinguish instances that provide high-leverage opportunities to advance students’ mathematical understanding from those that do not, the MOST construct has the potential to become a tool to make sense of classroom interactions. In particular, the construct provides both a means for systematically analyzing instances of classroom discourse and a vocabulary for discussing the mathematical and pedagogical importance of student thinking that arises within such discourse. Engaging in this analysis provides a mechanism for researchers and teacher educators to frame teachers’ practice in terms of their use of high-leverage instances of student mathematical thinking. This framing shifts the focus of the work from *whether* a teacher is using student thinking, to *what* student thinking a teacher is incorporating into a lesson and *why* that incorporation is valuable.

MDISC Project Overview

The Mathematics Discourse in Secondary Classrooms (MDISC) project seeks to provide tools to secondary mathematics teachers to enhance the quality and power of their classroom mathematics discourse through the design of professional development materials. The set of tools at the core of the materials are the *Teacher Discourse Moves* (TDMs), revised from Chapin, O’Connor, and Anderson’s (2009) construct of talk moves, extensively researched and disseminated at the elementary level (see Herbel-Eisenmann, Steele & Cirillo, 2013, for additional detail on the TDMs). A suite of concepts from sociolinguistics and discourse studies situate the TDMs in ways that help teachers consider how their principled use can influence students’ conceptions of the purpose of classroom mathematics discourse, their communication development, and their identities as mathematical learners and doers. These ideas illuminate aspects of discourse related to both productive and powerful discourse, where *productive* relates to mathematical goals and *powerful* relates to social goals.

Teacher Discourse Moves (TDMs)

A suite of six TDMs (see Table 1) are introduced as a part of the professional development materials, each serving a variety of roles in shaping classroom discourse related to mathematical and social goals.

These moves, used individually or in combination, can advance a teacher’s mathematical goals (productive discourse), social goals (powerful discourse), or both. The frames of productive and powerful discourse help teachers analyze the ways in which their use of the TDMs influences students’ opportunities to learn mathematics and their developing mathematical identities.

Table 1: The Teacher Discourse Moves

TDMs and Intended Purposes	Examples
Waiting can... ● Provide students with time to process a question or response ● Hold students accountable for thinking and doing mathematics ● Encourage broader participation	● [pause without saying anything] ● I want you to think individually, without saying anything yet. ● Think about this for a few seconds and write down any questions you have.
Inviting Student Participation can... ● Initiate a discussion ● Elicit multiple student perspectives ● Position a student as someone whose ideas are worth considering	● Who is ready to share their thinking? ● What do other people think? ● Does anyone have a question for <student>?
Revoicing can... ● Amplify or draw attention to an idea ● Provide more mathematically precise or correct language ● Mark the value of individual students' thoughts for exploration	● The way I'm interpreting what you are saying is.... . Is that what you meant? ● So, I heard you say two things: _____ and _____. ● For those of you who may not have heard, <student> was just saying... .
Asking Students to Revoice can... ● Amplify student ideas ● Encourage students to use more or less mathematical language or precision ● Position a student as someone whose ideas are worth considering	● Can someone else say that in his or her own words? ● What did you hear <student> just say? ● Can everyone look at what <student> just wrote on the board? I want someone else to explain her strategy.
Probing a Student's Thinking can... ● Allow the learner to transform, modify, or correct their contribution/thinking ● Assist students in further articulating ideas by elaborating or justifying ● Make a student's thinking available to other students for follow-up	● Why does that work? ● Can you say a little bit more about your thinking? I am not sure that we are all clear on what you are trying to say. ● Can you come up to the board and show us what you mean?
Creating Opportunities to Engage with Another's Reasoning can... ● Allow students to take someone else's approach and use it for yourself ● Allow students to disagree/agree with someone else's idea, approach, or explanation	● Do you agree or disagree? Why? ● Can someone continue with <student's> train of thought? ● In what situations do you think that <student's> method would be more efficient than the one we discussed yesterday?

