Viewing Classroom Mathematics Discourse Through Two Complementary Lenses

Building on Mathematical Opportunities in Student Thinking

Shari L. Stockero
Blake E. Peterson
Keith R. Leatham
Laura R. Van Zoest

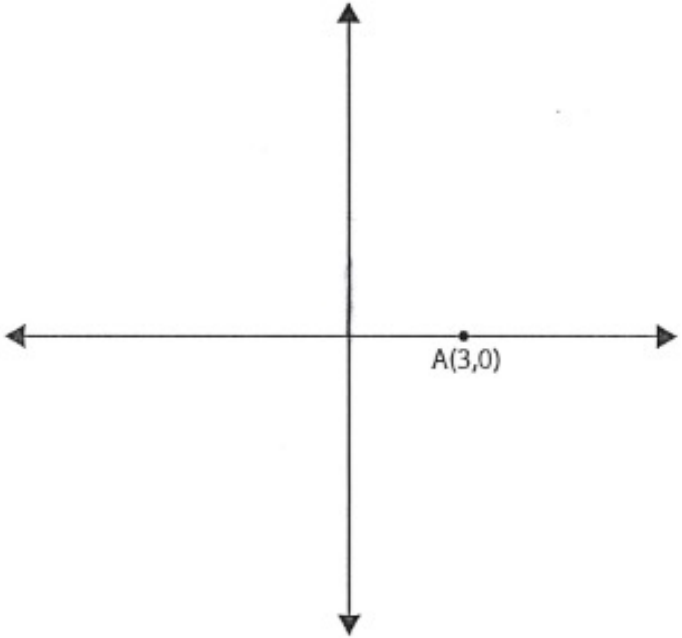
Learning to Support Productive Collective Argumentation in Secondary Mathematics Classrooms

AnnaMarie Conner
Jonathan Foster
Laura Singletary
Hyejin Park
Yuling Zhuang

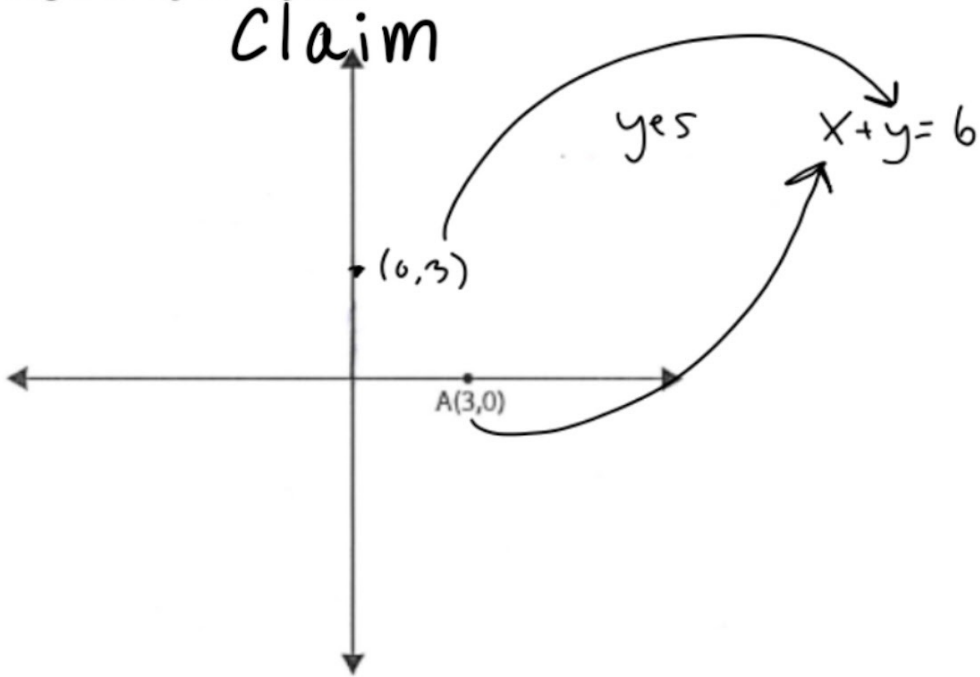
AMTE 2023

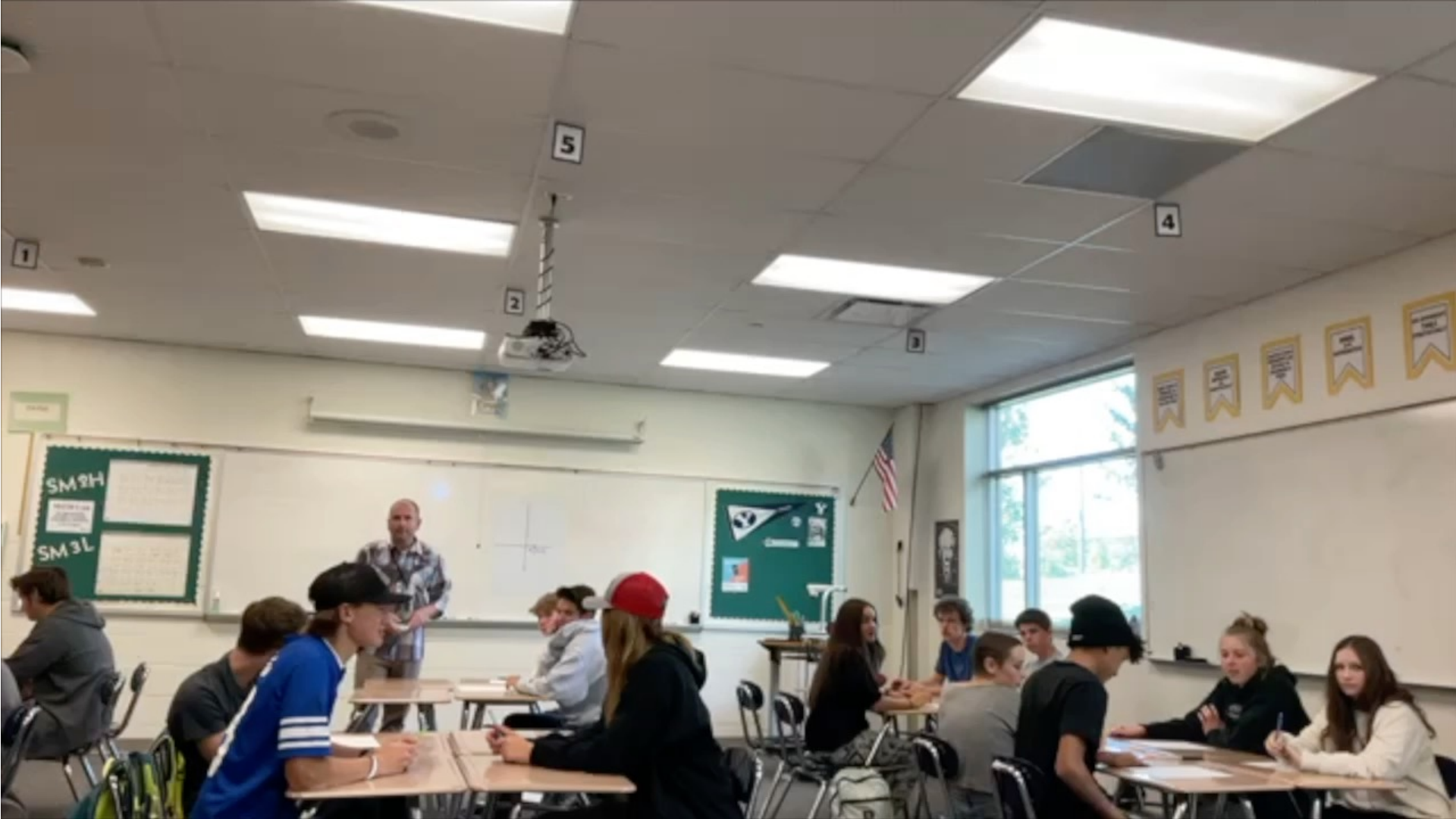
Setting up the clip

Is it possible to select a point B on the y-axis so that the line $x + y = 6$ goes through both points A and B? Explain why or why not.



Is it possible to select a point B on the y-axis so that the line $x + y = 6$ goes through both points A and B? Explain why or why not.




5
1
2
3
4
SM 9H
SM 3L

Building on Mathematical Opportunities in Student Thinking

Shari L. Stockero

Blake E. Peterson

Keith R. Leatham

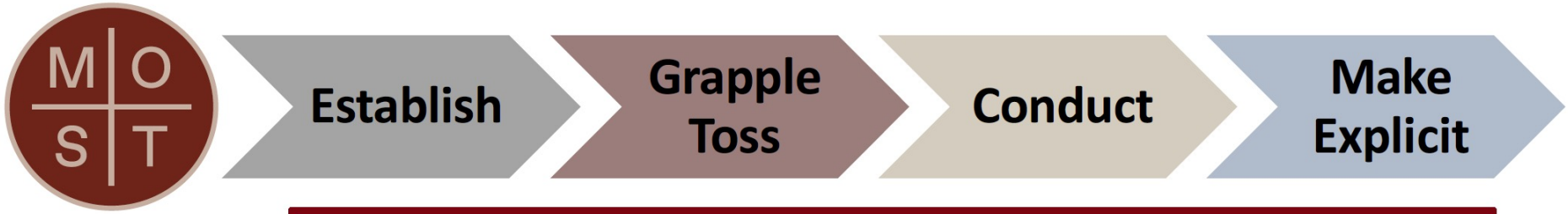
Laura R. Van Zoest

Michigan Technological University

Building on MOSTs: Investigating Productive Use of High-Leverage Student Mathematical Thinking

This research report is based on work supported by the U.S. National Science Foundation (NSF) under Grant Nos. DRL-1720410, DRL-1720566, and DRL-1720613. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the NSF.

