BuildingOnMOSTs.org

Establishing Student Mathematical Thinking as an Object of Class Discussion

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Building on MOSTs: Investigating Productive Use of High-Leverage Student Mathematical Thinking

Using Student Thinking …

Mathematical Opportunities in Student Thinking

to have an "intentional discussion of selected and sequenced student approaches ... to move students through a trajectory of sophistication toward the intended mathematics learning goal of the lesson" (AMTE Standards, p. 17)

Using Student Thinking to …

Mathematical Opportunities in Student Thinking

"to build on and honor student thinking while ensuring that the mathematical ideas at the heart of the lesson remain prominent in class discussions." (A summary of the work of Engle and Conant, 2002 in Principles to Action, NCTM, 2014, p. 30)

Using Student Thinking to …

Mathematical Opportunities in Student Thinking

"to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments." (NCTM, 2014, pg. 29)

Establishing Student Thinking

Mathematical Opportunities in Student Thinking

In order to

- have a discussion about student thinking
- build on and honor student thinking
- compare student approaches and arguments

the student thinking itself must be **established** as the object of discussion.

What does it mean to "establish"?

How does one do this "establishing"?

Data and Analysis Goals

Mathematical Opportunities in Student Thinking

- 6 middle school teachers
  - two different tasks
  - 14 classroom episodes
- 6 high school teachers
  - two different tasks
  - 13 classroom episodes

Looked for effective and ineffective ways in which the student mathematical thinking was established to be the object of discussion.

Two Aspects of Establishing

Mathematical Opportunities in Student Thinking

- **Make Precise**

  Make the content of the student mathematical contribution clear—clarify what they said, not what they mean.

- **Objectify**

  Establish the clarified student mathematical contribution as the object to be discussed.

MAKE PRECISE

Mathematical Opportunities in Student Thinking

There are several ways in which the student mathematical contribution can be made more precise

- Clarify (+ Confirm)
- Expand
- Hone

MAKE PRECISE

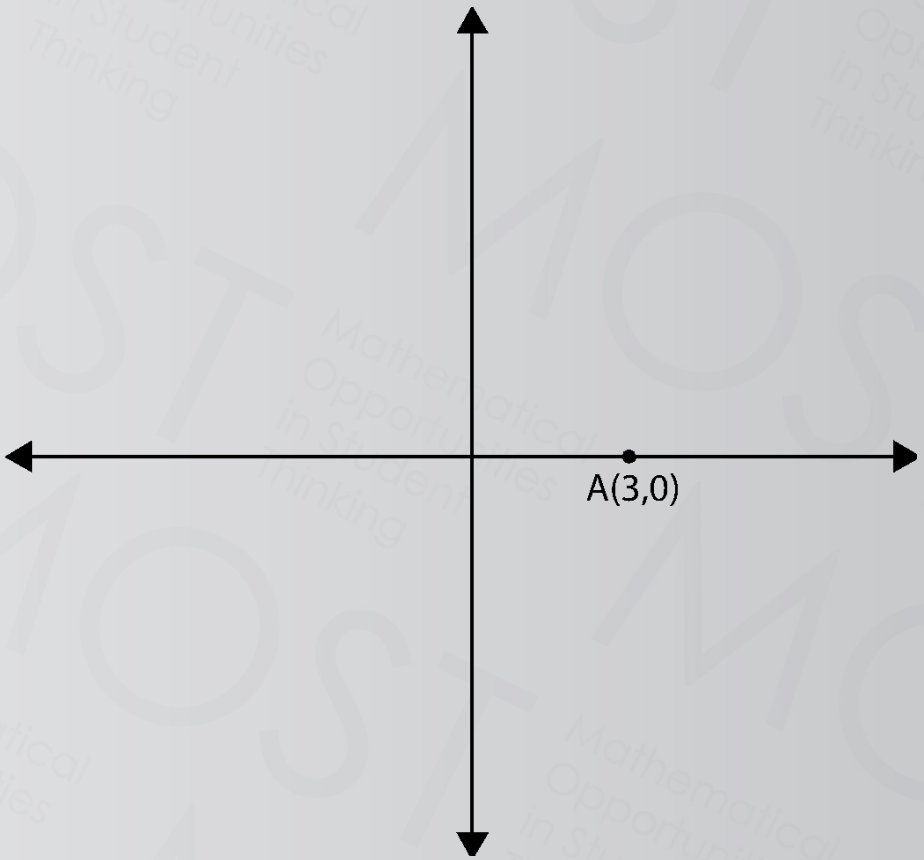
Mathematical Opportunities in Student Thinking

- A claim or answer is shared
- If necessary, teacher elicits reasoning for the claim/answer (this is a form of expanding)
- Teacher follows-up the student's explanation to make precise the contribution by
  - Clarifying (+ Confirming)
  - Expanding

MAKE PRECISE

Mathematical Opportunities in Student Thinking

Students were given the graph at the right. They were asked "Is it possible to find a point B on the y-axis so that the line $x + y = 6$ goes through both points A and B? Explain why or why not.



