BuildingOnMOSTs.org

Examining Which Student Thinking is Considered in Responsive Teaching

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Building on MOSTs: Investigating Productive Use of High-Leverage Student Mathematical Thinking

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Responsive Teaching

Mathematical Opportunities in Student Thinking

- *responsiveness to students' mathematical thinking*: "a characteristic of interactions wherein **students' mathematical ideas are present, valued, attended to, and taken up** as the basis for instruction. In responsive interactions **students help to determine the direction** of mathematics lessons and **teachers build on student ideas**" (Bishop, Przybyla-Kuchek, & Hardison, 2018, p. 2).
- responsive teaching **focuses on and pursues the substance of students' ideas**, and recognizes disciplinary connections within these ideas (Robertson, Atkins, Levin, & Richards, 2016)

Which particular instances of student thinking should the teacher be responsive to?

To what extent do instances of student thinking that have been shared need to remain under consideration as new ideas are shared?

Characterize the responsiveness of the teacher moves in this vignette

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- **teachers focus on and pursue the substance of students' ideas**
- **teachers build on student ideas**

Vignette 1

Students had individually considered a task in which a price was first increased 50% and later decreased 50%, and whether the original and final prices would be the same. The teacher began the whole-class discussion by noting that many students said the two prices would be the same and inviting Dan to explain. He responded that 50 minus 50 is 0, so it would stay the same price. José was asked to re-explain the same idea, and the teacher summarized that idea. She then asked others what they thought about this idea and Jen disagreed, explaining that the percent of a value depends on what the value is, illustrating with a numerical example. The teacher surfaced that multiple students had used a similar numerical example to conclude that the prices would not be the same. Students were prompted to explain this phenomenon. Rob explained that 50% of a larger number, after the price increase, is bigger than 50% of the original number. The teacher asked what others thought about this idea; Josh agreed and re-explained. The teacher invited other comments and Zoe contributed an idea about why the first students who thought the original and final prices were the same might have been confused. The teacher highlighted a key idea from Zoe that you are taking 50% of different values and called on Sai, who returned to Rob's idea.

Characterize the responsiveness of the teacher moves in this vignette

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Vignette 2

Students had individually considered the question "Which is larger, x or $x + x$? Explain your reasoning." The teacher began a whole-class discussion by inviting two students to share their claims (both of which argued that $x + x$ was larger) and the teacher recorded their ideas next to each other on the board. The teacher then asked, "Does anyone agree and want to add on to what they have, or respectfully disagree?" Two more arguments, in basic agreement with the first two, were then volunteered and recorded near the first two. At this point a student shared that he disagreed (that you can't tell because, for example, you could put 100 in for the first x and 50 in for each of the xs in $x + x$). The teacher recorded this answer in a separate space on the board. The teacher then explained to the class that they now had a new situation, where they could agree or respectfully disagree with either the first group of answers or with this new contrary claim.

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Principles underlying productive use of student thinking

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- Mathematics Principle: Student mathematics is at the forefront
- Legitimacy Principle: Students are positioned as legitimate mathematical thinkers
- Sense-making Principle: Students are engaged in sense making
- Collaboration Principle: Students are working collaboratively

-aligned with *Principles to Actions* (NCTM, 2014)

Building on Student Thinking

Mathematical Opportunities in Student Thinking

Making student thinking an object of consideration for the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

1. Make Precise: Make the object to be considered clear
2. Grapple Toss: Turn the object of consideration over to the students with parameters that put them in a sense-making situation
3. Orchestrate: Engage the class in collaborative sense making of the object of consideration that includes a whole-class discussion of that object
4. Make Explicit: Facilitate the extraction and articulation of the mathematical point of the object that was considered

Discussion Questions

Mathematical Opportunities in Student Thinking

- What are your reactions to these examples of responsiveness and our interpretations of them?
- What other variations in responsiveness have you observed that might be useful to consider?
- How might these ideas influence us as mathematics teacher educators as we help teachers become more responsive to student mathematical thinking?