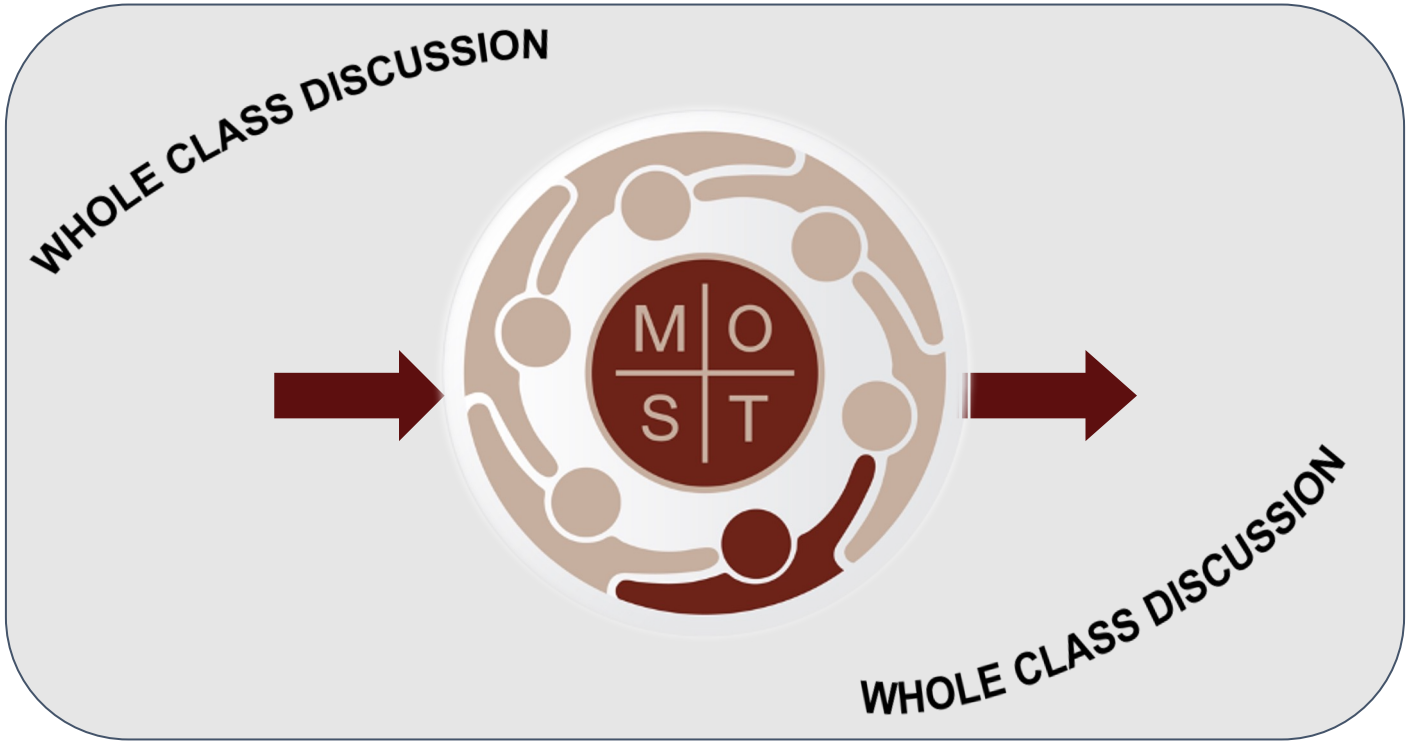
A MOST is a **M**athematical **O**pportunity in **S**tudent **T**hinking



Building on a MOST

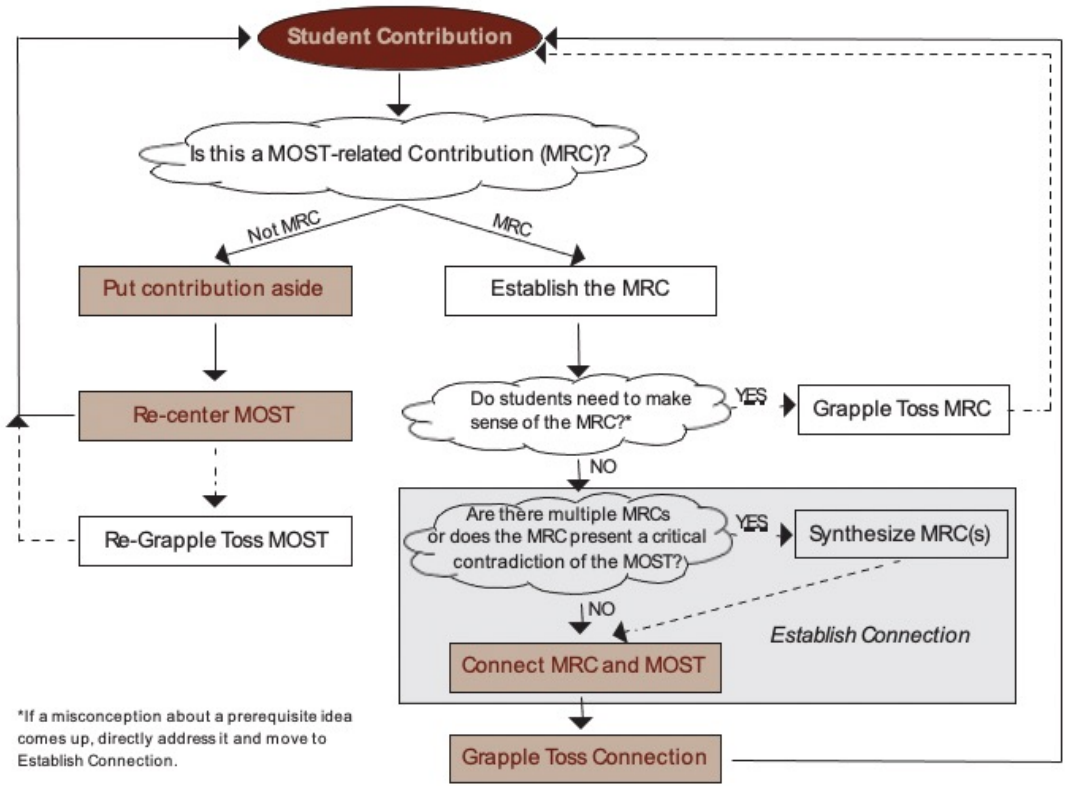
Building on a MOST is engaging the class in making sense of the MOST to better understand the mathematics of the MOST.

CONVERSATIONAL BUBBLE

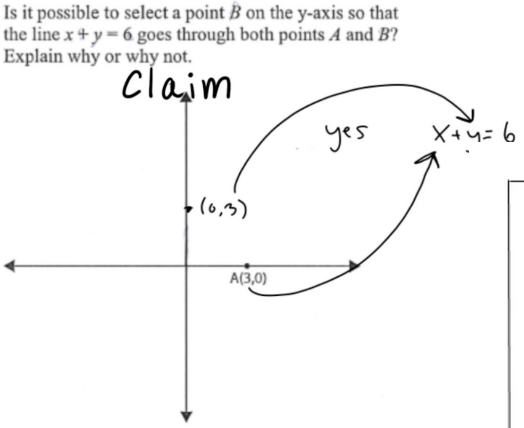


BUILDING

CONDUCT



*If a misconception about a prerequisite idea comes up, directly address it and move to Establish Connection.