MAKE PRECISE - Clarifying, Confirming, Expanding

Mathematical Opportunities in Student Thinking

Jose: I put a 3 in for x in the equation and solved for y and got 3 so I got the point $(0,3)$.

MAKE PRECISE - Clarifying, Confirming, Expanding

Mathematical Opportunities in Student Thinking

Jose: I put a 3 in for x in the equation and solved for y and got 3 so I got the point $(0,3)$.

Teacher: So you put a 3 in the equation $x + y = 6$. Where did the first 3 come from?

MAKE PRECISE - Clarifying, Confirming, Expanding

Mathematical Opportunities in Student Thinking

Jose: I put a 3 in for x in the equation and solved for y and got 3 so I got the point $(0,3)$.

Teacher: So you put a 3 in the equation $x + y = 6$. Where did the first 3 come from?

Clarifying + Confirming

MAKE PRECISE - Clarifying, Confirming, Expanding

Mathematical Opportunities in Student Thinking

Jose: I put a 3 in for x in the equation and solved for y and got 3 so I got the point $(0,3)$.

Teacher: So you put a 3 in the equation $x + y = 6$. Where did the first 3 come from?

Expanding

MAKE PRECISE - Clarifying, Confirming, Expanding

Mathematical Opportunities in Student Thinking

Jose: I put a 3 in for x in the equation and solved for y and got 3 so I got the point $(0,3)$.

Teacher: So you put a 3 in the equation $x + y = 6$. Where did the first 3 come from?

Jose: It was the x-value of the point A.

MAKE PRECISE - Clarifying, Confirming, Expanding

Mathematical Opportunities in Student Thinking

Jose: I put a 3 in for x in the equation and solved for y and got 3 so I got the point $(0,3)$.

Teacher: So you put a 3 in the equation $x + y = 6$. Where did the first 3 come from?

Jose: It was the x-value of the point A.

Teacher: So you put a 3, the x-value of the point A, in the equation $x + y = 6$ and solved for y to get 3. You are claiming $B = (0,3)$ and that the line $x + y = 6$ goes through A and B.

MAKE PRECISE - Clarifying, Confirming, Expanding

Mathematical Opportunities in Student Thinking

Jose: I put a 3 in for x in the equation and solved for y and got 3 so I got the point $(0,3)$.

Teacher: $x + y = 6$. Where did the first 3 come from?

Expanded by incorporating elements of the problem statement into the student statement.

Jose: It was the x-value of the point A.

Teacher: So you put a 3, the x-value of the point A, in the equation $x + y = 6$ and solved for y to get 3. You are claiming $B = (0,3)$ and that the line $x + y = 6$ goes through A and B.

MAKE PRECISE – Honing

Mathematical Opportunities in Student Thinking

Allie: Well, we already know what the x is to plug into the equation because of the 3 in point *A*. So we get three plus y equals six. We did the usual thing to get the y all by itself and subtracted 3 from both sides of the equation and got $y = 3$. Since *B* has to be on the y -axis, x is going to be 0. Putting the $y = 3$ together with the $x = 0$, we got the point (0, 3) and the line would go through both points.

MAKE PRECISE – Honing

Mathematical Opportunities in Student Thinking

Allie: Well, we already know what the x is to plug into the equation because of the 3 in point A. So we get three plus y equals six. We did the usual thing to get the y all by itself and subtracted 3 from both sides of the equation and got y = 3. Since B has to be on the y-axis, x is going to be 0. Putting the y = 3 together with the x = 0, we got the point (0, 3) and the line would go through both points.

Teacher: So you put the 3 from A into the equation for x and solved for y to get 3. You're claiming that the point B = (0, 3) is on the line of the equation.

OBJECTIFY

Mathematical Opportunities in Student Thinking

There are two main ways in which a student mathematical contribution that has been made precise can be made an object of discussion.

- Re-present
- Referring to the object

OBJECTIFY – Re-present

Mathematical Opportunities in Student Thinking

- Repeat or Revoice
  - Verbal
    - Teacher or student restates the contribution with no replacement of terms (repeat)
    - Teacher or student paraphrase the contribution (revoice)

OBJECTIFY – Verbal Repeating and Revoicing

Mathematical Opportunities in Student Thinking

Jose: I put a 3 in for x in the equation and solved for y and got 3 so I got the point $(0,3)$.

Teacher: So you put a 3 in the equation $x + y = 6$. Where did the first 3 come from?

Jose: It was the x-value of the point A.