Productive Discourse

Productive discourse supports students' opportunities to engage with mathematical content and mathematical ways of understanding. The Language Spectrum, which contains four Communication Contexts, helps teachers analyze their use of the TDMs in promoting productive discourse. Communication Contexts describe the settings in which classroom mathematics discourse is produced (e.g., small group, whole class, written solutions, textbook excerpts). The Language Spectrum, developed from Gibbons' (2009) mode continuum, considers the relationship between communication context and expected characteristics of a text that would be produced in that context. For example, in small groups, a typical text features context-dependent language (e.g., here and here, this, it) and gesture. Students might use less formal language, such as, "on the top," or "under here." Their common small-group experiences facilitates shared understanding in this context. In contrast, textbook excerpts draw heavily on the mathematics register (Halliday, 1978; Pimm, 1987) the style of communication valued by the mathematics community. Human actors are rare; instead, mathematical objects or processes serve as the subject of sentences which are constructed in passive voice using timeless present tense. The textbook necessitates communication to an audience who is not present, which leads to language that is not reliant on immediate context. The examples given here are two of the set of four Communication Contexts described in the Language Spectrum. Increasing awareness of the relationship between communication context and the expected kind of text that is produced is important as teachers are more likely to evaluate student work as correct if it includes characteristics of the mathematics register (Morgan, 1998). It is important to note that, in this range of texts, one way of communicating is *not* better than another. Instead, the Language Spectrum illustrates how communication context affects the kind of language that students use, and, by extension, illuminates how important it is to provide communication contexts in which students can use mathematically complex language.

The Language Spectrum and Communication Contexts are important aspects of communication to make explicit because teachers and other students also have been shown to treat students differently depending on whether or not they consistently use features of the Mathematics Register correctly (e.g., Esmonde, 2009). So, it is important for participants to be aware of the particularities of the Mathematics Register and to openly support students' use of it.

Powerful Discourse

Powerful discourse attends to how students are positioned both socially and as knowers and doers of mathematics. Messages about how students are perceived by others, themselves, and what they come to understand about what it means to know and do mathematics are embedded implicitly and explicitly in the discourse. The idea teachers use to analyze how use of the TDMs influences powerful discourse is *positioning*, "the ways in which people use action and speech to arrange social structures" (Harré & van Langenhove, 1999). That is, students, teachers, and the content of mathematics are being positioned through the interactions in the classroom all of the time.

We introduce the concept of positioning in three different ways. Teachers are asked to attend to student-to-student interactions to consider positioning. For example, when other students say, "let's try that," or "that will never work," they are positioning one another or indicating one another's status (Featherstone, Crespo, Jilk, Oslund, Parks, & Wood, 2011). Teachers are asked to consider teacher-student interactions because these also position people in classrooms in various ways. Teachers are both *in* authority in their classrooms and *an* authority in their classrooms. Teachers are also asked to consider how the practice of mathematics is positioned through a consideration of the kinds of tasks, activities, process, and practices students engage in during mathematics class. For example, students might come to see mathematics as an individual endeavor or a collaborative one. Questions, such as, "Who is considered 'smart' in my classroom?," "Who is talking (the teacher, which students specifically)?," and "What kind of mathematical practices (e.g., argumentation, explanation, just answers) do we engage in?" provide teachers with an opportunity to surface ideas related to positioning about their own classroom practices.

Similar to the roles of the Language Spectrum and the Communications Contexts for productive discourse, teachers can consider positioning as they analyze the ways in which their use of the TDMs might support students' identity development (both as people and as mathematical knowers and doers). These three primary concepts, along with the TDMs, comprise a set of analytical tools teachers can use to better understand, plan for, and implement rich classroom mathematics discourse practices.

Plan for Working Group Sessions

At the beginning of the first session Dan Heck of Horizon Research will briefly introduce the purpose of the overall working group. As a lead external evaluator on each of the NSF projects from which these tools are taken, Dan is positioned well to facilitate discussion about the potential advantages to analyzing the tools in tandem.

The first two sessions of the working group will then be organized around the tools the two projects have developed--the MOST project in the first session (Keith, Blake, Shari, Laura, Isaí and Lindsay) and the MDISC project in the second (Kate, Mike, and Beth). Each of these sessions will consist of a) a description of the framework for analyzing classroom mathematics discussion, b) a coding activity in which all participants use the framework to analyze an excerpt of classroom mathematics discourse, and c) a group discussion of participants' answers to the following questions:

1. What aspects of classroom mathematics discourse does this tool foreground and background?
2. What are the affordances and constraints of foregrounding and backgrounding these aspects of classroom mathematics discourse?
3. How and under what conditions could this tool be used by mathematics educators in their work, both as researchers and as mathematics teacher educators?