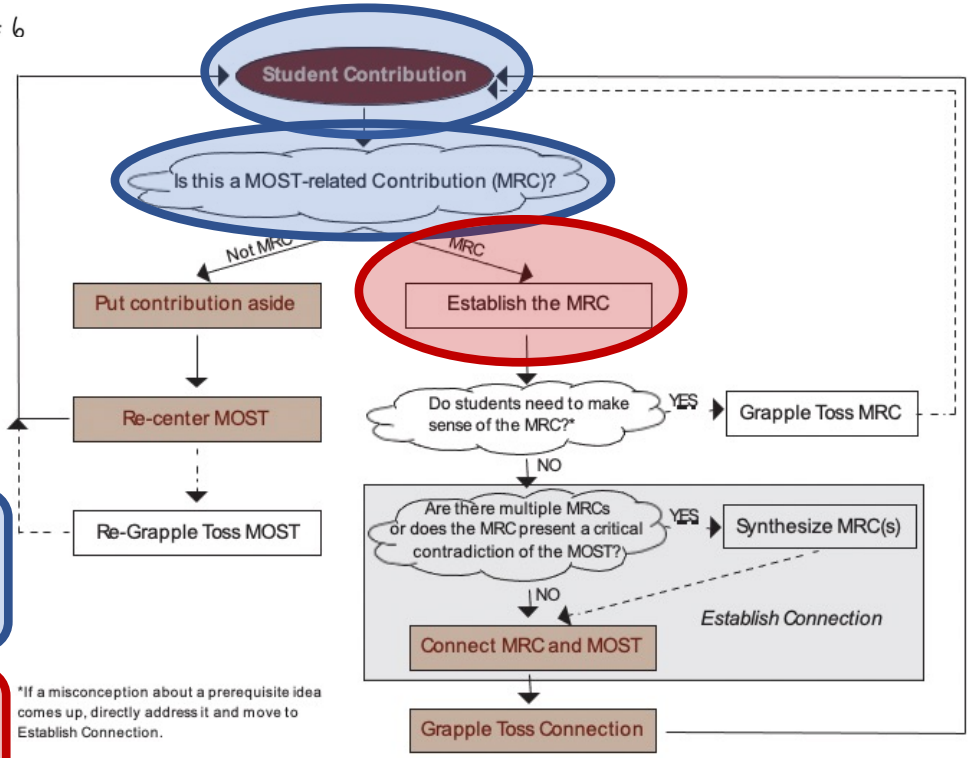


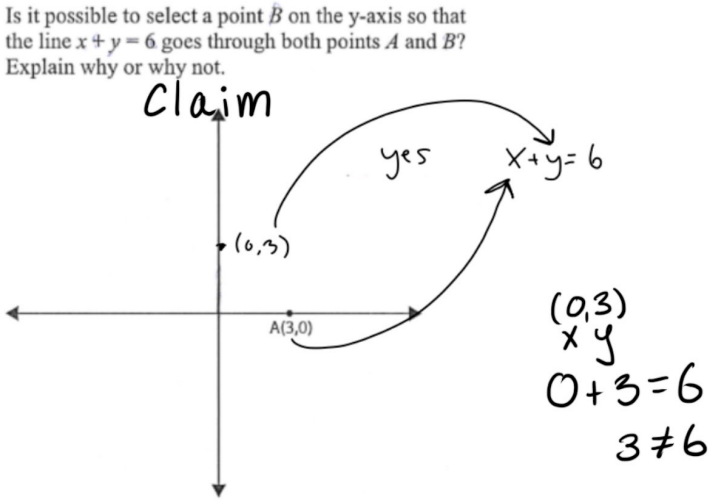
7:40 Mr. Kennedy: So the question is now, how do you think-does that-does that claim hold up mathematically? {long pause} Does anyone have a response to that, that they can share aloud? Andre, what do you think?

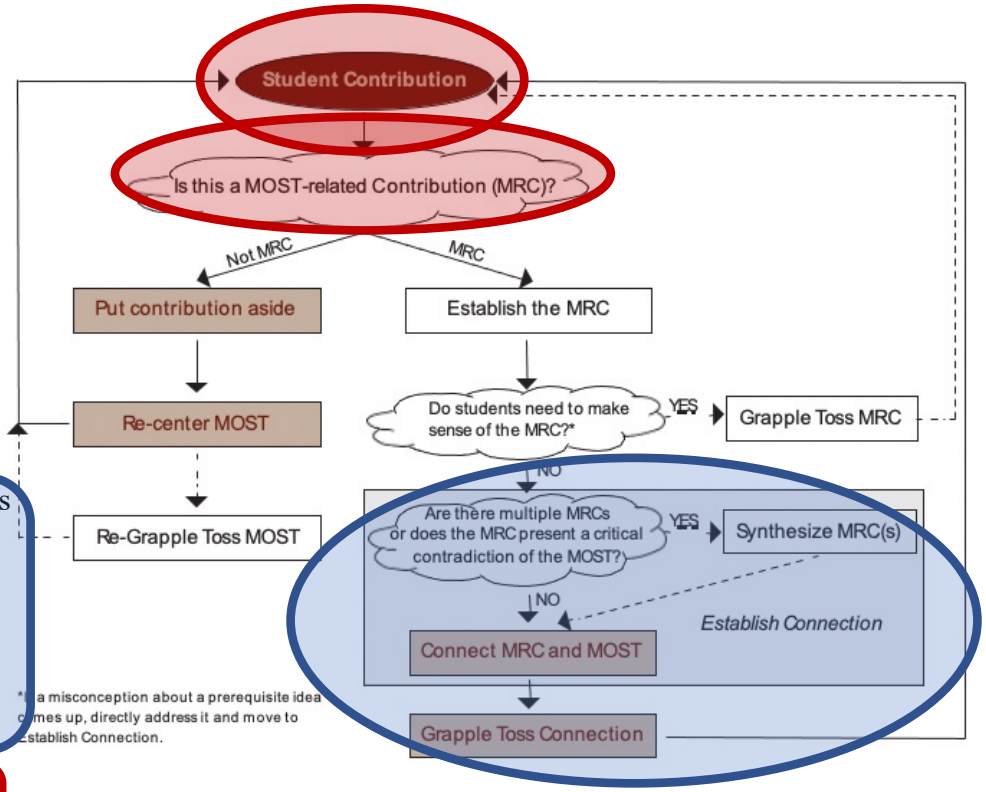
8:04 Andre: The thing is that the point is (0,3) and if you plug it into that equation it comes out with 3 equals 6 which is false. Basically <Mr. Kennedy: Okay> 'kay so the point (0,3) wouldn't work.

8:16 Mr. Kennedy: Okay so um so you're referring to this point B [points to (0,3) on the board.] < Andre: Yeah.> Right? And-and you're saying the point (0,3) wouldn't work. Can you tell us more about like why that wouldn't work?

8:29 Andre: Can I use the marker and do it?





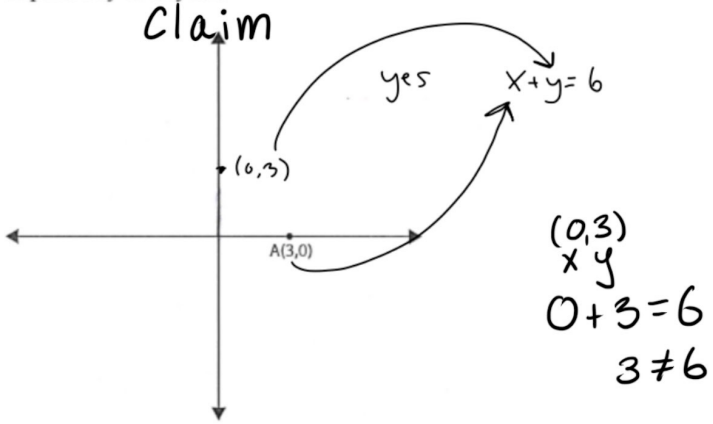


8:50 Mr. Kennedy: So Andre <Student: wow> Andre's saying that this reasoning [points to original claim] um, does-does not hold up mathematically because of this reasoning here. [points to "0+3=6 3/=6"] So here we have 3 plus 3 equals 6 [points as he speaks to original claim] here we have 0 plus 3 does not equal 6. [points to Andre's work] So what do you think about- what do you think about that? Olivia.

9:19 Olivia: Um sorry I just had something about the original one, just something to add to it.

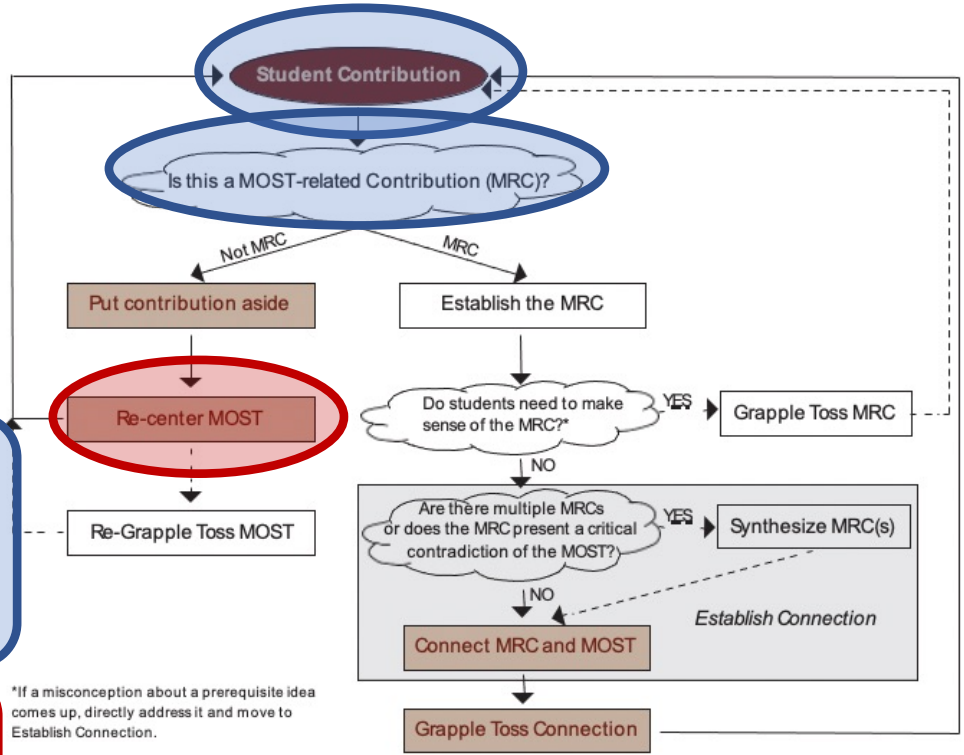
9:22 Mr. Kennedy: Okay go for it.