Re-present

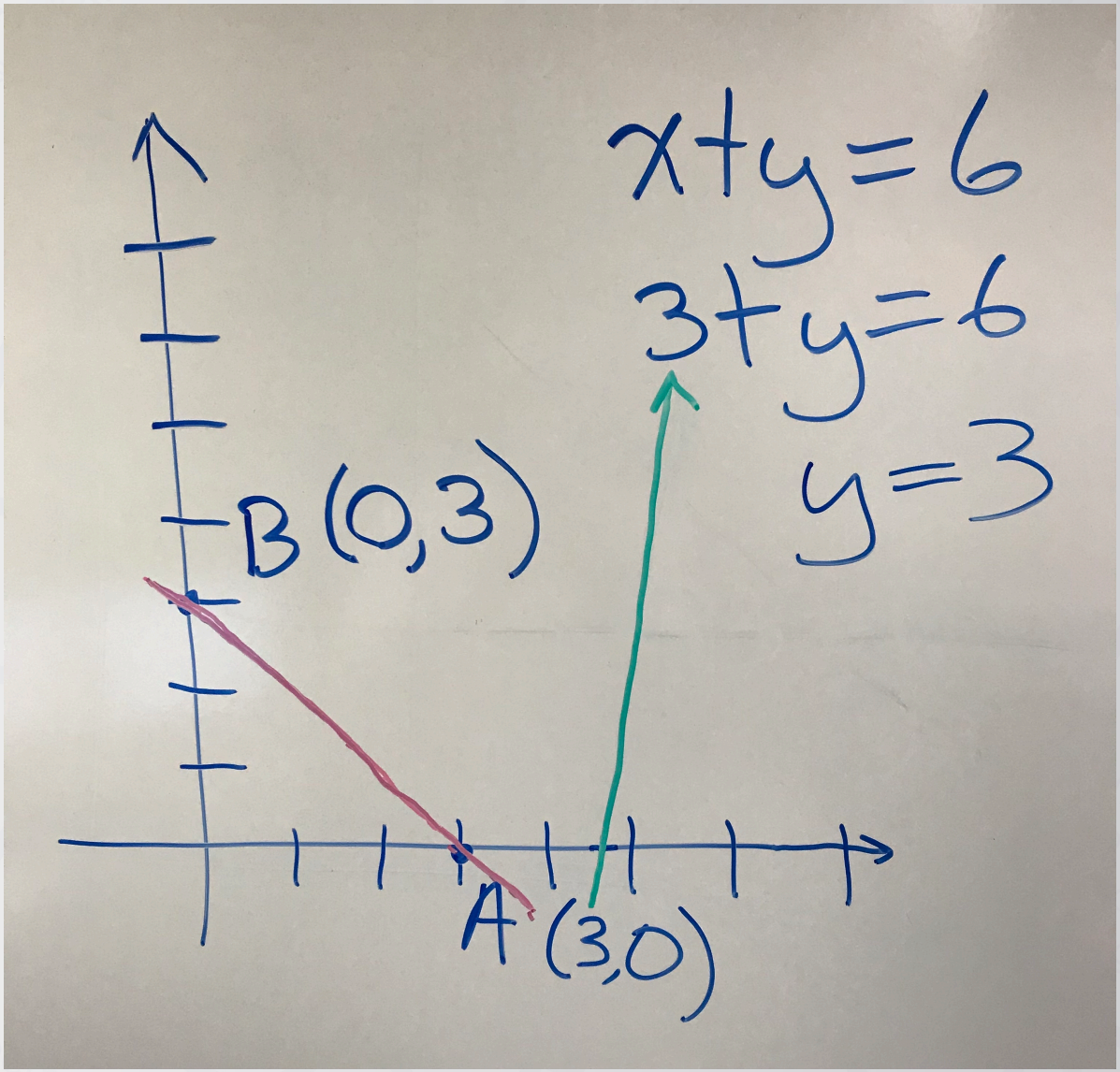
Teacher: So you put a 3, the x-value of the point A, in the equation $x + y = 6$ and solved for y to get 3. You are claiming $B = (0,3)$ and that the line $x + y = 6$ goes through A and B.

OBJECTIFY – Re-present

Mathematical Opportunities in Student Thinking

- Repeat or Revoice
  - Verbal
    - Teacher or student restates the contribution with no replacement of terms (repeat)
    - Teacher or student paraphrase the contribution (revoice)
  - Written
    - Teacher or student creates a public record of what was said either word for word (repeat) or paraphrase (revoice)

OBJECTIFY – Re-present



OBJECTIFY - Referring

Mathematical Opportunities in Student Thinking

- Verbal pronoun reference
- Naming (e.g. “claim”, “strategy”, ”Jose’s Thinking”, ”Allie’s claim”)
- Reference to public record (pronoun, gesture, location)
- Written highlight (underlining or boxing)

These are almost always used in some kind of combination with each other.

Establishing Student Thinking

Mathematical Opportunities in Student Thinking

- Make Precise
  - Clarifying (+ Confirming)
  - Expanding
  - Honing
- Objectify
  - Re-present (verbally or in writing)
  - Referring (naming, gesturing, written highlight)

Clearly establishing the student mathematical thinking that will be the object of discussion improves the capacity to have a productive discussion about that thinking.

Discussion

Mathematical Opportunities in Student Thinking

- What are your reactions to the practice of establishing student thinking?
- What are the benefits of unpacking this practice?
- What aspects of the practice might be natural to teachers? What aspects might be challenging?
- What might MTEs do to help teachers at all levels develop this practice?

Contact Information

Mathematical Opportunities in Student Thinking

BuildingOnMOSTs.org

Post-Session Discussion

https://byu.zoom.us/my/bepeterson

Link in chat

THANK YOU!