During the third session Dan will present summaries of the previous sessions' discussions. Participants will then engage in re-analyzing the excerpts from the previous two sessions with each tool. The aforementioned summaries and the coding experience will then be fodder for discussion around the following questions:

1. In what substantive ways are these two tools similar and different?
2. What are potential advantages or disadvantages to attempting to use these research tools in tandem?

Anticipated Follow-up Activities

We anticipate two types of follow-up activities as a result of the working group - additional activities on the part of each of the two projects, and further engagement of other researchers and research groups interested in analyzing classroom mathematics discourse. Through engagement in the working group discussions, each of the two project groups will refine their suite of tools based on the discussions across the three days. In addition, the two projects will identify intersection points in the two sets of tools and frameworks and continue dialogue about ways in which the projects can work together to better understand classroom mathematics discourse.

In addition, the discussions are likely to spark interest from other researchers in making use of and connections to the projects' tools and frameworks. The work of analyzing classroom mathematics discourse has been an increasing focus of research and practice in the last two decades. The discussions in the working group will contribute to this research agenda by engaging participants in the consideration of two sets of tools that can be mobilized in a wide range of research and teacher education settings.

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References

Chapin, S. H., O'Connor, M. C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn* (2nd ed.). Sausalito, CA: Math Solutions Publications.

Davies, B., & Harré, R. (1990). Positioning: The discursive production of selves. *Journal for the Theory of Social Behavior, 20*(1), 43-63.

Davies, N., & Walker, K. (2005). Learning to notice: One aspect of teachers' content knowledge in the numeracy classrooms. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce & A. Roche (Eds.), *Building connections: Theory, research and practice (Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia)* (pp. 273-280). Sydney, Australia: MERGA.

Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching. *Journal for Research in Mathematics Education, 28*, 355-376.

Esmonde, I. (2009). Explanations in mathematics classrooms: A discourse analysis. *Canadian Journal of Science, Mathematics, and Technology Education, 9,* 86-99.

Featherstone, H., Crespo, S., Jilk, L., Oslund, J., Parks, A., & Wood, M. (2011). *Smarter together! Collaboration and equity in the elementary math classroom*. Reston, VA: NCTM.

Gibbons, P. (2009). *English learners, academic literacy, and thinking*. Portsmouth, NH: Heinemann.

Halliday, M. (1978). *Language as social semiotic: The social interpretation of language and meaning*. Baltimore, MD: University Press.

Harré, R., & van Langenhove, L. (1999). *Positioning theory: Moral contexts of intentional action*. Oxford, UK: Blackwell.

Herbel-Eisenmann, B. A., Staples, M., Choppin, J., & Wagner, D. (2007). Working group on mathematics classroom discourse. In Lamberg, T. & Wiest, L. R. (Eds.), *Proceedings of the 29th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Lake Tahoe, NV: University of Nevada, Reno.

Herbel-Eisenmann, B. A., Steele, M. D., & Cirillo, M. (2013). (Developing) teacher discourse moves: A framework for professional development. *Mathematics Teacher Educator*, *1*(2), 181-196.

Jaworski, B. (1994). *Investigating mathematics teaching: A Constructivist Enquiry*. London: Falmer Press.

Kim, O.-K., Remillard, J., Steele, M. D., Blunk, M., Piecham, M. B., & Lewis, J. M. (2012). Measuring instruction in relation to curriculum use. In Van Zoest, L. R., Lo, J.-J., & Kratky, J. L. (Eds.). *Proceedings of the 34th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1237-1244). Kalamazoo, MI: Western Michigan University.

Leatham, K. R., Stockero, S. L., Van Zoest, L. R., & Peterson, B. E. (2010). Investigating mathematically important pedagogical opportunities. In Brosnan, P., Erchick, D. B., & Flevares, L. (Eds.)*, Proceedings of the 32nd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1599-1605). Columbus, OH: The Ohio State University.

Morgan, C. (1998). *Writing mathematically: The discourse of investigation*. Bristol, PA: Falmer.

Pimm, D. (1987). *Speaking mathematically*. New York, NY: Routledge and Kegan Paul.

Schoenfeld, A. H. (2008). On modeling teachers' in-the-moment decision making. In A. H. Schoenfeld (Ed.), *A study of teaching: Multiple lenses, multiple views, JRME monograph #14* (pp. 45-96). Reston, VA: National Council of Teachers of Mathematics.

Walshaw, M., & Anthony, G. (2008). The teacher's role in classroom discourse: A review of recent research into mathematics classrooms. *Review of Educational Research*, *78*, 516–551.