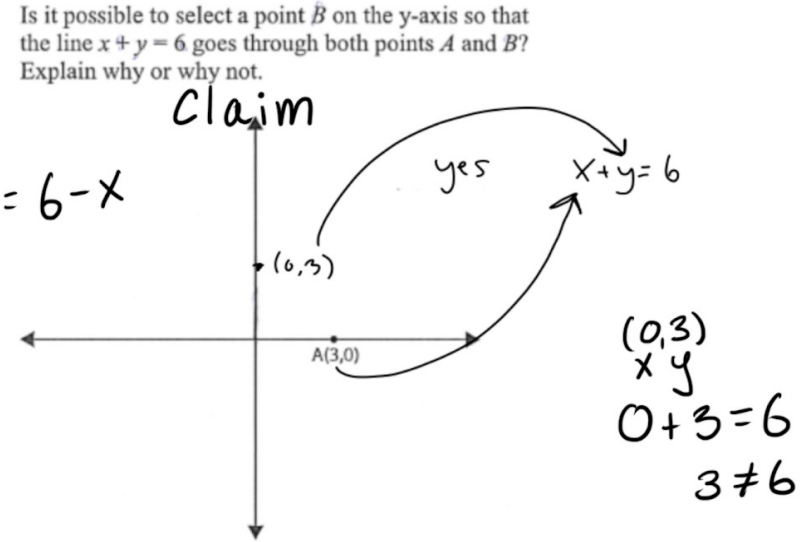
Is it possible to select a point B on the y-axis so that the line $x + y = 6$ goes through both points A and B? Explain why or why not.



9:23 Olivia: Um well I think it's a lot easier for us to comprehend like stuff in slope-intercept form because that's what we've been learning about for a while. So, if you just convert, like x plus y equals 6 into slope-intercept form then its y equals negative x plus 6 and then that doesn't work with the two points cause the slope wouldn't go through those.

9:48 Mr. Kennedy: Kay so you're saying if we took this equation and switched it to [writes equation in slope-intercept form] converted it to slope-intercept form < Olivia: Mhm> it would be y equals negative x plus 6. < Olivia: Yep.> and then tell me more about um about this original claim.[circle motion to board around point B on graph]

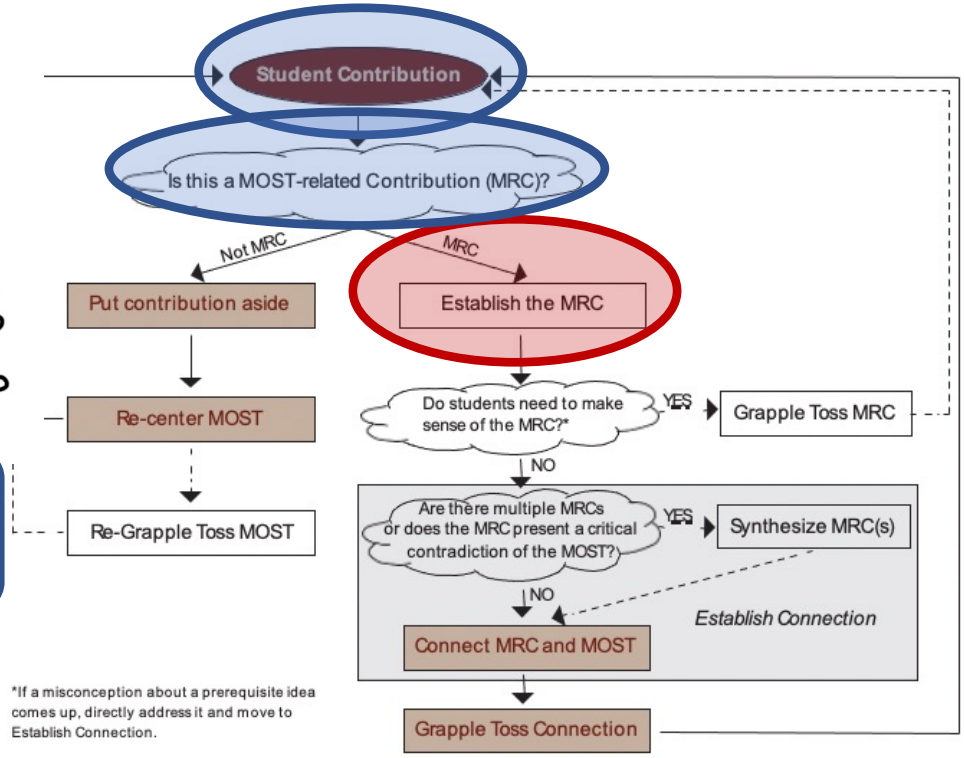




10:03 Olivia: Well you can't like pick and choose which x and y values you take to put into your equation. You can only take like the one point if you're doing it the way they are.

10:15 Mr. Kennedy: So uh are you uh, refuting this claim [points to graph on board] or agreeing with the claim?

10:19 Olivia: I'm refuting it cause you can't just add 3 plus 3 cause they're not on the same point of the graph, they're two separate points.



Thank you!

BuildingOnMosts.org

Learning to Support Productive Collective Argumentation in Secondary Mathematics Classrooms

AnnaMarie Conner
Jonathan Foster
Laura Singletary
Hyejin Park
Yuling Zhuang

Collective Argumentation

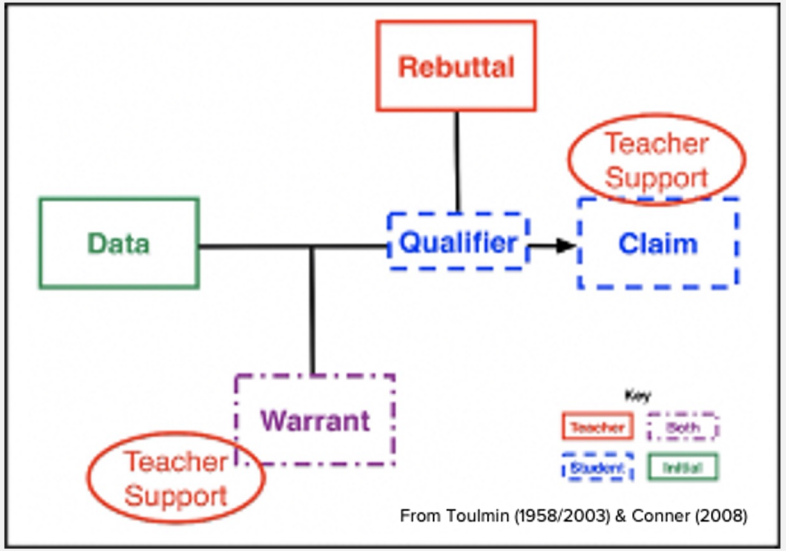
Collective argumentation: a process by which a group of people establishes the veracity of a claim

- Argumentation and proof are critical in mathematics (foregrounds reasoning and sense making in mathematics)
- Literature contains multiple examples of teachers facilitating collective argumentation (e.g., Krummheuer, 1995)

Collective Argumentation in Classrooms

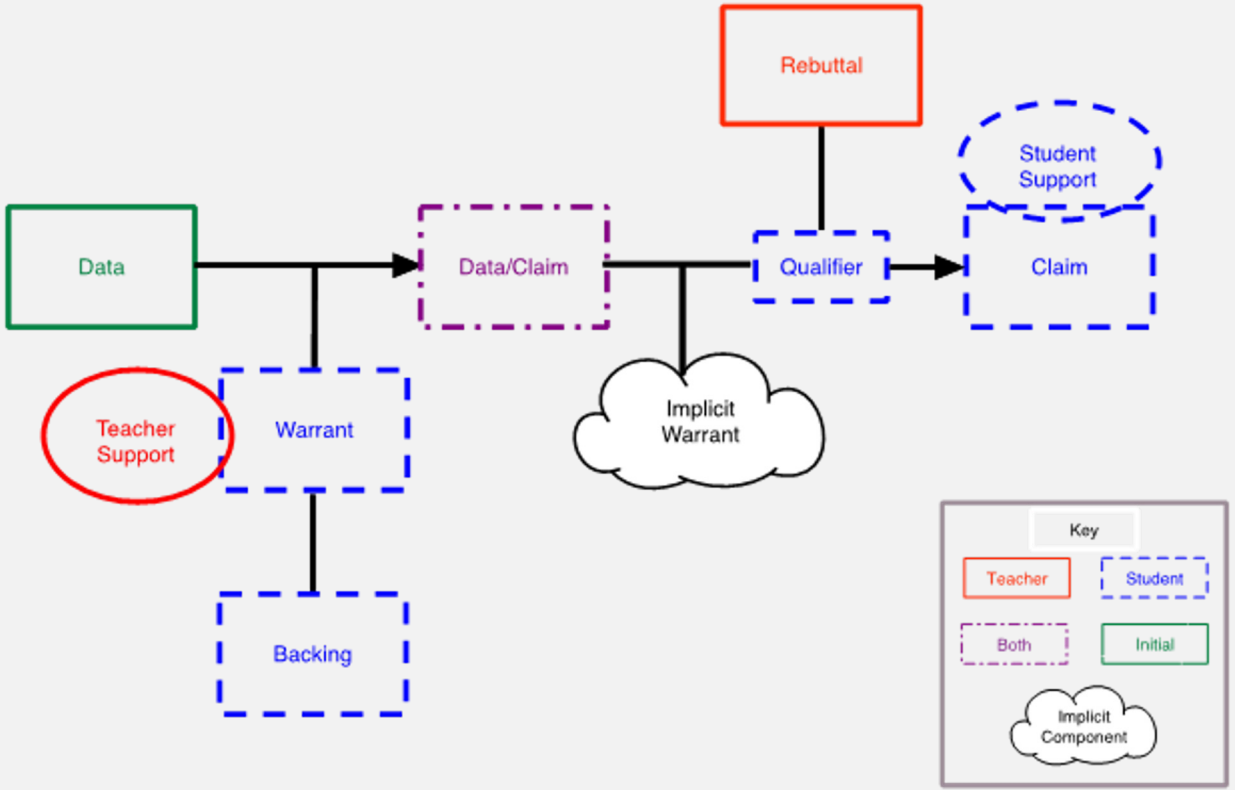
Arguments consist of...

- *Claims*, statements whose validity is being established
- *Data*, support provided for the claim
- *Warrants*, statements that connect data with claim
- *Qualifiers*, statements that show the strength of the warrant
- *Rebuttals*, statements that provide circumstances under which a warrant would not hold



Adaptation of Toulmin's diagrams

Extended Toulmin Diagrams



Teacher Support for Collective Argumentation (TSCA) Framework

Table 1 Teacher support for collective argumentation framework

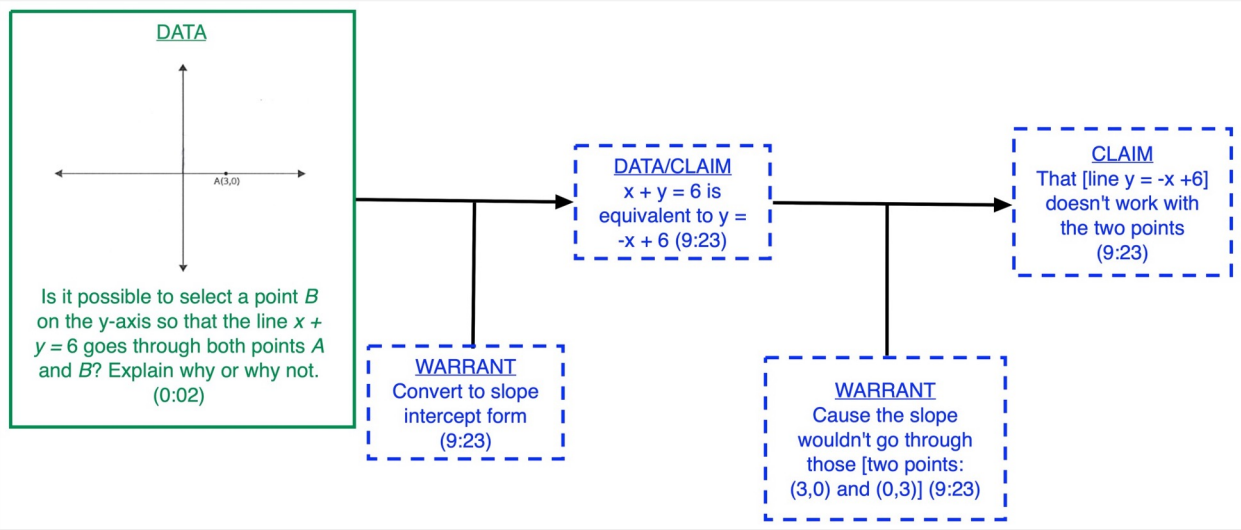
Direct contributions		Questions		Other supportive actions	
Claims	Statements whose validity is being established	*Requesting a factual answer*	Asks students to provide a mathematical fact	*Directing*	Actions that serve to direct the students' attention and/or the argument
Data	Statements provided as support for the claims	*Requesting a method*	Asks students to demonstrate or describe how they did or would do something	*Promoting*	Actions that serve to promote mathematical exploration
Warrants	Statements that connect data with claims	*Requesting an idea*	Asks students to compare, coordinate, or generate mathematical ideas	*Evaluating*	Actions that center on the correctness of the mathematics
Rebuttals	Statements describing circumstances under which the warrants would not be valid	*Requesting elaboration*	Asks students to elaborate on some idea, statement, or diagram	*Informing*	Actions that provide information for the argument
Qualifiers	Statements describing the certainty with which a claim is made	*Requesting evaluation*	Asks students to evaluate a mathematical idea	*Repeating*	Actions that repeat what has been or is being stated
Backings	Usually unstated, dealing with the field in which the argument occurs				

Example Analysis of Sub-Argument

9:10 Teacher: So what do you think about- what do you think about that? Olivia.

9:23 Olivia: Um well I think it's a lot easier for us to comprehend like stuff in slope-intercept form because that's what we've been learning about for a while. So, if you just convert, like x plus y equals 6 into slope-intercept form then its y equals negative x plus 6 and then that doesn't work with the two points cause the slope wouldn't go through those.

9:48 Teacher: Okay, so you're saying if we took this equation and switched it to [*writes equation in slope-intercept form*] converted it to slope-intercept form < Olivia : Mhm> it would be y equals negative x plus 6. < Olivia : Yep.> and then tell me more about um about this original claim [*circle motion to board around point B on graph*].

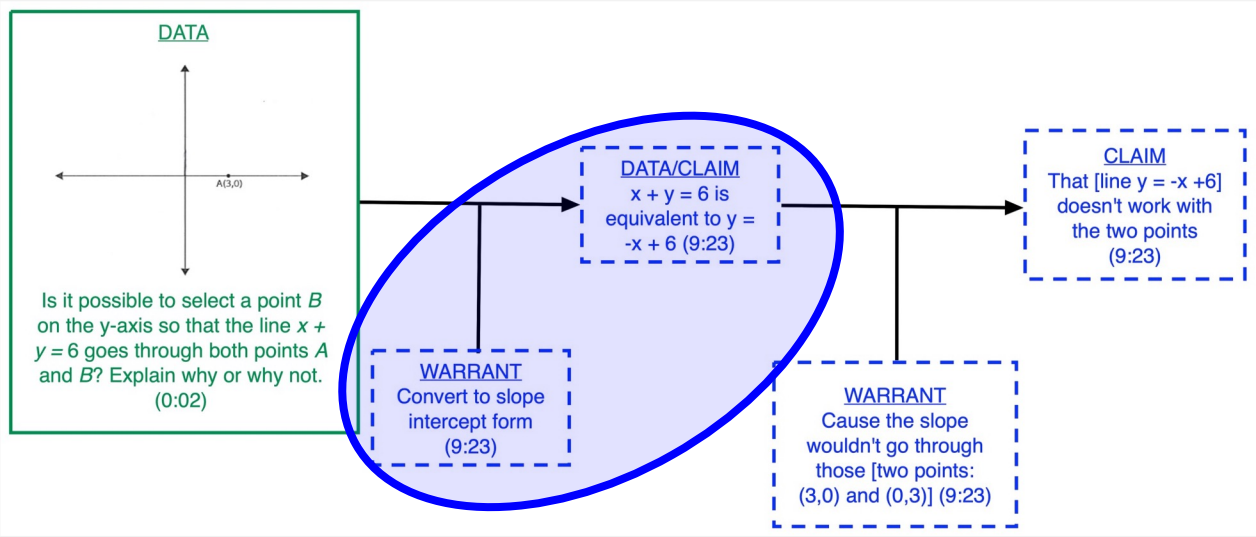


Example Analysis of Sub-Argument

8:50 Teacher: So what do you think about- what do you think about that? Olivia.

9:23 Olivia: Um well I think it's a lot easier for us to comprehend like stuff in slope-intercept form because that's what we've been learning about for a while. So, if you just convert, like **x plus y equals 6 into slope-intercept form then its y equals negative x plus 6** and then **that doesn't work with the two points cause the slope wouldn't go through those**.

9:48 Teacher: Okay, so you're saying if we took this equation and switched it to [*writes equation in slope-intercept form*] converted it to slope-intercept form < Olivia : Mhm> it would be y equals negative x plus 6. < Olivia : Yep.> and then tell me more about um about this original claim [*circle motion to board around point B on graph*].

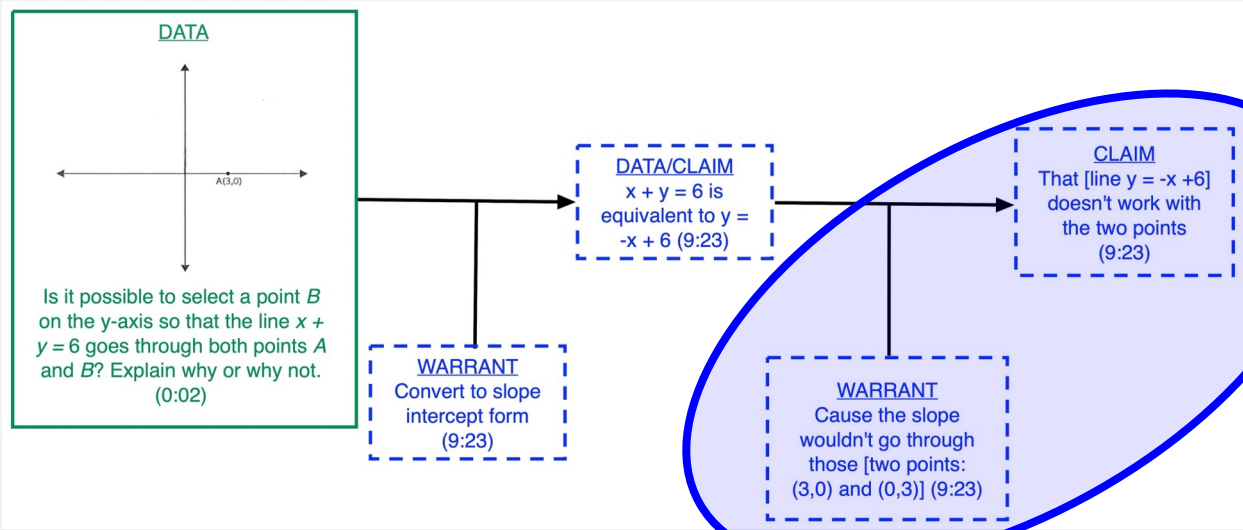


Example Analysis of Sub-Argument

8:50 Teacher: So what do you think about- what do you think about that? Olivia.

9:23 Olivia: Um well I think it's a lot easier for us to comprehend like stuff in slope-intercept form because that's what we've been learning about for a while. So, if you just convert, like x plus y equals 6 into slope-intercept form then its y equals negative x plus 6 and then **that doesn't work with the two points cause the slope wouldn't go through those**.

9:48 Teacher: Okay, so you're saying if we took this equation and switched it to [*writes equation in slope-intercept form*] converted it to slope-intercept form < Olivia : Mhm> it would be y equals negative x plus 6. < Olivia : Yep.> and then tell me more about um about this original claim [*circle motion to board around point B on graph*].

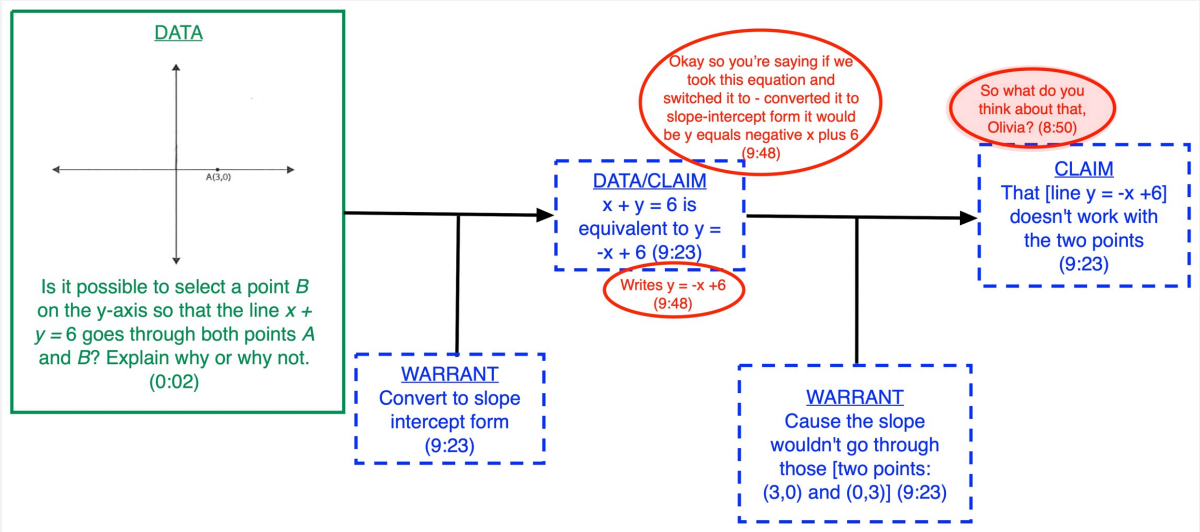


Example Analysis of Sub-Argument

8:50 Teacher: So what do you think about- **what do you think about that?** Olivia.

9:23 Olivia: Um well I think it's a lot easier for us to comprehend like stuff in slope-intercept form because that's what we've been learning about for a while. So, if you just convert, like **x plus y equals 6 into slope-intercept form then its y equals negative x plus 6** and then **that doesn't work with the two points cause the slope wouldn't go through those**.

9:48 Teacher: Okay, so you're saying if we took this equation and switched it to [*writes equation in slope-intercept form*] converted it to slope-intercept form < Olivia: Mhm> it would be y equals negative x plus 6. < Olivia: Yep.> and then tell me more about um about this original claim [*circle motion to board around point B on graph*].

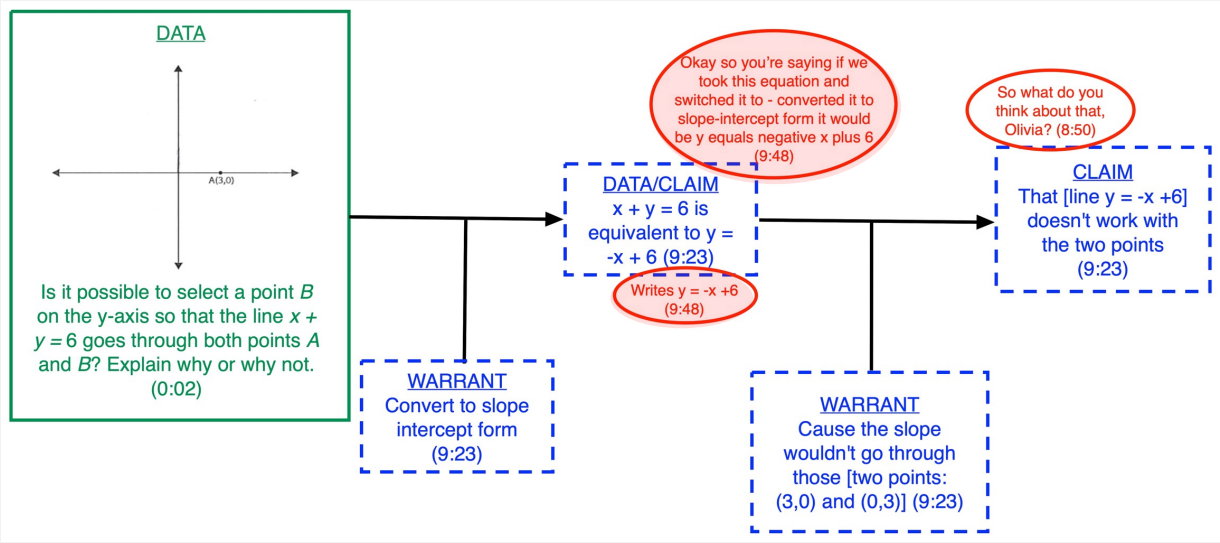


Example Analysis of Sub-Argument

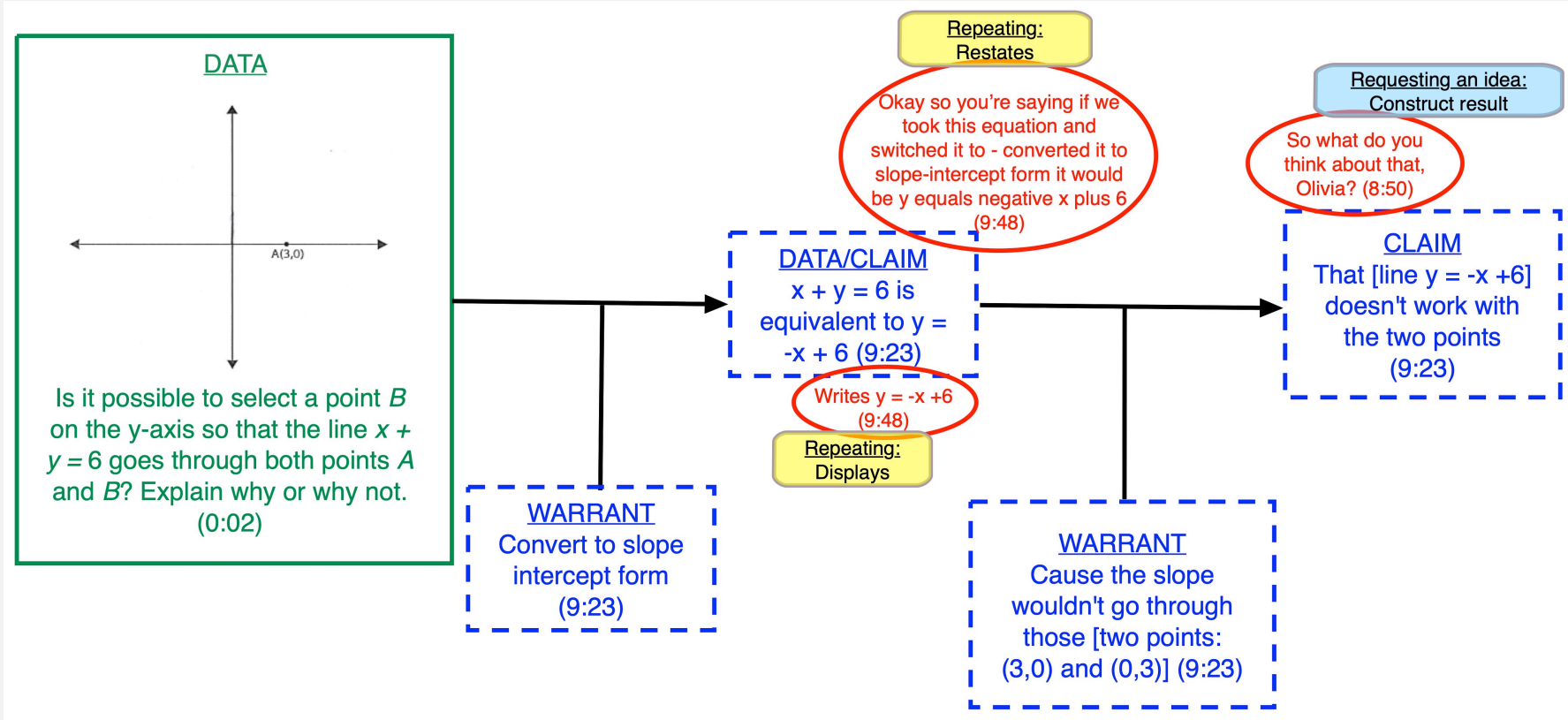
8:50 Teacher: So what do you think about- **what do you think about that?** Olivia.

9:23 Olivia: Um well I think it's a lot easier for us to comprehend like stuff in slope-intercept form because that's what we've been learning about for a while. So, if you just convert, like **x plus y equals 6 into slope-intercept form then its y equals negative x plus 6** and then **that doesn't work with the two points cause the slope wouldn't go through those**.

9:48 Teacher: **Okay, so you're saying if we took this equation and switched it to [*writes equation in slope-intercept form*] converted it to slope-intercept form** < Olivia: Mhm> **it would be y equals negative x plus 6.** < Olivia: Yep.> and then tell me more about um about this original claim [*circle motion to board around point B on graph*].

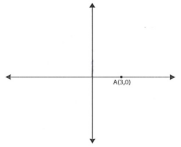


Example Analysis of Sub-Argument



Extended Toulmin Diagram for Points on a Line

DATA



Is it possible to select a point *B* on the y-axis so that the line $x + y = 6$ goes through both points *A* and *B*? Explain why or why not. (0:02)

Okay so the point (0,3) wouldn't work (8:16)

Okay so um you're referring to this point B…and you're saying the point (0,3) wouldn't work. (8:16)

REBUTTAL/CLAIM
The thing is that the point is (0,3) and if you plug it into that equation it comes out with 3 equals 6, which is false (8:04)

Does that claim hold up mathematically? Does anyone have a response? What do you think? (6:03)

Points to original claim (8:50)

Can you tell us more about like why that wouldn't work? (8:16)

WARRANT

Alright so you got the point here, (0,3) x and y, if you plug it into this equation, you get 0 plus 3 equals 6, which doesn't exactly work. (8:33)

So Andre's saying that this reasoning um, does-does not hold up mathematically because of this reasoning here. So here we have 3 plus 3 equals 6. Here we have 0 plus 3 does not equal 6 (8:50)

Points to new boardwork that Andre wrote. (8:50)

Points as he speaks to original claim (8:50)

Points to equation x+y=6 (8:33)

Writes "0 + 3 = 6" and "3 != 6" (8:33)

Writes "(0,3)" with "x" and "y" under neath the point (8:33)

So are you refuting this claim or agreeing with the claim? (10:15)

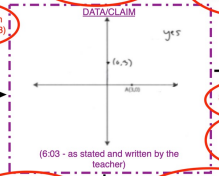
REBUTTAL/DATA
I'm refuting [this claim]…cause you can't just add 3 plus 3 cause they're not on the same point of the graph, they're two separate points (10:19)

Draws point B at (0,3) on the board and labels it (0,3)| (6:03)

So um one answer I saw um again a few times was uh, if I have point B at (0,3) then the answer would be yes (6:03)

Writes yes on the board (6:03)

DATA/CLAIM



(6:03 - as stated and written by the teacher)

So the point would be B at (0,3) (6:03)

Points to point B (6:03)

Writes *claim* on the board (6:03)

So we'll call this a claim… That point B at (0,3) would work…(6:03)

Tell me more about this original claim (9:48)

Okay so Olivia says you can't pick and choose. (10:26)

CLAIM
Well you can't like pick and choose which x and y values you take to put into your equation. You can only take like the one point if you're doing it the way they are (10:03). You have to stick to the numbers inside each point (10:37)

Pick and choose wh Olivia? The x's and y's? (10:26)

Writes "you can't pi choose" on the b (10:26)

What do you mean by the numbers inside the point? (10:45)

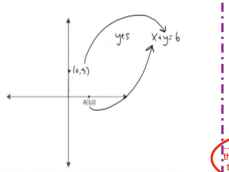
Are we in consensus that um, you can't pick and choose the x's and y's? (11:16)

WARRANT
Well like the coordinates for each point are like (3,0) and (0,3) so there has to be a 3 and a 0 whichever one you choose. [KP continues to write on the board] You can't just take a 3 and a 3 or a 0 and a 0. Cause if you're doing it that way you could also take 0 and 0 and 0 plus 0 doesn't equal 6. (10:51)

So you're saying you need to stick to one coordinate (11:12)

Teacher writes on board (10:51)

WARRANT



If x + y = 6, then the 3 from the x-value and the 3 from the y-value would make it true (6:03 - as stated and written by the teacher)

…because we're taking um the x-value and the y-value to make a x plus y equals 6 there (6:03)

Writes x+y=6 and draws an arrow from the 3 in point A to the x in the equation (6:03)

Points to point A (6:03)

So what do you think about that, Olivia? (8:50)

Writes y = -x +6 (9:48)

DATA/CLAIM
x + y = 6 is equivalent to y = -x + 6 (9:23)

Okay so you're saying if we took this equation and switched it to - converted it to slope-intercept form it would be y equals negative x plus 6 (9:48)

COUNTERCLAIM
That [line y = -x +6] doesn't work with the two points (9:23)

WARRANT
Convert to slope intercept form (9:23)

WARRANT
Cause the slope wouldn't go through those [two points: (3,0) and (0,3)| (9:23)

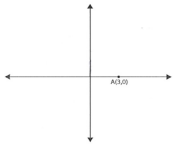
Student

Both

Teacher

Extended Toulmin Diagram for Points on a Line

DATA



Is it possible to select a point *B* on the y-axis so that the line *x* + *y* = 6 goes through both points *A* and *B*? Explain why or why not. (0:02)

Okay so the point (0,3) wouldn't work (8:16)

Okay so um you're referring to this point B...and you're saying the point (0,3) wouldn't work. (8:16)

REBUTTAL/CLAIM
The thing is that the point is (0,3) and if you plug it into that equation it comes out with 3 equals 6, which is false (8:04)

Does that claim hold up mathematically? Does anyone have a response? What do you think? (8:

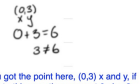
Points to original cla (8:50)

So are you refuting this claim or agreeing with the claim? (10:15)

REBUTTAL/DATA
I'm refuting [this claim]...cause you can't just add 3 plus 3 cause they're not on the same point of the graph, they're two separate points (10:19)

Can you tell us more about like why that wouldn't work? (8:16)

WARRANT



Alright so you got the point here, (0,3) x and y, if you plug it into this equation, you get 0 plus 3 equals 6, which doesn't exactly work. (8:33)

So Andre's saying that this reasoning um, does-does not hold up mathematically because of this reasoning here. So here we have 3 plus 3 equals 6. Here we have 0 plus 3 does not equal 6 (8:50)

Points to new boardwork that Andre wrote. (8:50)

Points as he speaks to original claim (8:50)

Points to equation x+y=6 (8:33)

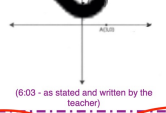
Writes "0 + 3 = 6" and "3 /= 6" (8:33)

Writes "(0,3)" with "x" and "y" under neath the point (8:33)

Draws point B at (0,3) on the board and labels (0,3)] (6:03)

So um one answer I saw um again a few times was uh, if I have point B at (0,3) then the answer would be yes (6:03)

Writes yes on the board (6:03)



(6:03 - as stated and written by the teacher)

So the point would be B at (0,3) (6:03)

Points to point B (6:03)

Writes *claim* on the board (6:03)

So we'll call this a claim... That point B at (0,3) would work...(6:03)

Okay so Olivia says you can't pick and choose. (10:26)

Tell me more about this original claim (9:48)

Pick and choose what Olivia? The x's and y's? (10:26)

CLAIM
Well you can't like pick and choose which x and y values you take to put into your equation. You can only take like the one point if you're doing it the way they are (10:03). You have to stick to the numbers inside each point (10:37)

Writes "you can't pick and choose" on the board (10:26)

Are we in consensus that um, you can't pick and choose the x's and y's? (11:16)

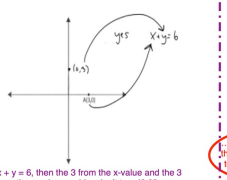
What do you mean by the numbers inside the point? (10:45)

WARRANT
Well like the coordinates for each point are like (3,0) and (0,3) so there has to be a 3 and a 0 whichever one you choose. [KP continues to write on the board] You can't just take a 3 and a 3 or a 0 and a 0. Cause if you're doing it that way you could also take 0 and 0 and 0 plus 0 doesn't equal 6. (10:51)

So you're saying you need to stick to one coordinate (11:12)

Teacher writes on board (10:51)

WARRANT



If x + y = 6, then the 3 from the x-value and the 3 from the y-value would make it true (6:03 - as stated and written by the teacher)

...because we're taking um the x-value and the y-value to make a x plus y equals 6 there (6:03)

Writes x+y=6 and draws an arrow from the 3 in point A to the x in the equation (6:03)

Points to point A (6:03)

So what do you think about that, Olivia? (8:50)

Writes y = -x +6 (9:48)

DATA/CLAIM
x + y = 6 is equivalent to y = -x + 6 (9:23)

Okay so you're saying if we took this equation and switched it to - converted it to slope-intercept form it would be y equals negative x plus 6 (9:48)

COUNTERCLAIM
That [line y = -x +6] doesn't work with the two points (9:23)

WARRANT
Convert to slope intercept form (9:23)

WARRANT
Cause the slope wouldn't go through those [two points: (3,0) and (0,3)] (9:23)

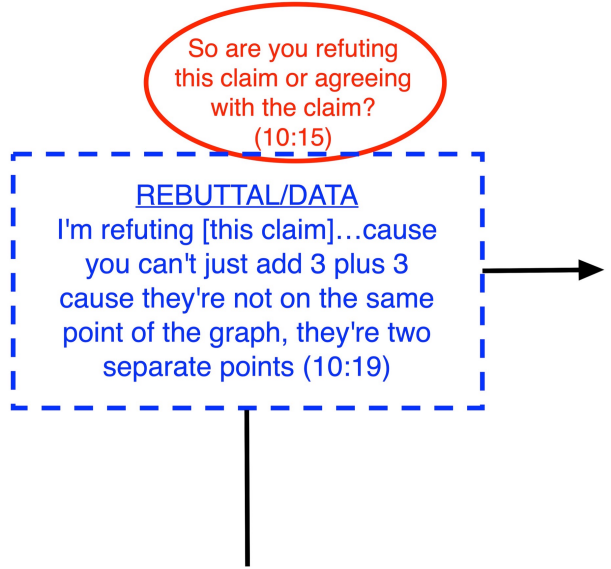
Student

Both

Teacher

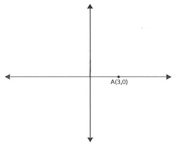
“So are you refuting this claim or agreeing with this claim?”

- This question seems similar to asking for agreement
- Our framework has a consensus question in the evaluation category that captures questions asking if students agree.
- However, this question is both asking the student to state whether they agree or disagree and asking them how their line of reasoning relates to the larger argument.
- Thus, the question is asking the student to position their response in relation to the reasoning **within the desired topic of conversation**.



Extended Toulmin Diagram for Points on a Line

DATA



Is it possible to select a point *B* on the y-axis so that the line $x + y = 6$ goes through both points *A* and *B*? Explain why or why not. (0:02)

REBUTTAL/CLAIM
The thing is that the point is (
and if you plug it into that equation
comes out with 3 equals 6, which is
false (8:04)

Okay so the point
wouldn't work (8:16)

um you're referring to this point
saying the point (0,3)
ork. (8:16)

Does that claim hold up
matically? Does anyone have a
? What do you think? (6:03)

Po
claim

Can
ore about like
n't work? (8:16)

WARRANT

(0,3)
x y
0+3=6
3≠6

Alright so you got the point here, (0,3) x and y, if you plug it into this equation, you get 0 plus 3 equals 6, which doesn't exactly work. (8:33)

So Andre's saying that this reasoning um, does-does not hold up mathematically because of this reasoning here. So here we have 3 plus 3 equals 6. Here we have 0 plus 3 does not equal 6 (8:50)

Points to new boardwork that Andre wrote. (8:50)

Points as he speaks to original claim (8:50)

Points to equation x+y=6 (8:33)

Writes "0 + 3 = 6" and "3 /= 6" (8:33)

Writes "(0,3)" with "x" and "y" under neath the point (8:33)

So are you refuting this claim or agreeing with the claim? (10:15)

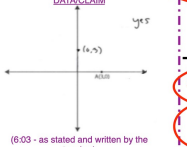
REBUTTAL/DATA
ing [this claim]...cause you
add 3 plus 3 cause they're
same point of the graph,
separate points (10:19)

Draws point B at (0,3) on the board and labels it (0,3)] (6:03)

So um one answer I saw um again a few times was uh, if I have point B at (0,3) then the answer would be yes (6:03)

Writes yes on the board (6:03)

DATA/CLAIM



(6:03 - as stated and written by the teacher)

So the point would be B at (0,3) (6:03)

Points to point B (6:03)

Writes *claim* on the board (6:03)

So we'll call this a claim... That point B at (0,3) would work...(6:03)

Tell me more about this original claim (9:48)

Okay so Olivia says you can't pick and choose. (10:26)

CLAIM
Well you can't like pick and choose which x and y values you take to put into your equation. You can only take like the one point if you're doing it the way they are (10:03). You have to stick to the numbers inside each point (10:37)

Pick and choose what Olivia? The x's and y's? (10:26)

Writes "you can't pick and choose" on the board (10:26)

Are we in consensus that um, you can't pick and choose the x's and y's? (11:16)

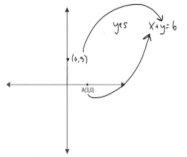
What do you mean by the numbers inside the point? (10:45)

WARRANT
Well like the coordinates for each point are like (3,0) and (0,3) so there has to be a 3 and a 0 whichever one you choose. [KP continues to write on the board] You can't just take a 3 and a 3 or a 0 and a 0. Cause if you're doing it that way you could also take 0 and 0 and 0 plus 0 doesn't equal 6. (10:51)

So you're saying you need to stick to one coordinate (11:12)

Teacher writes on board (10:51)

WARRANT



If x + y = 6, then the 3 from the x-value and the 3 from the y-value would make it true (6:03 - as stated and written by the teacher)

...because
the x-
l to
quals
:03)

x+y=6 and draws
the 3 in

Writes y = -x +6 (9:48)

DATA/CLAIM
x + y = 6 is equivalent to y = -x + 6 (9:23)

Okay so you're saying if we took this equation and switched it to - converted it to slope-intercept form it would be y equals negative x plus 6 (9:48)

So what do you think about that, Olivia? (8:50)

COUNTERCLAIM
That [line y = -x +6] doesn't work with the two points (9:23)

WARRANT
Convert to slope intercept form (9:23)

WARRANT
Cause the slope wouldn't go through those [two points: (3,0) and (0,3)] (9:23)

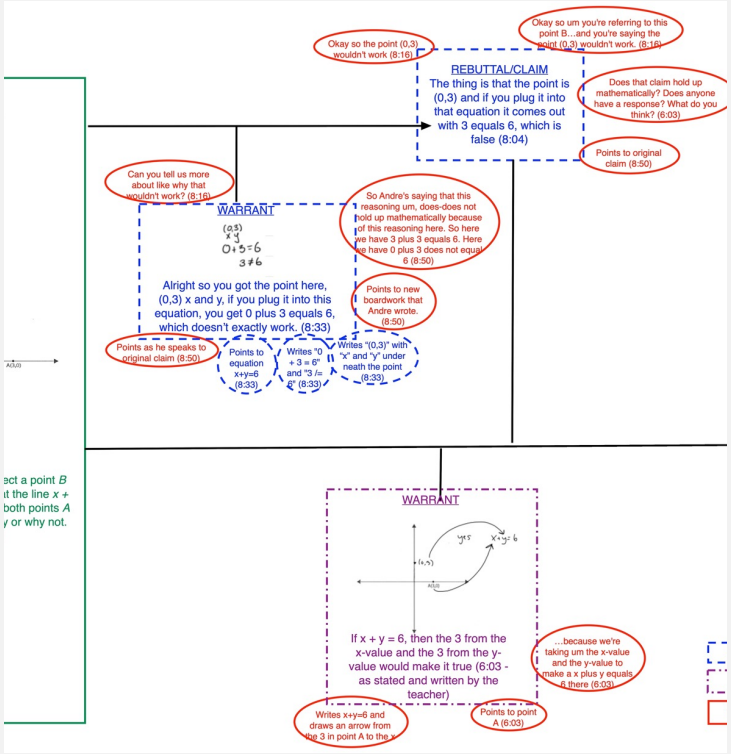
Student

Both

Teacher

Unexpected Structure: Rebuttal/Claim

- Extended Toulmin diagrams are useful for revealing structures of classroom discourse.
- Our diagram revealed an argument structure that we do not often encounter in novice teachers' practice: students constructed a rebuttal that also serves as a claim in the argument.
- This structure appeared because of the teacher's actions to make the students' thinking explicit by requesting warrants for the rebuttal.
- **Fostering productive mathematical discussions includes supporting students to contribute rebuttals.**



Connections across Projects

Using Extended Toulmin Diagrams and TSCA Framework, we noticed:

1. Unexpected teacher support: A teacher might ask a question to ask students to position a claim within the desired topic of conversation.

2. Unexpected diagram structure: rebuttal/claim, which appeared because of the teacher's actions to make the students' thinking explicit by requesting warrants for the rebuttal.

Discussion Questions

- What connections do you see between these lenses and your initial noticings?
- What connections do you see to other lenses for viewing mathematics teaching practice?
- What benefits do you see for viewing teaching through alternate/additional/other lenses?